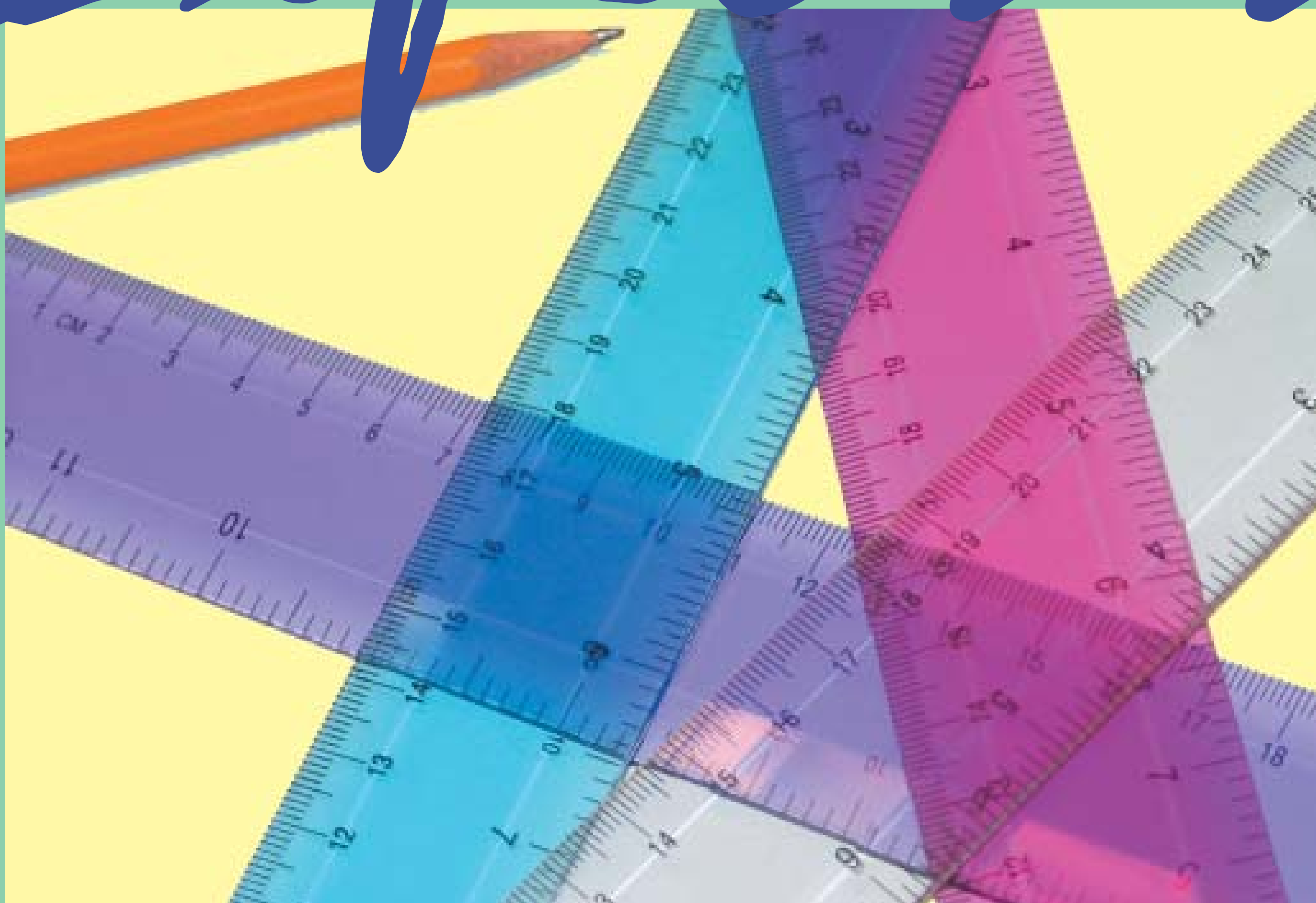
Math Explorer



MATHEMATICS AND MEASUREMENT

Sectors Add Up!

Rubik's Craze

Who's Walking the Dog?

Vol. 6.4 Summer 2004

Math Explorer

Contents

Executive Editor: Hiroko K. Warshauer
Senior Editors: Eugene Curtin, Terry McCabe, Max Warshauer
Special Writers: Heather Davis, Daniel Shapiro
Design: Jennifer LeGrévellec, Michelle Ovalle
Final Editing and Proofreading: David Nelson, Michael Kellerman
Administration: Lydia Carbuccia
Circulation: Lydia Carbuccia, Michelle Ovalle
Webmaster: Jennifer LeGrévellec

Math Explorer
Texas State University-San Marcos
San Marcos, TX 78666
Phone: (512) 245-3439, Fax: (512) 245-1469
e-mail: max@txstate.edu
Visit our website: www.txstate.edu/mathworks
Math Explorer is published by
Texas State University
Texas Mathworks.

