Math Explorer

MATH & MEASURING

Math Measures Up!

Ada Makes Math Poetic

No Spilling Allowed!!

Vol. 4.4 Summer 2002

Math Explorer

Contents

Executive Editor: Hiroko K. Warshauer
Senior Editors: Terry McCabe, Max Warshauer, Eugene Curtin
Special Writers: Jean Davis, Dan Shapiro
Design: Jennifer LeGrévellec, Amanda Eliasson
Final Editing and Proofreading: David Nelson, Michael Kellerman
Administration: Lydia Carbuccia
Circulation: Elma Falcon, Silvana Naguib
Webmaster: Jennifer LeGrévellec

Math Explorer

Southwest Texas State University

San Marcos, TX 78666

Phone: (512) 245-3439, Fax: (512) 245-1469

e-mail: max@swt.edu.

Visit our website: www.swt.edu/mathworks

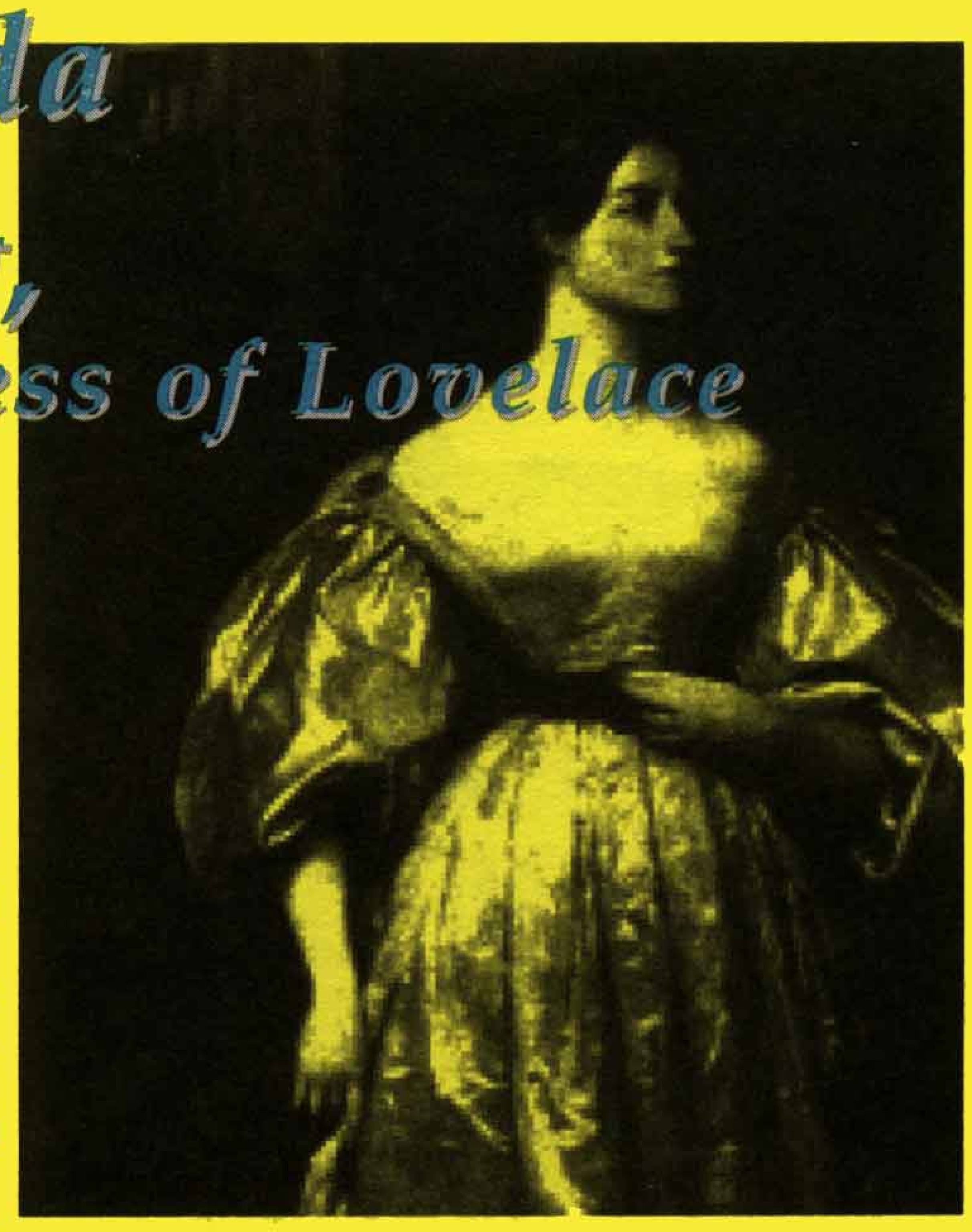
Math Explorer is published by

Southwest Texas State University

Mathworks.

