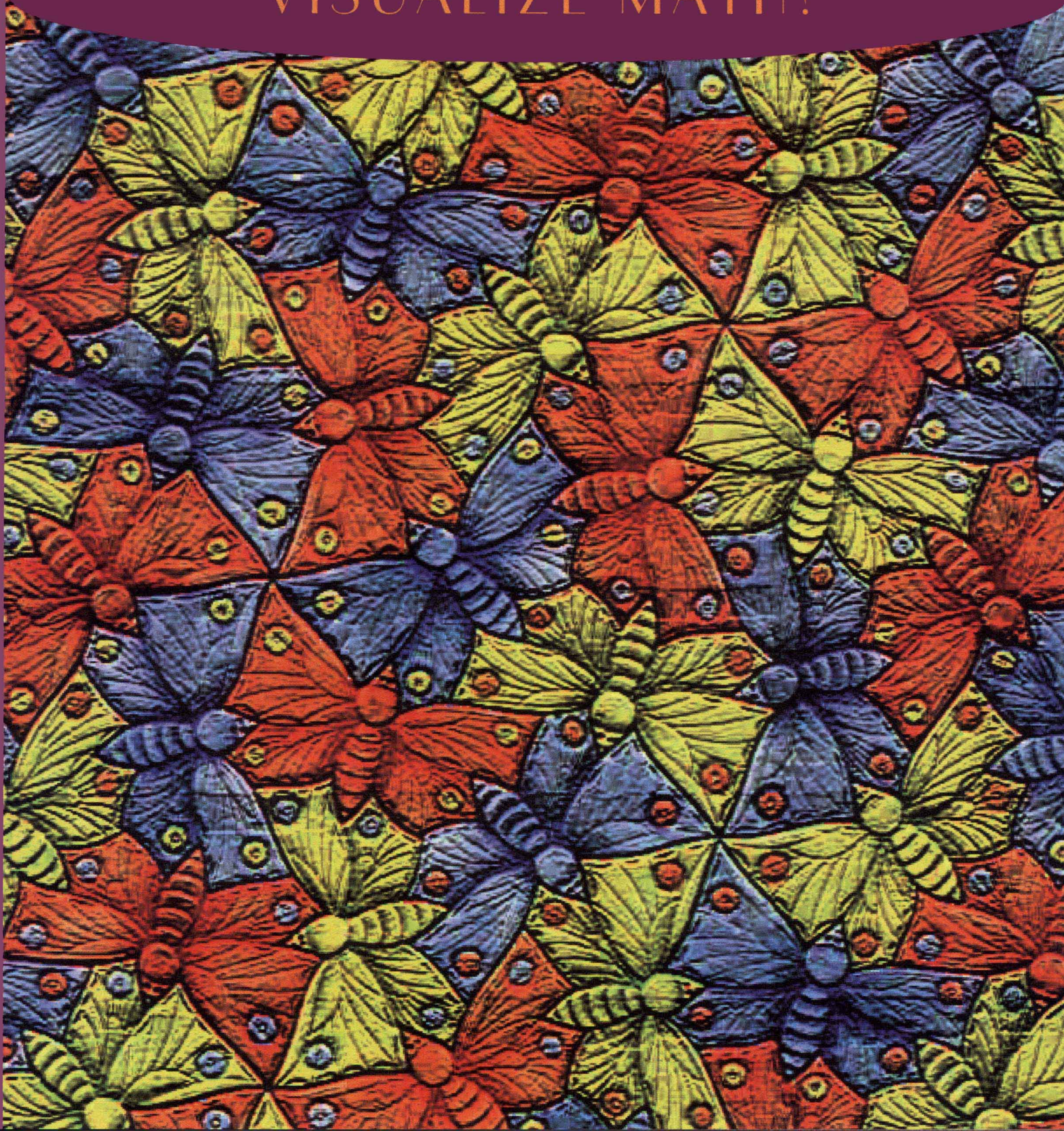


# Math Reader

VISUALIZE MATH!