Erno Rubik

by Heather Davis

Erno Rubik is a Hungarian educator whose love for geometry led to a world-wide craze. Born in 1944 in Budapest, Hungary to a mechanical engineer and an artist, that combination of technology and the arts became the building blocks for Rubik's creativity. As a student, he was passionate about learning and studied the arts, technology, architecture and design. Rubik felt that the best way to learn was to pass on knowledge as a teacher.

Rather than just lecturing, Rubik preferred to use actual models to help his students grasp concepts. He sought to create a three-dimensional puzzle that looked nice, was very challenging, and would be one self-contained cube throughout its different configurations. Inspiration for the design of this cube came while sitting on the banks of the Danube river. The smooth pebbles gave him the idea to use cylinders on the inside of the cube to make it rotate.

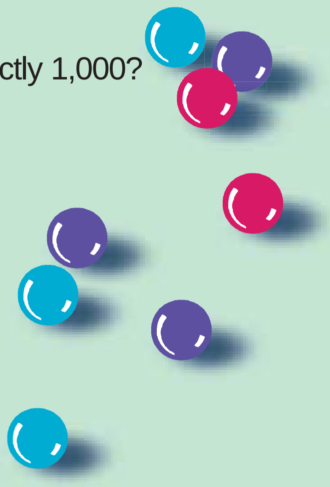
His creation was born in 1974, while Rubik was a lecturer at the Academy of Applied Arts in Budapest, Hungary. The puzzle, known initially as the Magic Cube, became very popular in Hungary and soon spread as Rubik traveled to toy fairs. In 1980, the cube was renamed Rubik's Cube, and became extremely popular in the United States and elsewhere. Between 1980 and 1982, Rubik sold 100,000 of his puzzles. Erno Rubik is still very involved in creating new puzzles and games, but he will be forever known for the creation of what many would call the best three-dimensional puzzle ever.

Reference:
www.Rubiks.com
http://cubeland.free.fr/infos/ernorubik.html

Heather Davis is a student at Texas State University-San Marcos. She is a political science major and is also pursuing a teacher certification.

1. Find the value of $$\frac{1}{2}+\left(\frac{1}{3}+\frac{2}{3}\right)+\left(\frac{1}{4}+\frac{2}{4}+\frac{3}{4}\right)+\left(\frac{1}{5}+\frac{2}{5}+\frac{3}{5}+\frac{4}{5}\right)+\ldots+\left(\frac{1}{100}+\frac{2}{100}+\ldots+\frac{99}{100}\right)$$

2. Becky and Jacob have marble collections. Becky has twice as many marbles as Jacob. One-third of Becky's marbles are blue. Half of the marbles in their collections combined are blue. What fraction of the marbles in Jacob's collection are blue?



3. What is the largest number of consecutive positive integers that add up to exactly 1,000?

4. What is the next term in the sequence: 1, 4, 13, 40, 121, 364?

5. In a mathematical Olympiad, 100 students were given four problems to solve. The first problem was solved by exactly 90 students, the second by exactly 80 students, the third by exactly 70 students and the fourth by exactly 60. No participant solved all four problems. How many students solved both the third and the fourth problems?

6. How many integers are there from 0-2004 (inclusive) that contain at least one digit 2 but do not contain any digit 7?

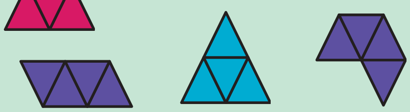
7. A clerk at a store owes you 25 cents change. The store is out of quarters, but has plenty of nickels, dimes and pennies. How many ways can he make the 25 cents to give you?

(Some of the problems above come from the 7th Primary Math World Contest.)

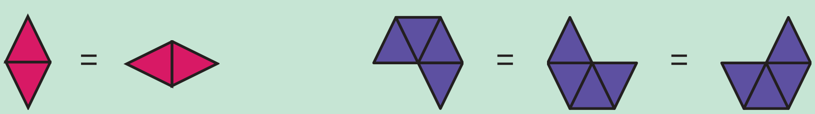
Ingenuity From an unlimited number of identical equilateral triangle tiles, we can put some of these tiles together to form different shapes, always having full sides touch and the corners meet. Then we find that:

using exactly 2 tiles, we can form only 1 shape, i.e. 

using exactly 3 tiles, we can form only 1 shape, i.e. 

using exactly 4 tiles, we can form only 3 shapes, i.e. 

