Question 1:

In a group of ten people, each person is asked to write the sum of the ages of all the other nine people. The ten sums are 82, 83, 84, 85, 87, 89, 90, 90, 91 and 92. Find the age of the youngest person.   Assume that the ages are all whole numbers.
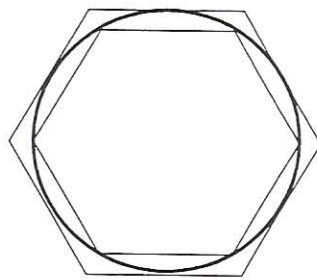
Question 2:

As shown in the figure, B is the midpoint of the line segment AD. The lengths of all the line segments AB, AC, AD, BC, BD and CD are integers. If the product of all the integers representing those lengths is 10500, find the length of AB.

$$A \quad\quad\quad\quad\quad\quad B \quad C \quad\quad D$$
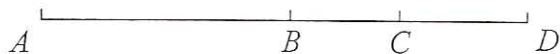
Question 3:

A small regular hexagon is inscribed in a circle. A large regular hexagon circumscribes the circle, as shown in the figure. If the area of the large regular hexagon is 10 square units, find the area of the small hexagon.

Question 4:

Peter plays a game of darts. On the target shown below each circle has a different score. What is the minimum number of hits that he must throw in order to score exactly 100 points?
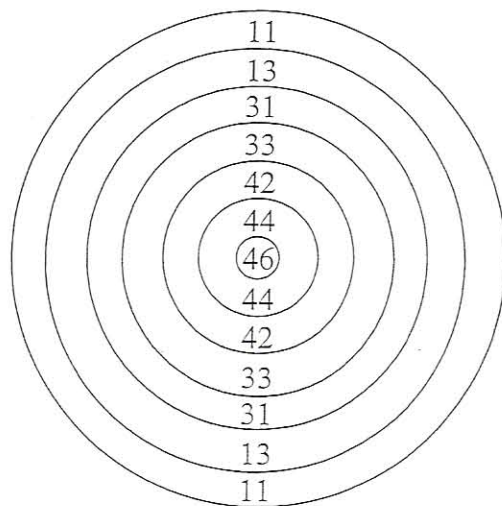
Question 5:

From the natural numbers 1, 2, 3, 4, 5, ..., delete the multiples of 2 and the multiples of 3, **but** keep the multiples of 5. The remaining numbers are:
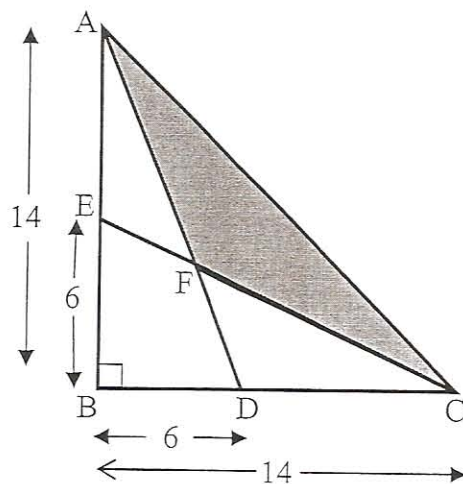
$$1, 5, 7, 10, 11, 13, 15, 17, 19, 20, 23, 25, 29, 30, \ldots\ldots$$

In this sequence, what is the 2006th number?

Question 6:

In the following figure, triangle ABC is a right-angled triangle, $AB = BC = 14$, $BE = BD = 6$, find the area of the shaded region.

A
14
E
6
F
B
6
D
14
C

Question 7:

A sealed bottle, which contains water, has been constructed by attaching a cylinder of radius 1 cm to a cylinder of radius 3 cm, as shown in Figure A. When the bottle is right side up, the height of the water inside is 20 cm, as shown in the cross-section of the bottle shown in Figure B. When the bottle is upside down, the height of the liquid is 28 cm, as shown in Figure C.   What is the total height of the bottle in cm?

Height of liquid

Figure A   Figure B   Figure C

20cm   28cm
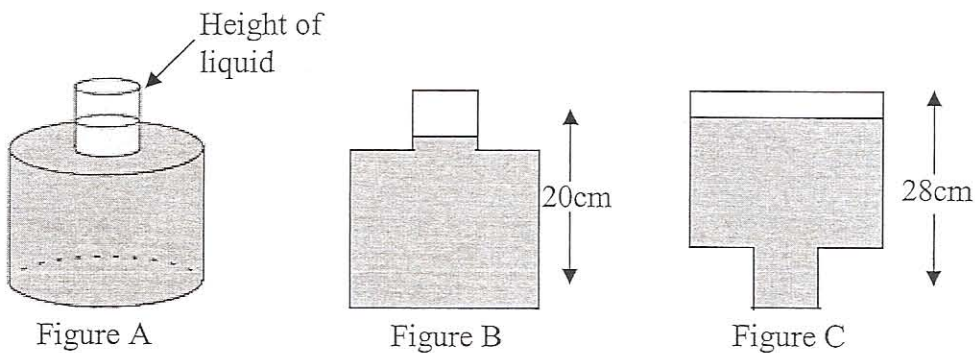
Question 8:

Let $N = \overline{abcde}$ be a five-digit number. For example, when $a = 4$, $b = 0$, $c = 4$, $d = 7$ and $e = 7$, $\overline{abcde} = 40477$.

If $\overline{ab} \times \overline{cde} = 42042$ and $\overline{abc} \times \overline{de} = 24642$, find N.

Question 9:

The numbers 1, 2, 3, …, 51 and 52 are written on 52 cards, one number on each card. We say that we have a "Lucky Set" if there are three cards in a hand (regardless of the total number of cards in the hand) such that the sum of the digits on each card is the same. What is the least number of cards we must have in order to ensure that we get a "Lucky Set" in the hand?

Question 10:

There are 7 students in a sport competition. Each one has his own number on his shirt labeled: 1, 2, . . ., or 7. The student who finished first took 1 minute, the student who finished second took 2 minutes, the student who finished third took 3 minutes and so on. Moreover,

   i. For each student, when the shirt number and the time used are added we
       find that:

      - The maximum sum is 13 (there is only one student);

      - The minimum sum is 4 (there is only one student);

      - There are exactly 3 students who have a sum of 9.

   ii. The shirt number of the student in first place is greater than the shirt
       number of the student in second place.

Write down the shirt number of each student and their positions.

Position	1	2	3	4	5	6	7
Shirt Number							