# Math Reader

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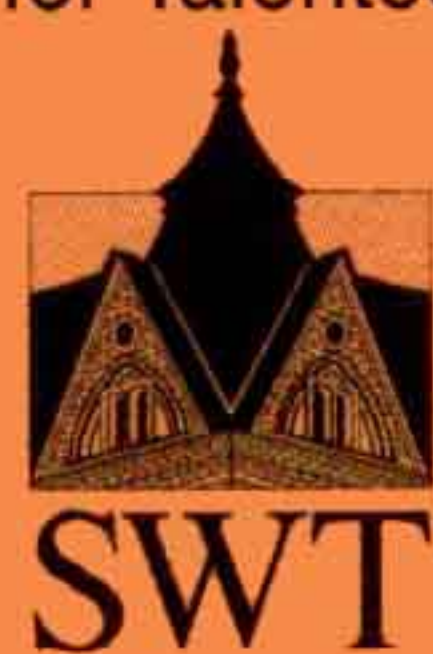
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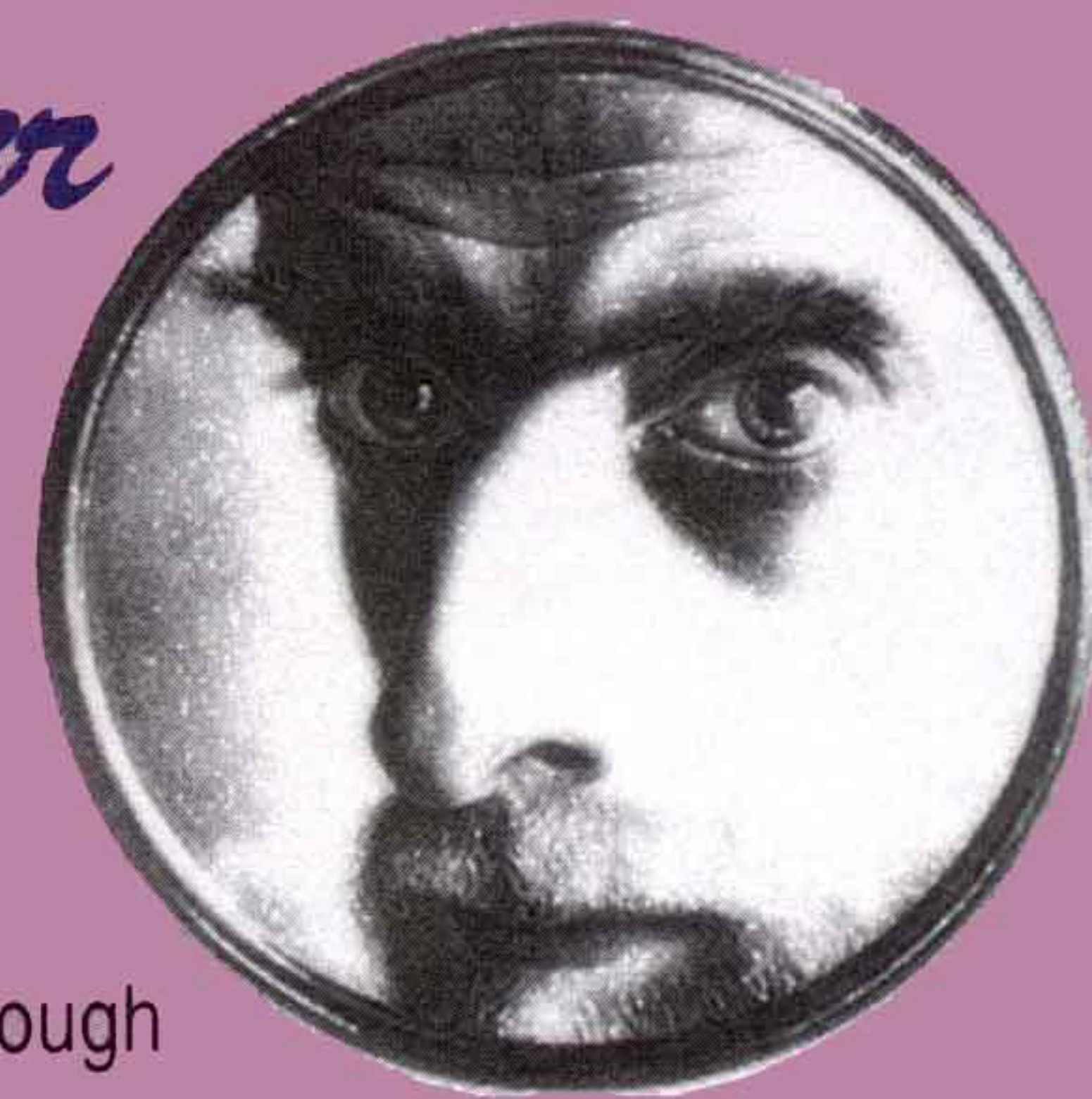
by Southwest Texas State University

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# M. C. Escher



Maurits Cornelius Escher, usually referred to as M. C. Escher, was born in Leeuwarden in The Netherlands in 1898, the youngest son of an engineer, G. A. Escher. As a student, the young Escher enjoyed art most of all. Though he studied architecture, Escher was encouraged by his teacher, Samuel Jesserun de Mesquita, in the direction of graphic art and woodcutting.

Escher never claimed to know a great deal about formal mathematics in the abstract sense, but from his numerous works of art, it is clear that he was adept at using mathematics concretely to look at perspective and two and three dimensions. In art, perspective means the spatial relation of objects and how we represent on paper (two dimensions) the way things look in real life (in our three dimensional world). Depth and distance are sometimes difficult to draw on paper. Escher was not only able to show perspective in three dimensions, he also drew shapes, structures and landscapes with impossible perspective. Using impossible perspective, Escher experimented with planes and space in ways that could not exist in real life. It was difficult for the viewer to know what part of the object was in front or in back, what was vertical or horizontal, what was going up or what was coming down. Like optical illusions, his art tricked the eye.

Another interesting aspect of Escher's work was his use of repeated shapes or figures to cover the paper (tessellation designs). By using a figure and transforming the shape with translation, rotation, reflection and glide reflection, he was able to cover a page completely, without any gaps. Even more interestingly, in his later works, he added metamorphoses (changes) in forms that might change fish to birds right in front of your eyes. Mathematicians often refer to Escher's prints, citing their many interesting geometric shapes. He began to write to the mathematicians Polya and Coxeter and stated "I have often felt closer to people who work scientifically than to my fellow artists". Towards the end of his life, Escher moved into a home for older artists in northern Holland. He continued to work on his art at the studio there until his death in 1972.

Source: [www.history.mcs.st-andrews.ac.uk/history/Mathematicians/Newton](http://www.history.mcs.st-andrews.ac.uk/history/Mathematicians/Newton)

by Laura Chavkin, who attends Yale University, and Hiroko Warshauer, who teaches mathematics at Southwest Texas State University.

## PROBLEMS OF THE MONTH

1. How many rectangles with area 72 square units can you make if you only use sides which have whole number valued lengths?
2. Draw a rectangle and divide it into 5 equal pieces using vertical lines. Then divide it into 3 equal pieces by drawing horizontal lines. Use this picture to explain what one-third of two-fifths is. (You can also ask this by saying: what is  $\frac{1}{3}$  of  $\frac{2}{5}$ ?)
3. A rectangle has perimeter 600 ft. It is twice as long as it is wide. What is the area?
4. Sandy pays 10 cents for a lollipop and a piece of bubble gum. Allison pays 24 cents for 2 lollipops and 3 pieces of bubble gum. How much does each lollipop and each piece of bubble gum cost?
5. Insert any combination of  $+$ ,  $-$ ,  $\times$ ,  $/$ ,  $=$ ,  $( )$  to make a true equation:

1      2      3      4      13

6. Use graph paper to draw at least 4 different triangles with area 12 square units. How many other triangles with area 12 square units can you draw?
7. It takes 1 pint of paint to cover a canvas which is 2 feet by 3 feet. How much paint would it take to cover a canvas that is 6 feet by 7 feet?