SWT

Augusta Ada Byron King,
Countess of Lovelace



Jean Davis

Whew! Quite a long name for one of the most celebrated and gifted women of the 19th century. Ada Byron was born Dec. 10, 1815 in London, England to Anna Isabella Milbanke and George Gordon Noel Byron. Her parents separated shortly after her birth, and she was raised by her mother. Once referred to by Ada's father as "the Princess of Parallelograms", Anna passed her interest in mathematics on to her daughter. She was tutored in mathematics by one of the eminent mathematicians of the time, Augustus DeMorgan.

Ada tended to look at mathematics in a very imaginative and poetic way, which is not surprising since her father was a famous poet, known to us as Lord Byron. She was also encouraged in her studies by Mary Somerville, an outstanding female mathematician of the day. At a dinner party at her house, a seventeen year old Ada heard Charles Babbage outline his ideas for a new calculating machine he called the "Analytical Engine". This was the forerunner of the computer. No one there was very excited about it - except Ada. She was struck by the "universality of his ideas", and from 1836 until her death she wrote many letters to Babbage.

At age 19, Ada married William King, Earl of Lovelace, with whom she had three children. In 1843 she read an article about the progress of Babbage's machine. Inspired, she translated it from French to English and added her own comments. She predicted how such a machine might be used to compose music, produce graphics, and perform functions that would have both practical and scientific uses. She was right! Her plan for how this machine could calculate certain types of numbers is regarded as the first computer program.

Ada died November 27, 1852 from cancer. She was only 36 years old. Though her life was short, she anticipated by more than a century what most of us consider to be recent discoveries in computing. A software language developed by the U. S. Department of Defense was named "Ada" in her honor in 1979.

(continued on page 7)

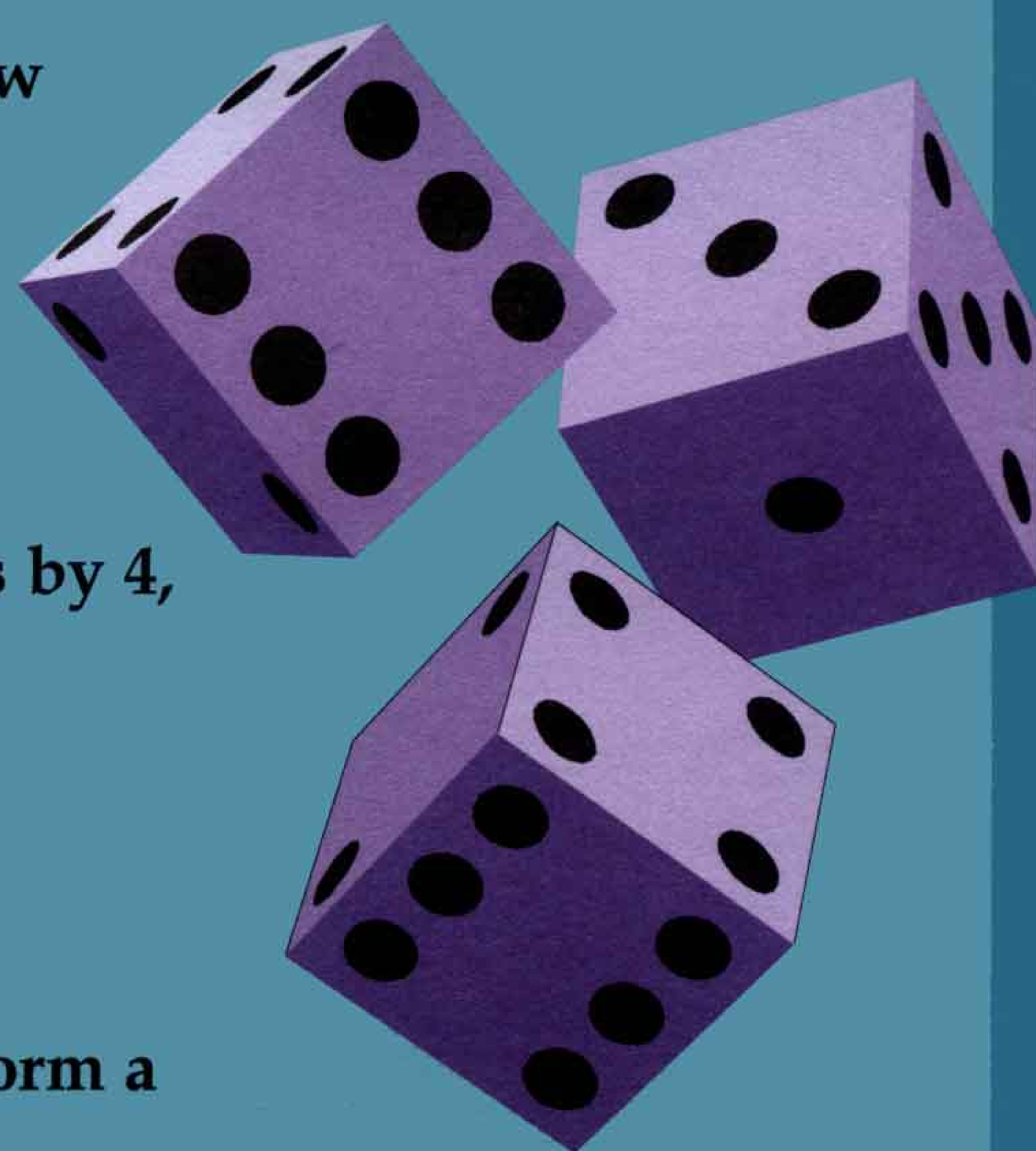
Jean Davis teaches mathematics at Southwest Texas State University.

