## Po Leung Kuk <br> $15{ }^{\text {th }}$ Primary Mathematics World Contest Individual Contest 2012

1. There are only white and red balls in a bag. Tom takes one ball, looks in the bag and says " $5 / 7$ of the remaining balls are white". After that he puts the ball back into the bag. Then Masha takes one of the balls, looks in the bag and says " $12 / 17$ of the remaining balls are white". How many balls in total were there in the bag initially?
2. Let $n$ denote the smallest positive integer that is divisible by both 4 and 9 , and whose digits are only 4's and 9's, with at least one of each. What are the last four digits of $n$ ?
3. In the figure below, a straight line $l$ divides the pattern formed by seven equal squares into two parts whose areas are equal. Let $E$ be the intersection point of $l$ and $A B$, and let $F$ be the intersection point of $l$ and $C D$. If the sum of the length of $C F$ and $A E$ is 91 cm , find the area of an individual square, in $\mathrm{cm}^{2}$.

4. Choose eight of the nine numbers 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011 and 2012 and write one number at each vertex of the cube shown in the figure below. Each number may only be used once, only one number can be used at each vertex and the sum of the four numbers on each side of the cube is 8034 . Which one of the given nine numbers is not used?

5. In the figure given below, the side of the square $A B C D$ is $2 \mathrm{~cm} . E$ is the midpoint of $A B$ and $F$ is the midpoint of $A D$. $G$ is a certain point on $C F$ and $3 C G=2 G F$. What is the area of the shaded triangle $B E G$, in $\mathrm{cm}^{2}$ ?

6. Consider the figure below


Find $\angle a+\angle b+\angle c+\angle d+\angle e+\angle f+\angle g+\angle h$, in degrees.
7. Find the last digit of $N$, where

$$
N=(1+2+3+4)+\left(1^{2}+2^{2}+3^{2}+4^{2}\right)+\cdots+\left(1^{2012}+2^{2012}+3^{2012}+4^{2012}\right) .
$$

8. We are given a two-digit number. If we divide this number by the product of its digits, we will get a quotient of 3 and a remainder of 9 . If we add the product of the digits to the sum of the squares of the digits, we get the given number. What is this given two-digit number?
9. What is the number and the letter in the $1000^{\text {th }}$ column in the following pattern?

| P | O | L | E | U | N | G | K | U | K | P | O | L | E | U | N | $\ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 0 | 1 | 2 | 7 | 1 | 8 | 2 | 0 | 1 | 2 | 7 | 1 | 8 | 2 | 0 | $\ldots$ |

10. The average of 20 numbers is 18 . The $1^{\text {st }}$ number is increased by 2 , the $2^{\text {nd }}$ number is increased by 4 , the $3^{\text {rd }}$ is increased by $6, \ldots$, the $20^{\text {th }}$ number is increased by 40 (that is, the $n^{\text {th }}$ number is increased by $2 n$ ). What is the average of the 20 increased numbers?
11. You must color each square in the figure below in red, green or blue. Any two squares with adjacent sides must be of a different color. In how many different ways can this coloring be done?

12. A "PoLeungKuk" number is a 9-digit number with the following properties:

- It is the product of the square of 4 different prime numbers; each prime number is less than 50 .
- The number formed by the first 3 digits and the number formed by the last 3 digits is the same.
- The number formed by the middle 3 digits is twice the number formed by the first 3 digits.
- The "PoLeungKuk" number is greater than $300,000,000$.

What is this "PoLeungKuk" number?
13. The figure below shows a square $A B C D . O$ is a point inside the square such that angle $B O C=90^{\circ}, O B=4 \mathrm{~cm}$ and $O C=7 \mathrm{~cm}$. Find the area of the shaded triangle $A O B$, in $\mathrm{cm}^{2}$.

14. In the figure below, four points $P, Q, R$ and $S$ are on a straight line, such that the distance between any two of them is a whole number. $Q$ is the mid-point of $P S$ and $P Q \times P R \times P S \times Q R \times Q S \times R S=10500$.


Find the length of $P S$.
15. Eleven consecutive positive integers are written on a board. Maria erases one of the numbers. If the sum of the remaining numbers is 2012, what number did Maria erase?