8. Using nickels and dimes, how many different ways can you make 50 cents?

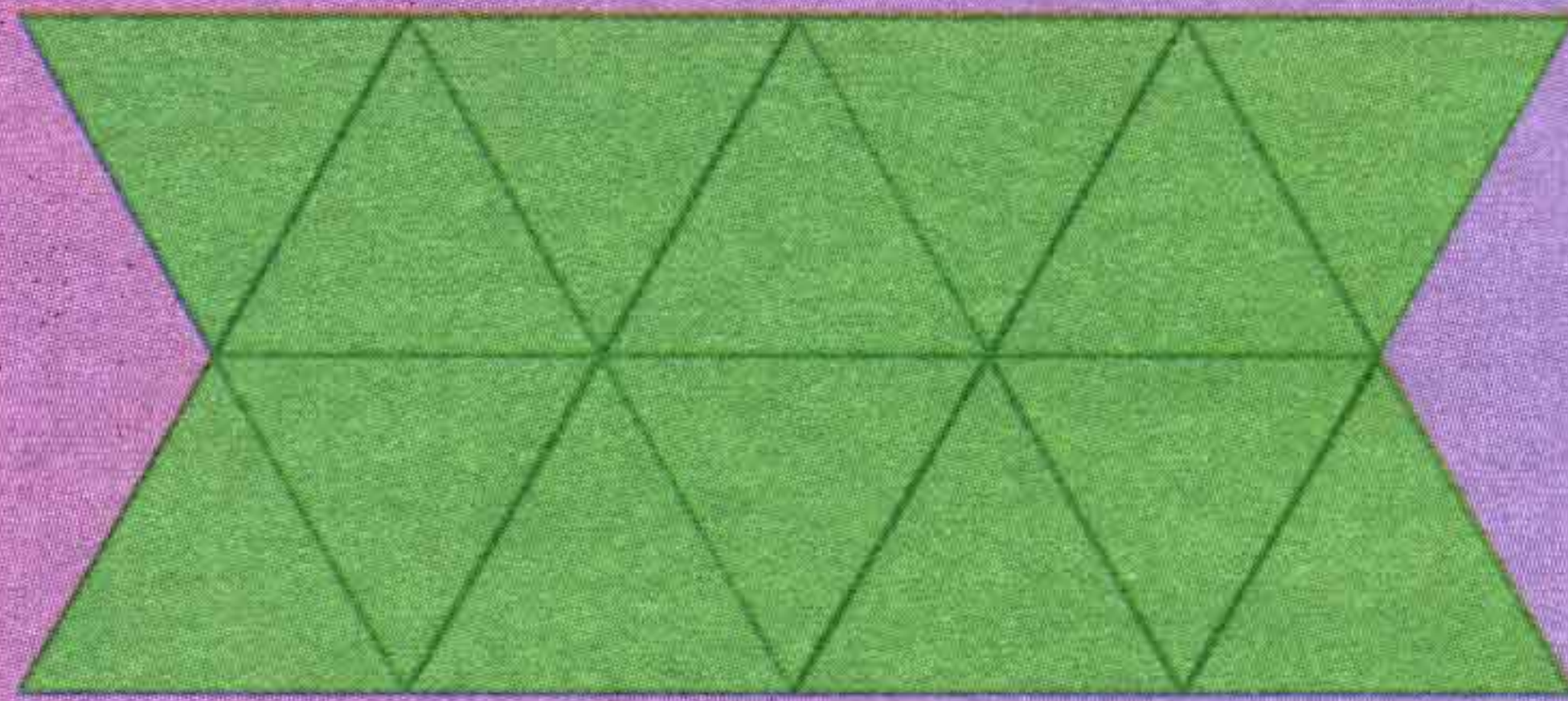
9. Fred wants to paint a picture with 3 colors: blue, green, and red. He has 4 shades of blue, 3 shades of green and 5 shades of red. If he wants to use exactly one shade of each color for his picture, how many options does he have?

10. **INGENUITY:** Suppose you have a 6 by 6 square and you want to cover it with non-overlapping triangles each with an area equal to a whole number. What is the most number of triangles that you can fit in this square? Do they form a nice tiling pattern? What about a 5 by 5 square?

Send us your solutions! Every month, we will publish the best solutions on our website: [www.mathexplorer.com](http://www.mathexplorer.com). If we print your solutions, we will send you and your teacher free **Math Reader** pens!