1. What is the smallest positive integer with exactly 10 divisors? With exactly 5 divisors?

2. Find the 100th term of the sequence 2, 5, 8, 11, ….

3. A theatre has 500 seats, and charges $6 for adults and $4 for children. It sold out on the Saturday afternoon show, and took in $2400 from the ticket sales. How many adult tickets were sold?

4. Becky rolls three dice: a red die, a green die and a blue die. How many ways can the sum of the numbers she rolls be 9?



5. Jacob has fewer than 100 books. If he divides the number of books by 3, the remainder is 2. If he divides the number of books by 4, the remainder is 1. If he divides the number of books by 5, the remainder is 3. How many books does Jacob have?

6. In how many ways can 10 tiles of size 3 X 1 be put together to form a 3 X 10 rectangle?

7. What is the units digit of 7^{95}?

8. How can you make 48 cents postage with only 5 cent and 7 cent stamps?

9. What is the smallest positive integer that is divisible by both 3 and 5 and is written using only ones and zeros in base 10?

Ingenuity: Draw 6 straight lines on a sheet of paper, so that each pair of lines intersect at a point and no three lines go through the same point. How many triangles are there?

1) 48, 16 2) 299 3) 200 4) 25 5) 53 6) 28 7) 3 8) 4 of each 9) 1110 Ingenuity) 20

PROBLEMS WITH MEASURING

by Daniel B. Shapiro

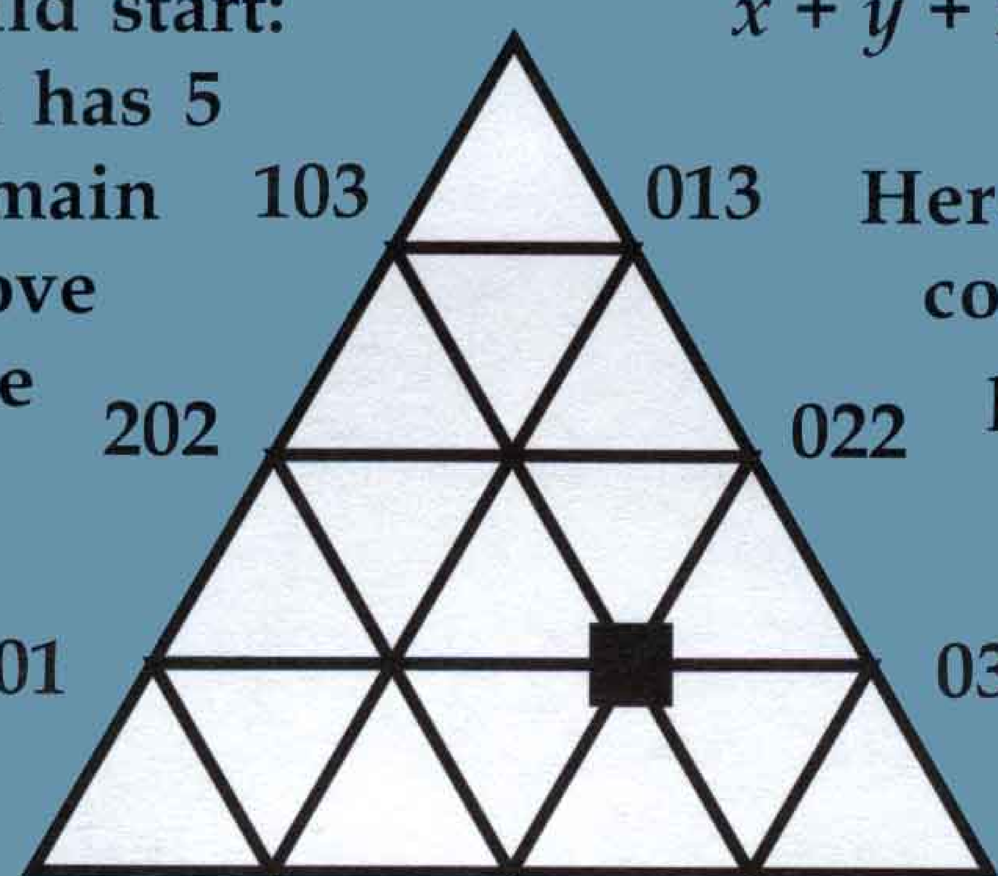
Two thieves broke into a wizard's workshop and stole a crystal decanter filled with 8 ounces of powerful magic potion. In their hideout they want to split the prize, but they can locate only two other containers: a flask that holds 5 ounces and a vase that holds 3 ounces. Can they split the potion equally using only those containers?

