Compound Interest

1. Simple Interest: Interest that is computed on the original principle only. I denotes the interest on a principle P (in dollars) at an interest rate of r per year for t years.

$$I = Prt$$

2. Accumulated Amount: is the sum of the principle and interest after t years, denoted by A

$$A = P(1 + rt)$$

3. Compound Interest: interest that is periodically added to the principle and then itself earns interest at the same rate.

Compound Interest Formula (Accumulated Amount)

$$A = P(1 + i)^n$$

where $i = \frac{r}{m}$, $n = mt$, and
A = Accumulated amount at the end of n conversion periods
P = Principle
r = Nominal interest rate per year
m = Number of conversion periods per year
t = Term (number of years)

Continuous Compound Interest Formula

$$A = Pe^{rt}$$

where
P = Principle
r = Nominal interest rate compounded continuously
t = Time in years
A = Accumulated amount at the end of t years

4. Conversion Period: The interval of time between successive interest calculations

Effective Rate of Interest Formula

The annual rate of interest that, when compounded annually, will yield the same accumulated amount as the nominal interest rate compounded m times a year (over the same term).

$$r_{eff} = \left(1 + \frac{r}{m}\right)^m - 1$$

where
r_{eff} = Effective rate of interest
r = Nominal interest rate per year
m = Number of conversion periods per year

5. Present Value: the principle P is often referred to as the present value

Present Value Formula for Compound Interest

$$P = A(1 + i)^{-n}$$

Present Value Formula for Continuous Compound Interest

$$P = Ae^{-rt}$$

6. Future Value: the accumulated amount A is often referred to as the future value

Annuities

1. Ordinary Annuity: An annuity in which the payments are made at the *end* of each payment period (annuities where payments are made at the beginning are called an *annuity due*)

2. Simple Annuity: An annuity in which the payment period coincides with the interest conversion period (if it differs, it is called a complex annuity)

Future Value of an Annuity

The future value S of an annuity of n payments of R dollars each, paid at the end of each investment period into an account that earns interest at the rate of i per period, is

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

Present Value of an Annuity

The present value P of an annuity consisting of n payments of R dollars each, paid at the end of each investment period into an account that earns interest at the rate of i per period, is

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

Amortization

Amortization Formula(Periodic Payment)
The periodic payment R on a loan of P dollars to be amortized over n periods with the interest charged at the rate of i per period is

$$R = \frac{Pi}{1-(1+i)^{-1}}$$

Amortization Formula (Amount Amortized)
By thinking of the monthly loan repayments R as the payments in an annuity, we see that the original amount of the loan is given by P, the present value of the annuity.

$$P = R\left[\frac{1-(1-i)^{-n}}{i}\right]$$

1. Sinking Fund: an account that is set up for a specific purpose at some future date

Sinking Fund Payment
The period payment R required to accumulate a sum of S dollars over n periods with interest charged at the rate of i per period is

$$R = \frac{iS}{(1+i)^n - 1}$$

Applications of the Definite Integral to Business and Economics

1. Consumers' Surplus: The difference between what the consumers *would* be willing to pay for $\bar{x}$ units of the commodity and what they *actually* pay for them

Consumers' Surplus
The consumers' surplus is given by

$$CS = \int_0^{\bar{x}} D(x)dx - \bar{p}\bar{x}$$

where *D(x)* is the demand function, $\bar{p}$ is the unit market price, and $\bar{x}$ is the quantity sold. The equation can also be written as

$$CS = \int_0^{\bar{x}} [D(x) - \bar{p}]dx$$

2. Producers' Surplus: The difference between what the suppliers actually receive and what they would be willing to receive

Producers' Surplus
The producers' surplus is given by

$$PS = \bar{p}\bar{x} - \int_0^{\bar{x}} S(x)dx$$

where *S(x)* is the supply function, $\bar{p}$ is the unit market price, and $\bar{x}$ is the quantity supplied. The equation can also be written as

$$PS = \int_0^{\bar{x}} [\bar{p} - S(x)]dx$$

Accumulated (Total) Future and Present Value of an Income Stream
The accumulated, or total, future value after T years of an income stream of *R(t)* dollars per year, earning interest at the rate of r per year compounded continuously, is given by

$$A = e^{r^T} \int_0^T R(t)e^{-rt}dt$$

The present value of an income stream of *R(t)* dollars per year, earning interest at the rate of r per year compounded continuously, is given by

$$PV = \int_0^T R(t)e^{-rt}dt$$

3. Amount of an Annuity: the sum of payments plus the interest earned

Amount of Annuity

The amount of an annuity is

$$A = \frac{mP}{r}\left(e^{rT} - 1\right)$$

where

P = Size of each payment in the annuity

r = Interest rate compounded continuously

T = Term of the annuity (in years) m = Number of payments per year

Present Value of an Annuity

The present value of an annuity is given by

$$PV = \frac{mP}{r}(1 - e^{-rT})$$

where P, r, T, and m are as defined earlier.

4. Lorenz Curve: a method used by economists to study the distribution of income in a society

Coefficient of Inequality of a Lorenz Curve

The coefficient of inequality, or **Gini Index**, of a Lorenz curve is

$$L = 2\int_0^1 [x - f(x)]dx$$