# Beneath

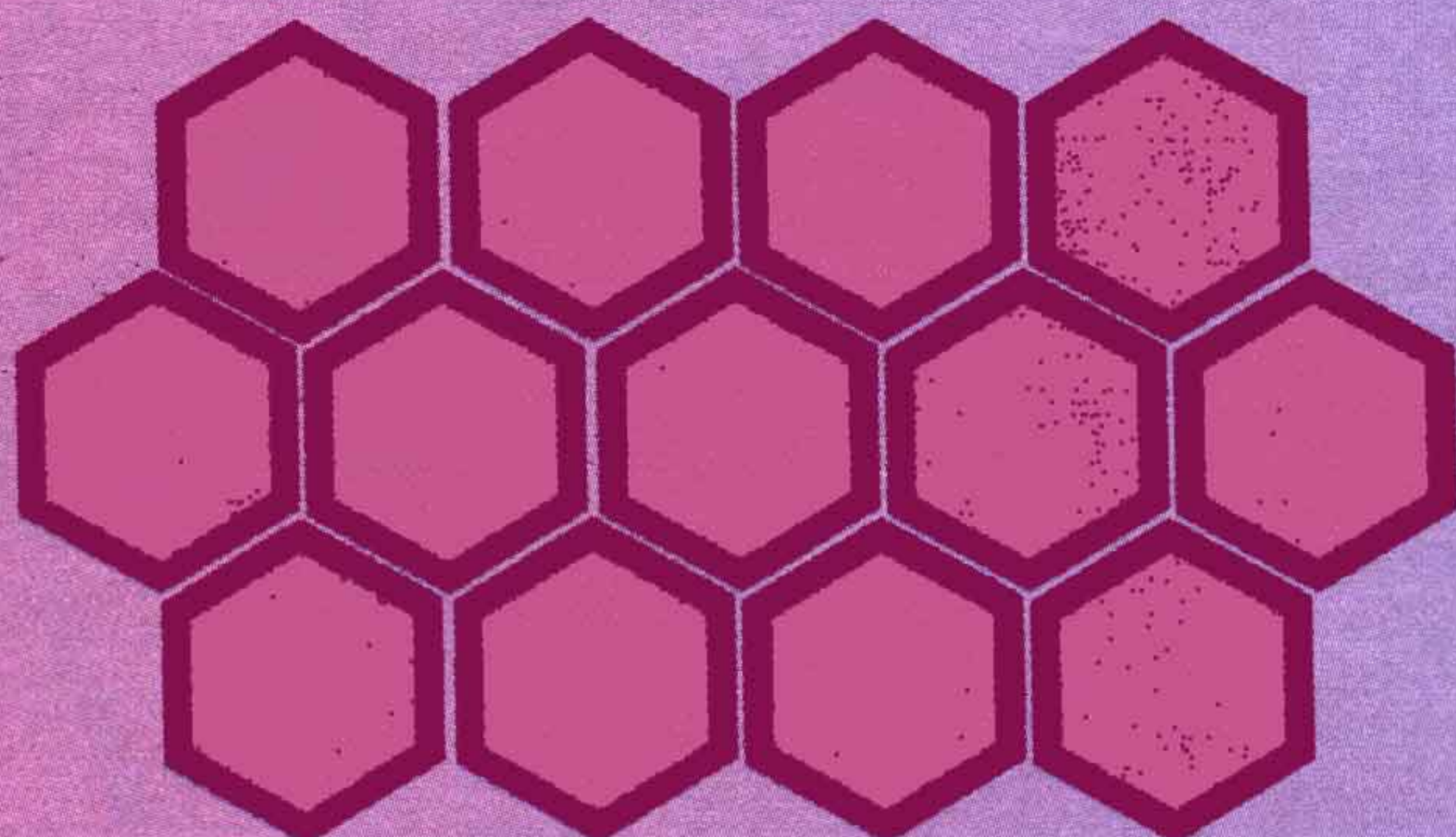
by Jean Davis



**Some** of the most wonderful combinations of mathematics and artistry are things we walk on every day—floor tilings. The beauty and complexity of these designs come from the interesting nature of the shapes used to tile the floor.

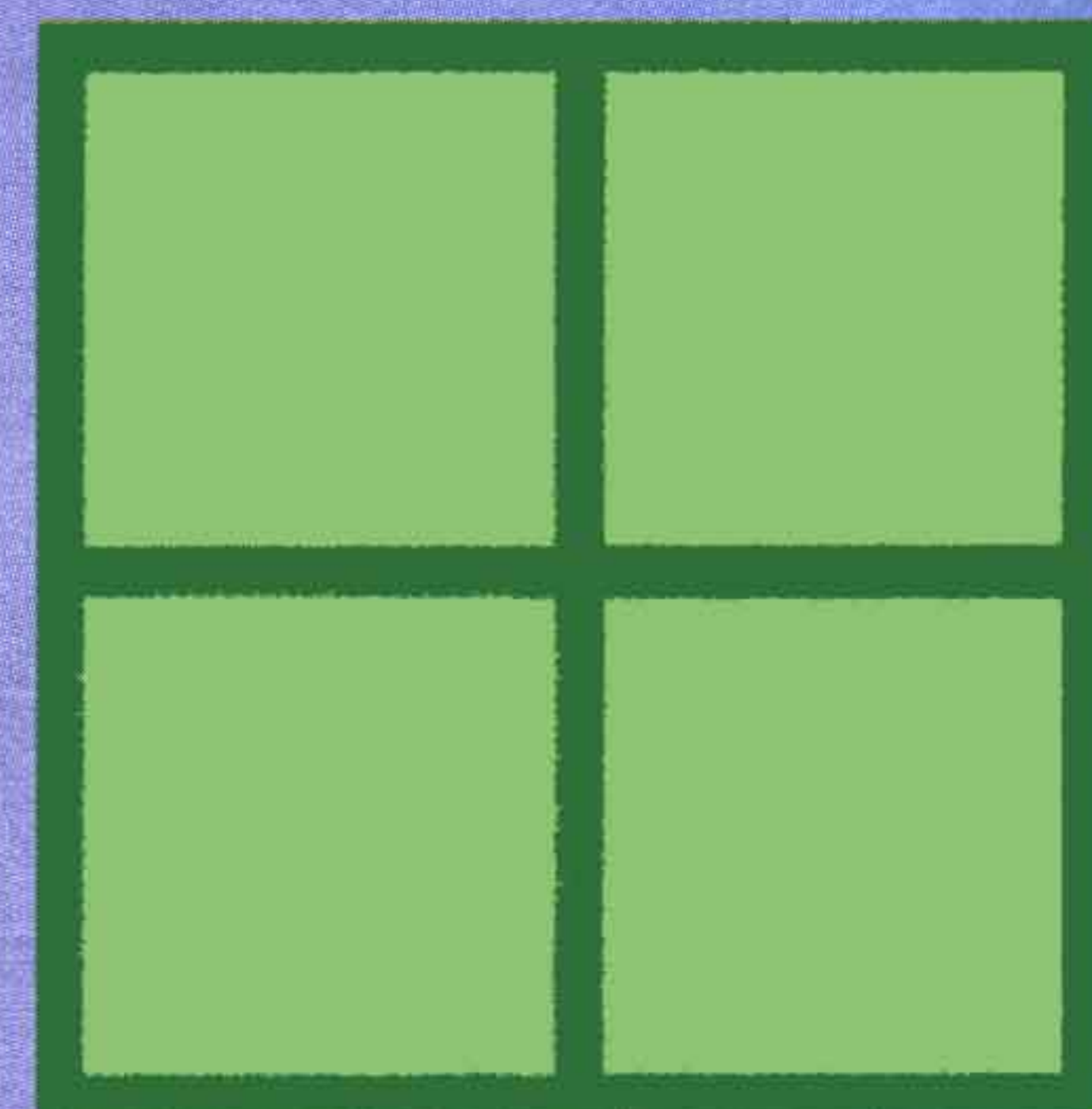
The next time you visit a store that sells flooring, take time to look at the different patterns available. What shapes do you see? There are many rectangles and triangles, but what other shapes seem to appear over and over?