This is a classic measuring problem. To model it mathematically we should consider only the key information, ignoring the rest. There are three containers: a decanter, a flask, and a vase, with capacities 8, 5, and 3 ounces. Starting with 8 ounces in the decanter, the thieves pour the liquid back and forth, trying to end up somehow with 4 ounces in one of the containers. Here's one way they could start: First they fill the flask, so the flask has 5 ounces of potion while 3 ounces remain in the decanter. For their second move they fill the vase by pouring from the flask. At that point there are 3 ounces in the vase, 2 ounces in the flask, with 3 ounces remaining in the decanter.

Explaining the steps with English words soon becomes confusing. To keep track of the moves let's write the symbol (x, y, z) to mean that there are x ounces in the decanter, y ounces in the flask, and z ounces in the vase. The story begins at (8, 0, 0). The first move above leads to (3, 5, 0) and after the second move they have (3, 2, 3). From there they could move to (0, 5, 3), or to (6, 2, 0), or back to (3, 5, 0). Are there other possibilities? There seem to be several ways to move after each step. The problem is to find a sequence of moves that ends with a 4 appearing somewhere. Take a minute now to see if you can discover the moves they need.

A position (x, y, z) in this game involves numbers x, y, z whose values are somewhat restricted. For instance, x represents the number of ounces in the decanter so it must be between 0 and 8. In symbols we write that as: $0 \le x \le 8$. Similarly there are y ounces in the flask, so y must lie between 0 and 5. That is: $0 \le y \le 5$. For the vase we find $0 \le z \le 3$. Finally since no spills are allowed (the potion is too valuable) there must be a total of 8 ounces at all times. In symbols:

$$x + y + z = 8.$$



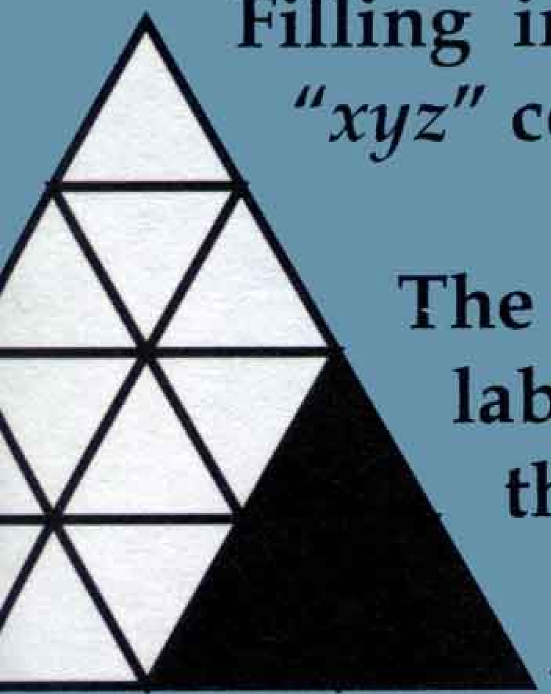
Here's a somewhat surprising connection with geometry. The points in a triangle are labeled so that each of those triples (x, y, z) corresponds to a point. For simplicity we start with a smaller example: Triples (x, y, z) where the entries are non-negative and $x + y + z = 4$. In the triangular grid pictured here, the labels are written in for the grid points around the outside. For example, the label "400" is shorthand for the triple (4, 0, 0) and represents the lower-left corner point. Can you see the pattern and figure out the labels for the unmarked interior points?

One explanation of this labeling pattern is to erase the "x" component and look only at the pair "yz". The labels along the bottom row become 00, 10, 20, 30, 40, and the labels along the left edge become 00, 01, 02, 03, 04. These behave like coordinates on the axes in standard plane geometry. For example the marked interior point is 2 steps to the right and 1 up, so its "yz" coordinates are "21". To find

Daniel B. Shapiro is a professor of mathematics at Ohio State University.

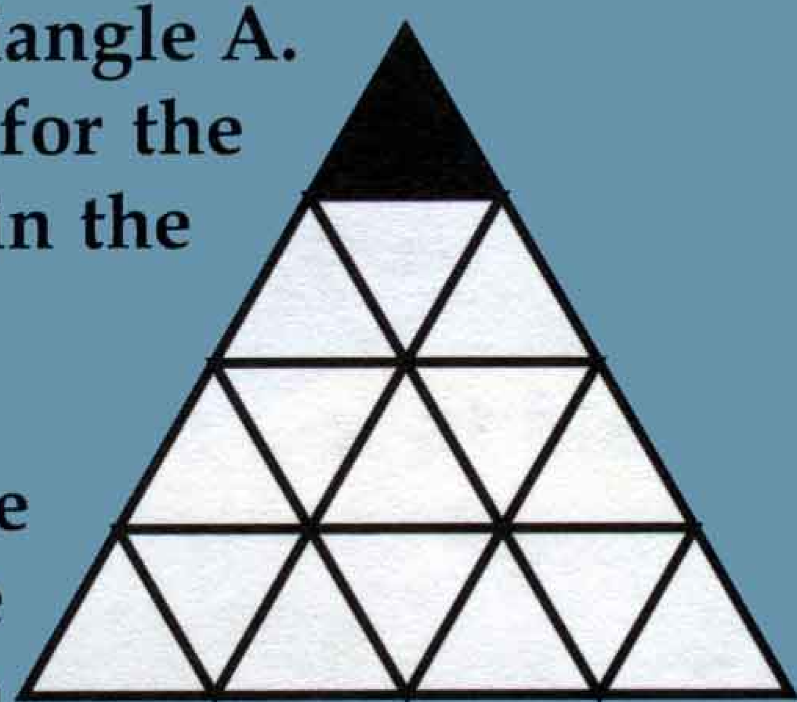
the missing "x" recall that $x + y + z = 4$. For this point, $x + 2 + 1 = 4$ so that $x = 1$ and the triple representing that point is: 121.

Alternatively, erase the "z" component and examine only "xy" to get a similar situation with the origin 00 at the top. The labels down-left are 00, 10, 20, ... while the labels down-right are 00, 10, 02, ... The marked interior point is then 1 step down-left and 2 steps down-right, so its "xy" coordinates are 12. Filling in the missing "z" we find the full "xyz" coordinates to be 121, as before.



A

The equation $y = 2$ represents all points labeled $x2z$ (where $x + 2 + z = 4$). From the triangle picture we see this is the diagonal line containing the grid points 220, 121, 022. Which points in the triangle satisfy the inequality $y \leq 2$? That set of points is white in triangle A. Similarly the solution set for the inequality $z \leq 3$ is white in the triangle B.