We often see hexagons (6-sided figures) and octagons (8-sided figures). Of all the patterns that I saw on a recent visit to a local home improvement center, not one had pentagons (5-sided figures).



**Why?** It's not because pentagons are not "pretty" to look at. There is a mathematical reason. In order to understand why certain shapes can be used and others can't, we need to know something about shapes and a little about angles.

When we design a pattern for flooring, we need shapes that fit together nicely -- no laps (overlaps), no gaps.

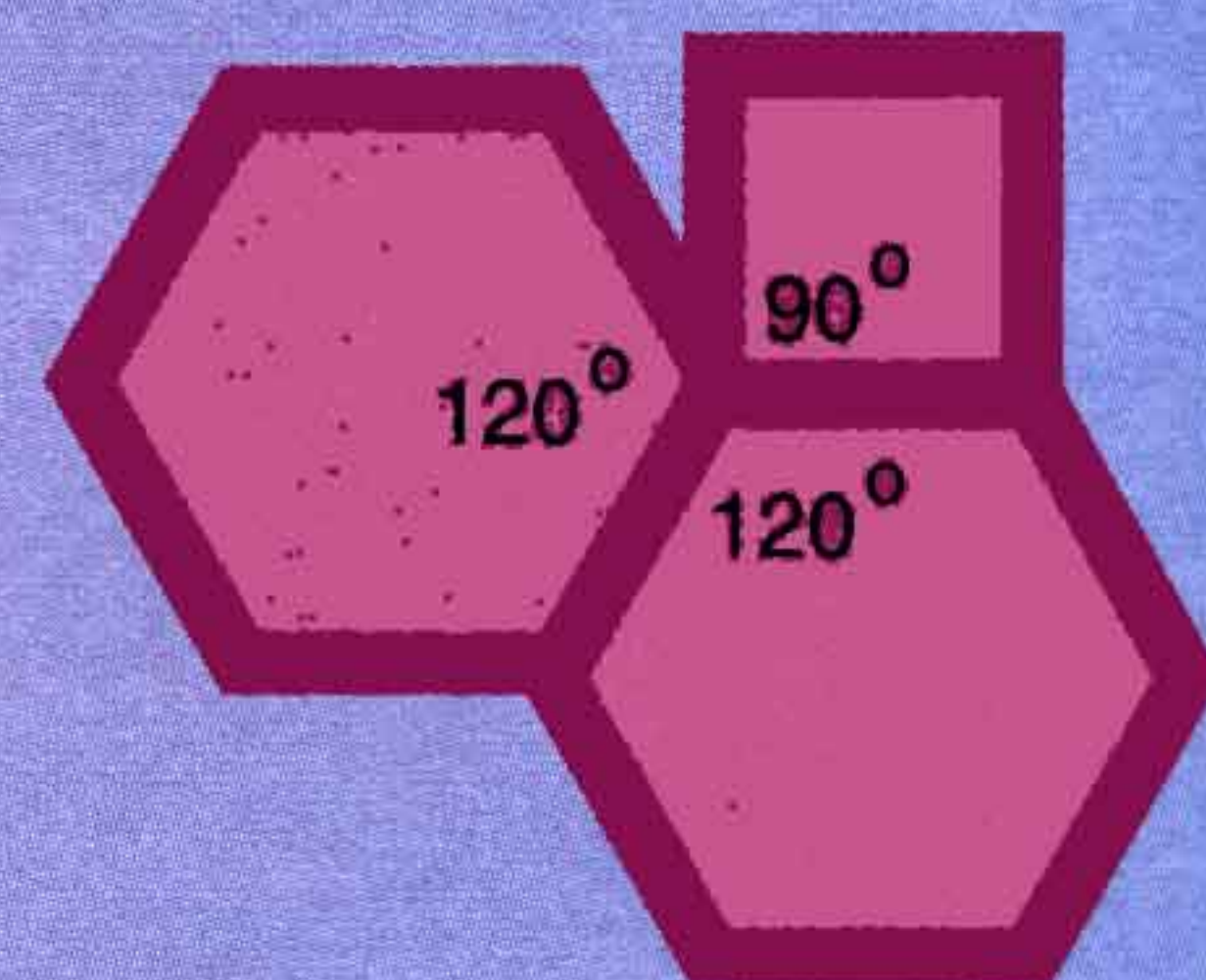


In mathematics, an arrangement of polygons that will "tile the floor" is called a "tessellation." Each angle of a square, for example, is  $90^\circ$ , so 4 squares fit together at one point. This gives a nice repeatable pattern.

Each angle of a regular hexagon measures  $120^\circ$ , so 3 hexagons fit together.

In each case, notice that the sum of the measures of the angles at each connection point is  $360^\circ$ . That's the key!

Less than  $360^\circ$ , and we have a "gap."



# Your Feet

More than  $360^\circ$ , and we have a "lap."

**What about using pentagons?**

The measure of each angle of a regular pentagon is  $108^\circ$ .

$3 \times 108^\circ = 324^\circ$   
which is less than  $360^\circ$ .  
 $4 \times 108^\circ = 432^\circ$   
which is greater than  $360^\circ$ .

Only three regular polygons will tile the floor by themselves: triangles, squares and hexagons.