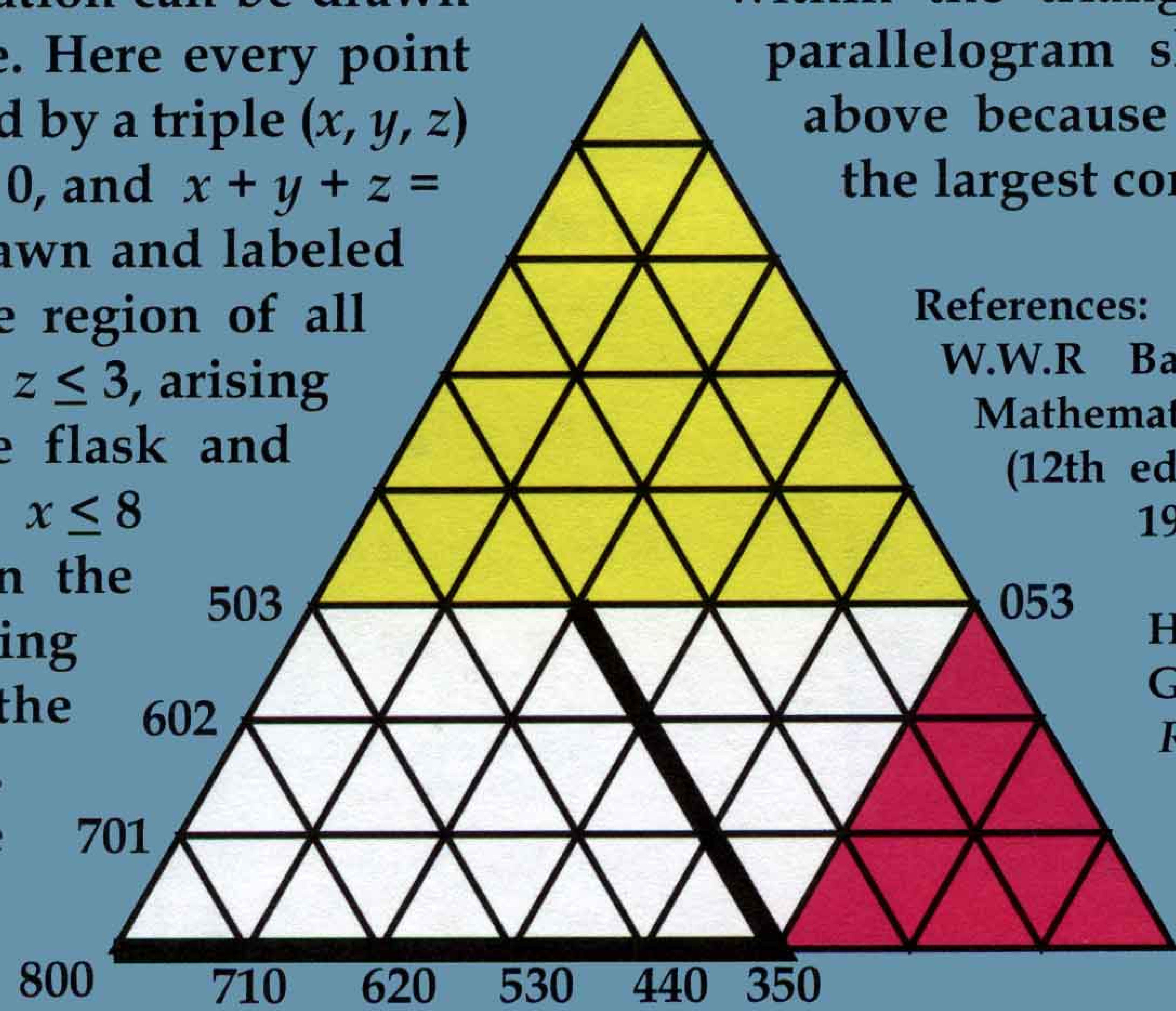
B

Can these coordinates be extended to points outside the triangle? Such an extension can be done provided negative coordinates are allowed. But we don't need those cases here.

Now let's return to the original problem with 8 ounces of potion. This situation can be drawn on a larger labeled triangle. Here every point in the triangle is represented by a triple (x, y, z) satisfying $x \geq 0$, $y \geq 0$, $z \geq 0$, and $x + y + z = 8$. Once that triangle is drawn and labeled we restrict attention to the region of all points satisfying $y \leq 5$ and $z \leq 3$, arising from the capacities of the flask and vase. (Why is the condition $x \leq 8$ automatically satisfied?) In the picture, the points satisfying those conditions lie in the white region of the triangle. The initial pour from the decanter to the flask is represented by the thickened segment from 800 to 350. The second move becomes the shaded segment from 350 to 323. Each step of pouring from one container to another appears in the picture as a line segment along the grid in the white region. The endpoint of each segment must always lie on the boundary of the white region. Those boundary points represent the situations where one (or more) of the containers is empty or full. At each step we have a choice of directions: we can move along any grid line as long as we stay within the white region and stop when we reach the boundary. After experimenting with ways to bounce around within that region, we do obtain a solution to the original problem: 800 → 350 → 323 → 620 → 602 → 152 → 143 → 440. Check that each step there represents a valid move, pouring the potion from one container to another. Here's a different solution: 800 → 503 → 530 → 233 → 251 → 701 → 710 → 134 → 440. Can you trace that path within the white region below?

That problem was pretty easy! Now that you understand this geometric method you can solve much more complicated measuring problems. The hardest part is usually the artwork: drawing and labeling the triangle picture.

Note: In three-bottle problems the restrictions on x, y and z might lead to a hexagonal shape within the triangular grid. We had a parallelogram shape in the example above because all the liquid fit into the largest container.



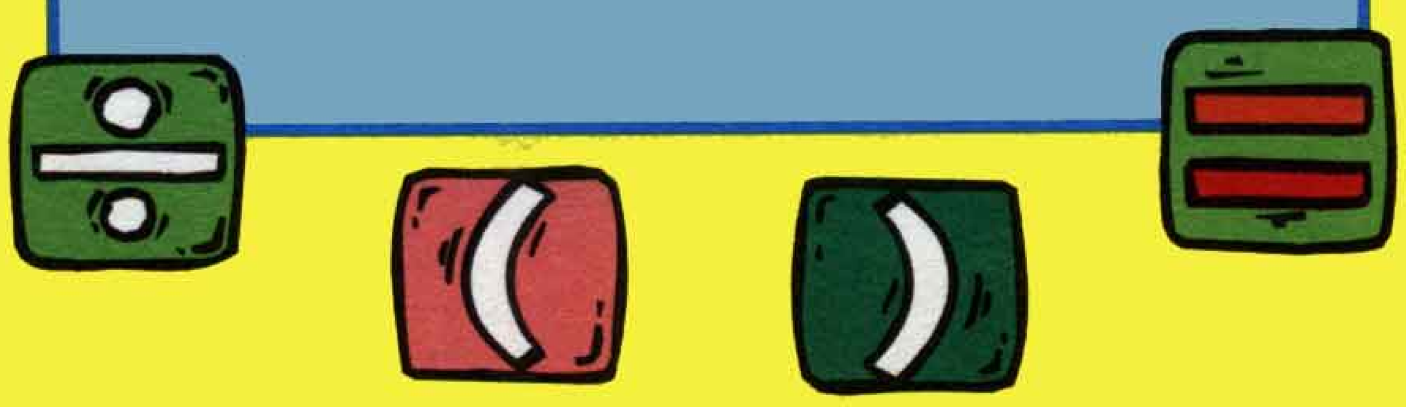
References:
W.W.R Ball and H.S.M. Coxeter, Mathematical Recreations and Essays (12th ed.), Univ. of Toronto Press, 1974. pp. 28, 40.