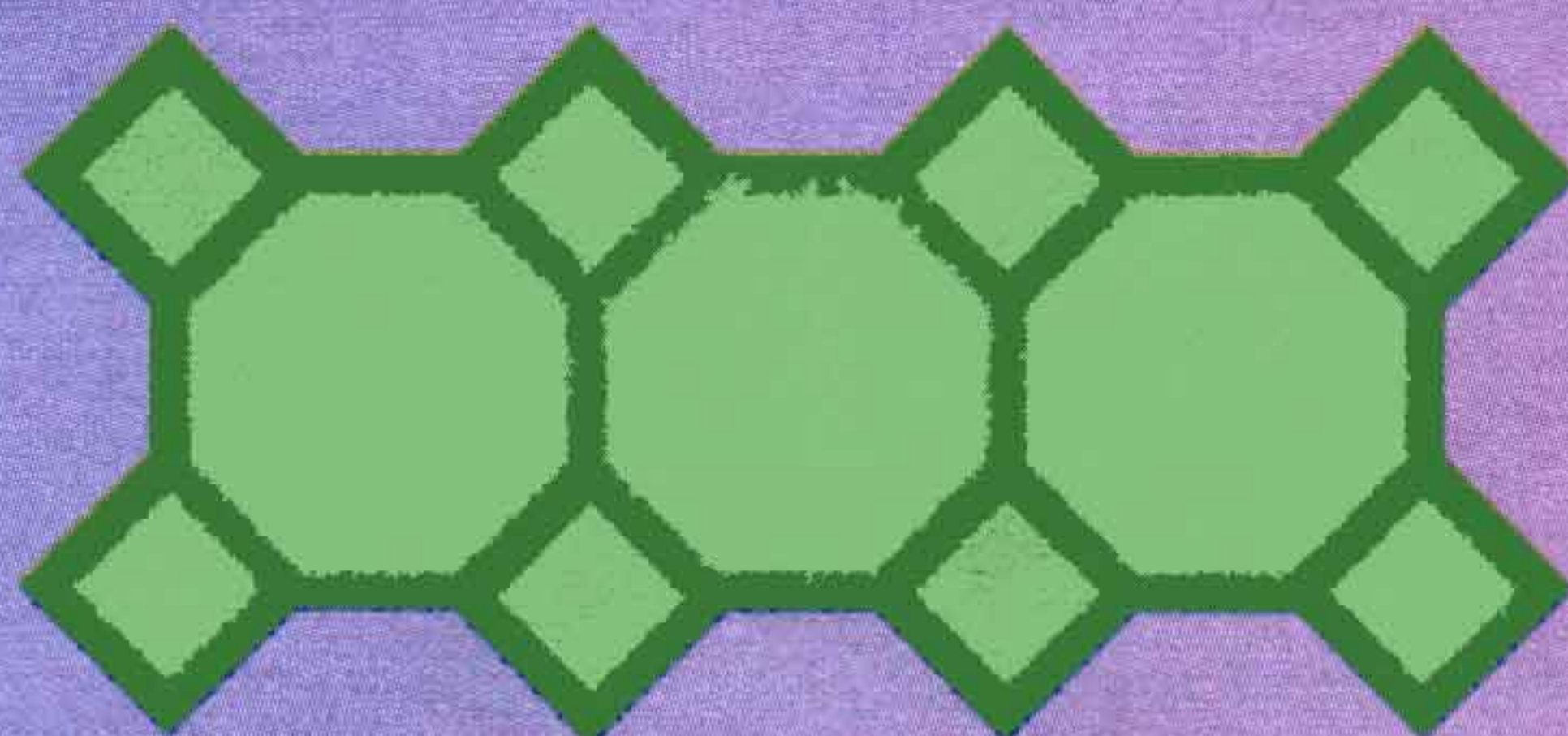
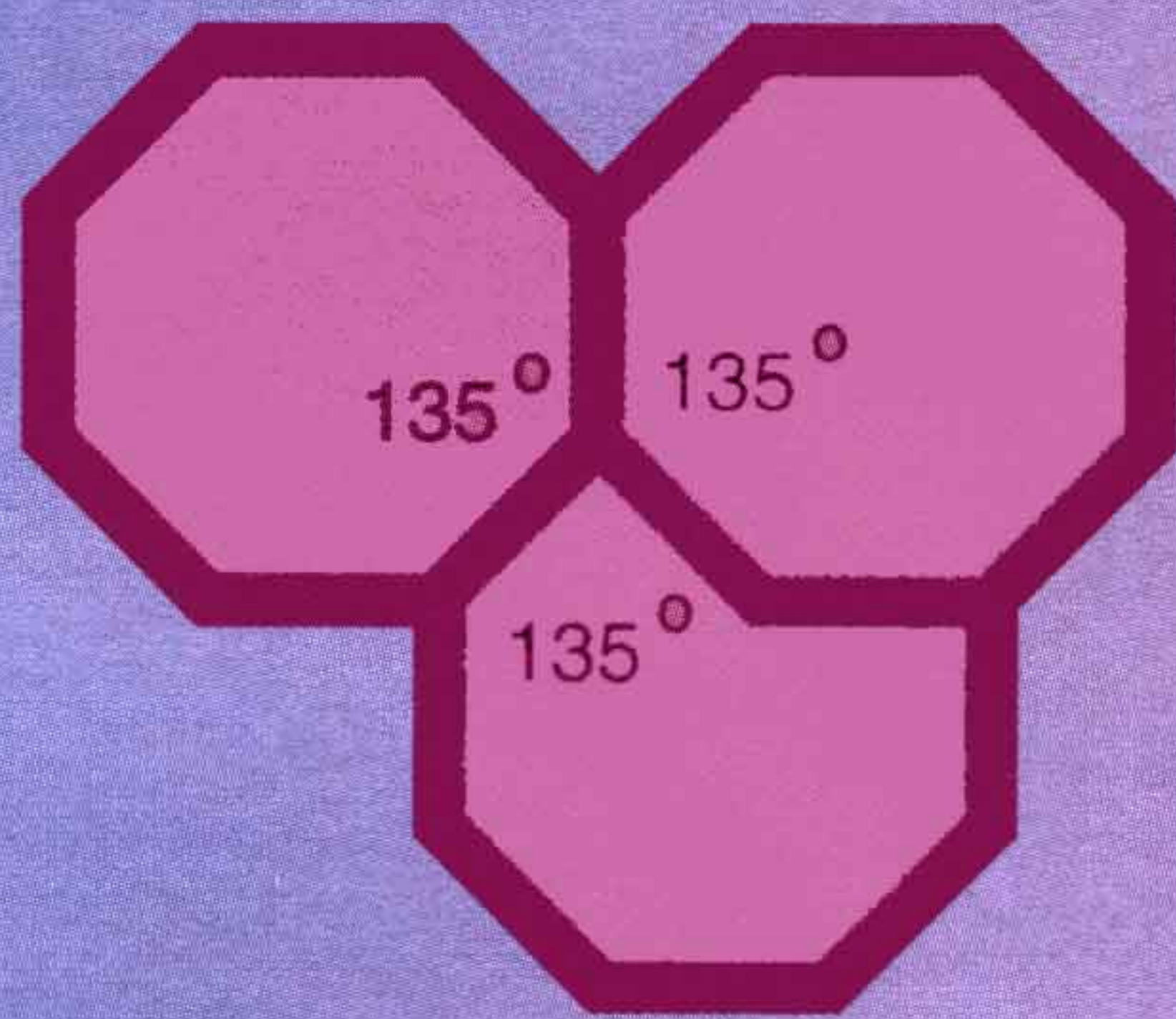
**Certain combinations**

of shapes fit together nicely. For example, squares and octagons. The measure of each angle of a regular octagon is  $135^\circ$ .

$$135^\circ + 135^\circ + 90^\circ = 360^\circ$$

So two octagons and one square will fit together. They form this pattern:

Two squares ( $90^\circ$  each) and three triangles ( $60^\circ$  each) combine to form  $360^\circ$ .



because their symmetry is appealing to the eye and creates a repeatable pattern. Using regular polygons, with the same number of each shape at each point, there are only 8 different tiling patterns that can be formed.

**No wonder we see so many of the same patterns on our floors!**

Different color choices can create many different versions of the same basic pattern.

Tessellations can also be made with irregular shapes. Any triangle and any quadrilateral will tile the floor. The work of the Dutch artist M. C. Escher, famous for his prints of interlocking animals such as the butterflies shown on the cover, illustrates well the link between mathematics and art.

**You can make your own design** by using

Pattern Blocks, dot paper, or by tracing shapes. Remember the rule: no laps, no gaps. Then color your design to make it your very own original floor tiling pattern.

Regular polygons appear frequently

# Puzzle Page

## Math Readers:

We want to print your work! Send us original math games, puzzles, problems, and activities. If we print them, we'll send you and your math teacher free **Math Reader** pens.

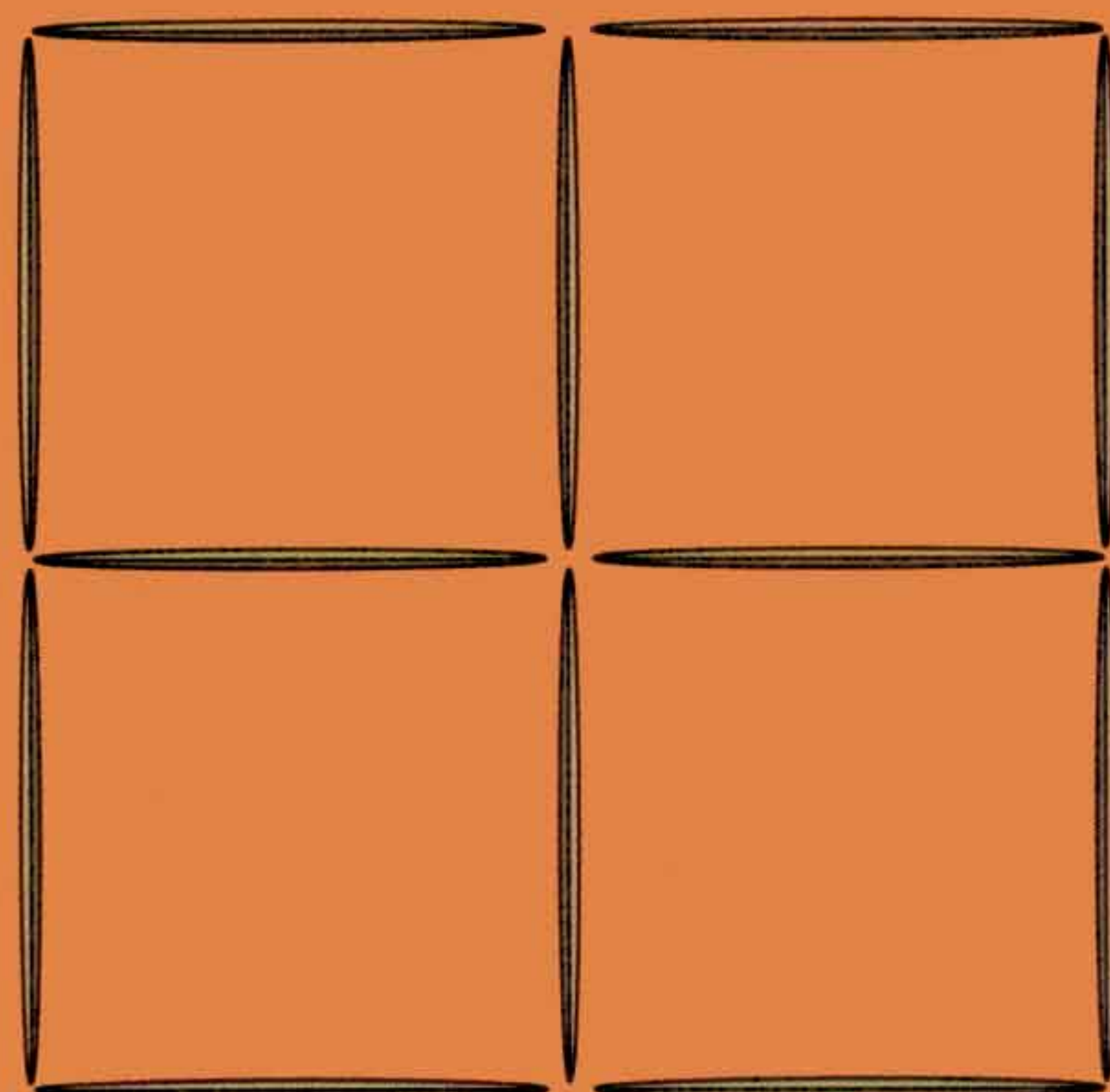
## Word Search

Forwards or backwards, up, slanted, or down.

Where can the words in this puzzle be found?