H.S.M. Coxeter and S.L. Greitzer, *Geometry Revisited, Math. Assoc. of Amer., 1967. Section 4.6*

Puzzle Page

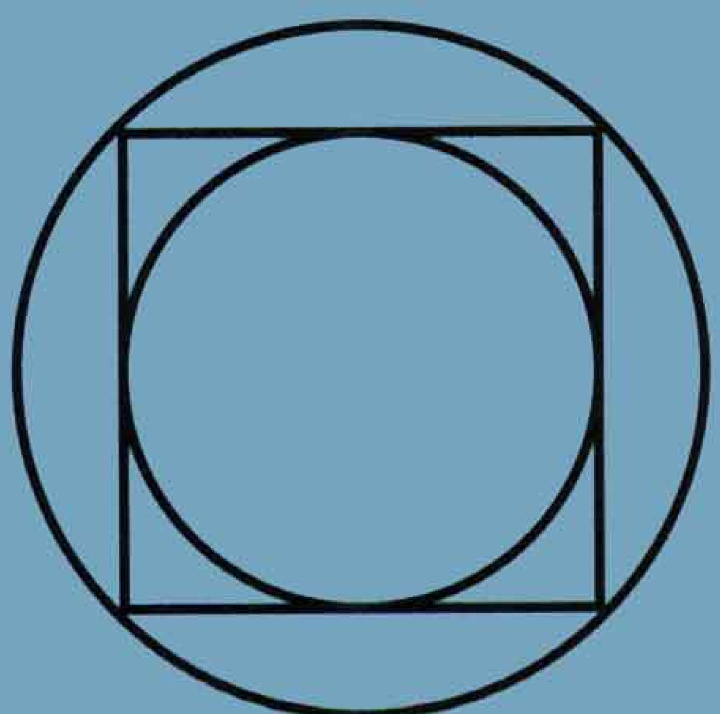
Math Explorers: We want to print your work! Send us original math games, puzzles, problems, and activities. If we print them, we'll send you and your math teacher free *Math Explorer* pens.

Using only the number 4, +, - , x , / and parentheses, can you write the numbers 1 through 10?



On a circle there is 1 red point and 2000 blue points. Consider all polygons with vertices at these points. Which polygons are more- those that have only blue vertices or those that have a red vertex? What is the difference between the number of polygons of both types?

What is the relationship between the area of the small circle inside the square and the large circle outside the square?



Word Search

Forwards or backwards, up, slanted, or down.

Where can the words in this puzzle be found?

- BOUNDARY
- GEOMETRY
- MEASURING
- TRIANGLE
- INTERIOR
- INEQUALITY
- SEGMENT
- REGION
- HEXAGONAL
- PARALLELOGRAM

P	L	L	G	E	O	M	E	T	R	Y	S	L	R
O	A	E	R	S	E	G	M	E	N	T	E	Q	D
E	A	R	M	L	R	D	Y	E	A	I	Y	A	C
I	O	E	A	Q	A	I	C	L	C	L	P	I	O
O	R	G	L	L	M	R	O	G	R	A	M	G	G
M	Y	I	G	N	L	N	L	N	S	U	Q	N	R
E	R	O	E	L	N	E	R	A	N	Q	S	I	R
A	A	N	O	A	M	E	L	I	N	E	E	R	Y
S	E	L	M	E	A	S	U	O	I	N	G	U	I
U	E	L	G	N	A	I	R	T	G	I	M	S	G
R	I	N	T	E	R	I	O	R	A	R	E	A	M
I	A	M	R	O	A	E	R	I	O	G	A	E	C
N	L	E	Y	R	A	D	N	U	O	B	T	M	G
G	L	A	N	O	G	A	X	E	H	R	E	L	R

Bulletin Board

President Bush Awards swtMathworks Director

Max Warshauer, Mathworks Director, was one of the 10 nationwide recipients of the Presidential Award for Excellence in Science, Mathematics, and Engineering Mentoring. The award includes a $10,000 grant which will be used to support student scholarships. The presentation was made by Rita Colwell, Director of the National Science Foundation, and John Marburger, Director of the Office of Science and Technology Policy for President Bush.