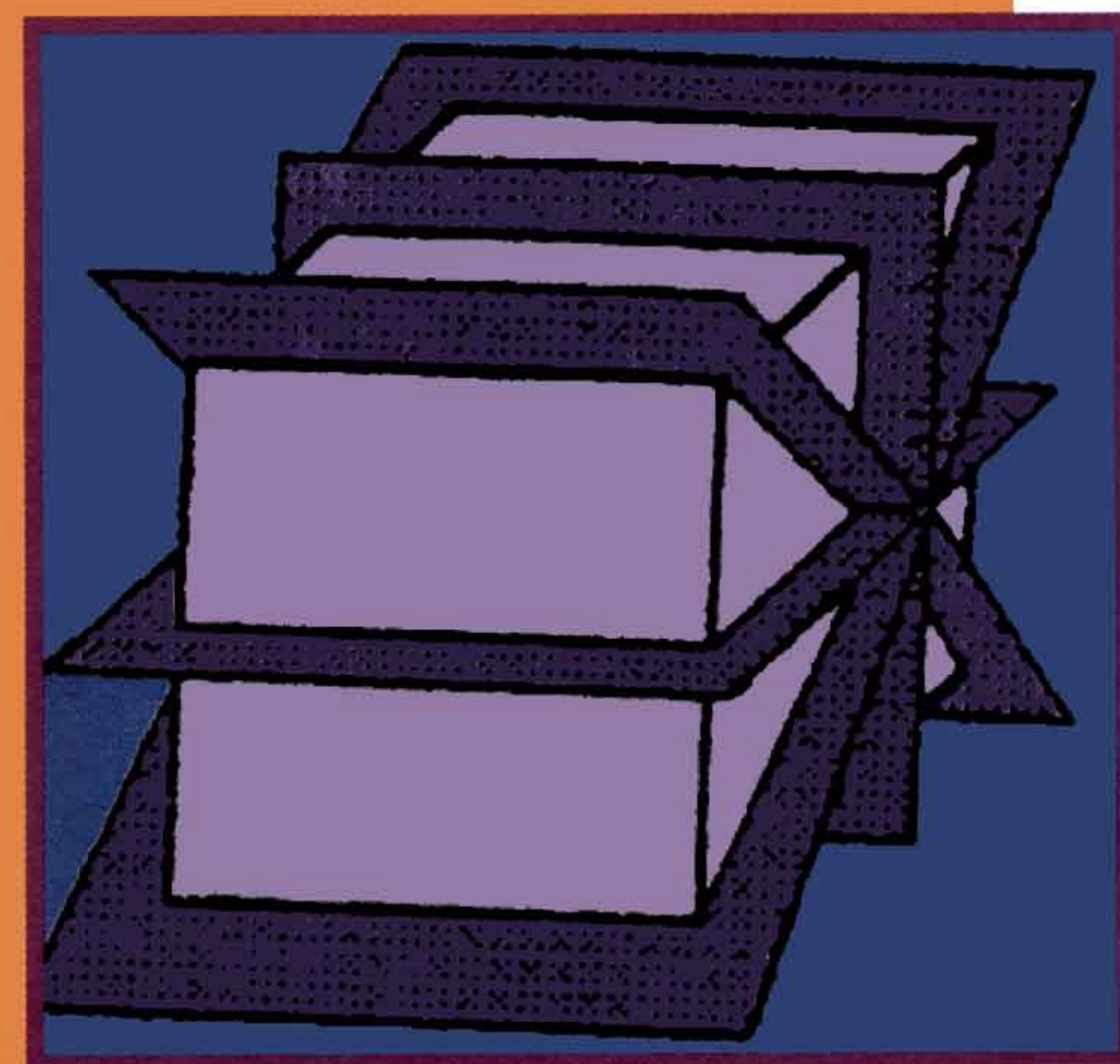
LINE  
CURVE  
SCALE  
POINT  
COLOR  
SHADE  
MAGNITUDE  
SIZE  
AREA  
PERIMETER

S S X L E E D A H S H S M B  
W T E I E I T I V E E P A T  
Z H T V S R I E N Q E A D R  
X G R M M U R A U R O E A P  
S U E N I P A E I S I R O J  
C E A R I A N M L E E I I E  
A H R F T C E C I R N U R Z  
L L E O E T L L P T G N E I  
E P A T E A I O E L G R K S  
P E R R M E I C R E L O L E  
R U V O A N L G N N R L A R  
T E E D U T I N G A M O W X  
D I R G B M U N C H A C R M  
Z E N I L I E L E S S U F I



## Cut up a cube

into 8 identical parts with 4 planes. We have done one for you. Can you find two other solutions to this puzzle?



## STICKY STUFF

Using 12 toothpicks, we made a big square containing 4 little squares. Relocate 3 toothpicks so that only three squares are in the picture.

# Bulletin Board

Yes! I want to subscribe.

## Words of Wisdom

"Everything should be made as simple as possible, but not simpler."

-- Albert Einstein

## Thanks to our Sponsors

**The Rockwell Fund, Inc.** is helping to underwrite Math Explorer and Math Reader magazines. Rockwell also sponsors scholarships for students to attend the SWT Junior Summer Math Camp and SWT Honors Summer Math Camp! Thanks to Rockwell.

## SWT Junior Summer Math Camp News

**The Fund for the Improvement of Postsecondary Education (FIPSE)** has awarded a 3 year grant to Southwest Texas State University. FIPSE is sponsoring a student and teacher training program for 672 students and 40 teachers. For more information, contact Elaine Hernandez from South Texas Community College (956-971-3712) who is coordinating the Rio Grande Valley Math Institute for Teachers and Summer Math Camps.

## Check it out!

Did you know that there are over 50 workshops for teaching elementary/middle school mathematics? You can check them out on the internet at <http://www.nctm.org/information/mathforumworkshops/workshops.html>.

## Did you know?

Did you know that the year 1999 using Roman numerals was MCMXCIX? Luckily, the year 2000 is a lot easier to write: MM.

The year 2000 is a leap year. Did you know that the years 2100, 2200, 2300 won't be leap years!? Do you know how to decide if a year is a leap year or not?

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Dear **Math Readers**,

Beneath the surface of art, in the beauty of its design and composition or its use of color, lies a mathematical principle. For example, using the golden ratio is pleasing to the eye, proportion and perspective add depth and dimension to the picture and geometry is an important part of many works of art.

We hope you will enjoy the articles and problems in this issue. We'd like to hear how you used mathematics in your next art project!

Sincerely,

*Hiroko K. Warshauer*

Hiroko K. Warshauer



**Math Reader**

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