Look it Up!

Math Puzzles is a collection of interactive games that give practice in basic logic and arithmetic.
www.mathispower.org/mathnav.html

Learn a Lot at Math Camp 2002

Send in your 2002 Summer Math Camp applications! Contact swtMathworks at (512) 245-3439 or our website at *www.swt.edu/mathworks*

Math Riddle

What keeps a square from moving?

Square roots

Ada (cont'd)

For more information on Ada Byron, refer to *Ada, The Enchantress of Numbers*, written by Betty Alexandra Toole and published by Strawberry Press.

"***The Analytical Engine weaves algebraic patterns just as the Jacquard-loom weaves flowers and leaves.***"

- Ada Lovelace

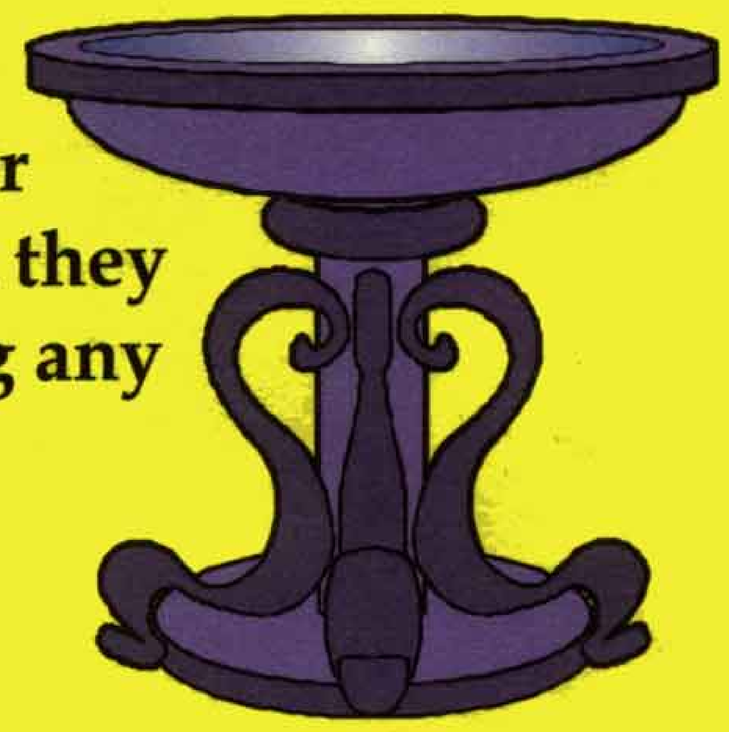
References:
www.cs.yale.edu/homes/tap/Files/ada-bio
cs.fit.edu/~ryan/ada/lovelace

MATH ODYSSEY

by Daniel B. Shapiro

Puzzling Problems with Potions

Here are a few puzzlers you can use to test your new skills.



1. The robbers return to the wizard's stronghold and steal a decanter filled with 12 ounces of water from the Fountain of Youth. They have two other containers: a flask that can hold 9 ounces and a vase that can hold 5 ounces. Can they equally split their ill-gotten gains, using only those containers and not spilling any of the precious liquid?

2. Bossie the cow has been biologically engineered to produce milk containing an antibiotic effective against the blue plague. The hospital needs exactly 10 pints of fresh milk for immediate use. Bossie's owner rushes out to the barn, but soon discovers that the only containers she has are three cans holding 19 pints, 13 pints and 7 pints. Can she milk Bossie and measure 10 pints using only those containers and without spilling any of the precious milk?

3. Pierre has three crystal bottles with sizes 8, 6 and 4 ounces. The first one is full of expensive French perfume and the others are empty. He needs to measure 5 ounces. Explain why this cannot be done with only those tools. What is the general rule here?

4. This time Pierre has bottles of sizes 8, 6 and 3 ounces. He needs to get 4 ounces of perfume from the large container in the store room. Pierre has mathematical training and writes out the triangular grid for this problem. However, if he fills the 8 ounce bottle and pours perfume back and forth, he cannot figure out how to measure out exactly 4 ounces! Can you help him?
[Hint. He might fill the other bottles instead.]

Now make up some similar problems of your own!

References:
M.C.K. Tweedie, Math. Gazette 23 (1939), 278-282.
T. H. O'Bierne, Puzzles and Paradoxes, Oxford Univ. Press, 1965. 49-75.

This issue of *Math Explorer* offers our readers many intriguing problems to work on during the summer months ahead. The main article on measuring looks at a well-known problem and illustrates the mathematics that helps to both analyze and solve it. The Math Odyssey will take you on a journey of better understanding the technique developed in the main article. Let us hear from you about suggestions or comments for the coming year. Don't forget to send in your renewals before the fall.

Have a wonderful and safe summer!

Sincerely,

Hiroko K. Warshauer