Shapes are considered the same if they can be formed by a sequence of translations, rotations and/or flips. For example,



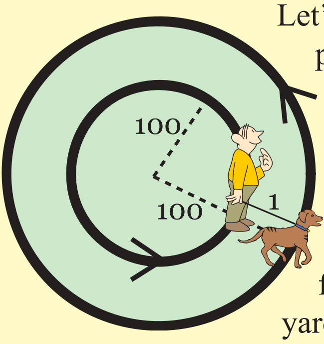
How many different shapes can be formed when we use exactly 6 tiles?

Answer: **1)** 2475 **2)** 5/6 **3)** 25 **4)** 1093 (multiply by three and add one) **5)** 30 **6)** 439 **7)** 12 **Ingenuity)** 12

WALKING THE DOG

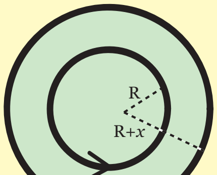
by Daniel Shapiro

I like to walk with my dog Phido on the path that goes around the nearby park. That dog is so well trained that he stays exactly one yard to my right at all times. Since I walk with the center of the park to my left, Phido's path is somewhat longer than mine. How much longer is it?

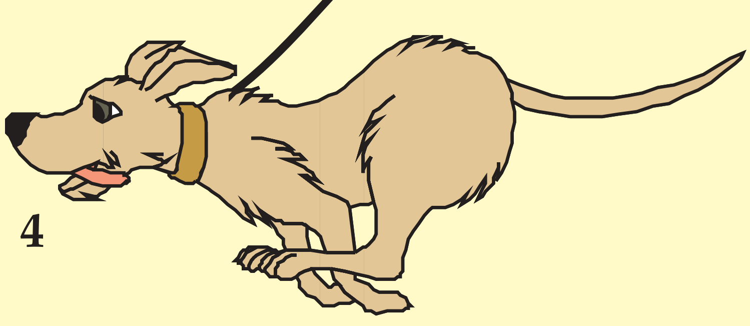


Let's think about the problem in a simplified (mathematical) world. Phido and I are represented by moving points and P is my path. For instance, if the path is a circle with a 100 yard radius, the total length L that I walk is the circumference $L = 2\pi\cdot 100$ or about 628.32 yards. Phido's path is a circle with a 101 yard radius, with total length $2\pi\cdot 101$ which is about 634.60. Then the difference in path lengths is about 634.60 – 628.32 = 6.28 yards.

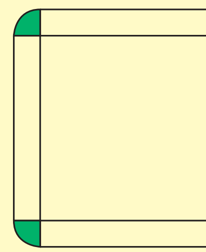
What about leashes of different length? For instance, if Phido walks x yards to my right, then Phido's path is a circle with radius $100 + x$ yards, and the distance he walks is the circumference $2\pi\cdot(100 + x)$, which equals $2\pi\cdot 100 + 2\pi\cdot x$. Since my path length is $L = 2\pi\cdot 100$, we find that Phido walks exactly $2\pi\cdot x$ yards farther than I do.

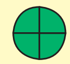


I was surprised to notice that the answer remains the same for circular paths of any size. If I go along a circle of radius R then Phido's path is a circle of radius $R + x$. Compare my path length $L = 2\pi\cdot R$ with Phido's path length $2\pi\cdot(R + x) = 2\pi\cdot R + 2\pi\cdot x$. The formula shows that Phido walks $2\pi\cdot x$ yards farther than I do. The answer doesn't depend on the size of the circle.

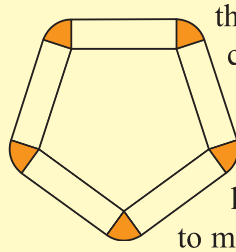


How about paths of other shapes, like a rectangle or hexagon? More generally, suppose my path P is a polygon, composed of several straight segments arranged in a roughly round shape. I walk with the center of the park to my left and Phido is always x yards to my right. When I go along a straight segment, the dog's path is a parallel segment of the same length. When I get to a corner, I stop and turn to face a new direction while Phido walks through an arc of a circle. Phido's path is made of straight sections (matching the straight sections of my path) together with several circular arcs of radius $\boldsymbol{x}$. (Remember, circumference equals two times the radius.)





For example, if my path is a square, those arcs make 4 quarter circles, which together form one full circle.



For the pentagon those 5 circular arcs also seem to fit together exactly to make a full circle.

Do the circular arcs always fit together perfectly? As I walk around the path suppose I hold a big arrow which points to my right at every instant. After I walk once around the park, the arrow ends up pointing in the same direction it did at the start (since I went in a loop). In the process the arrow turns through one full circle. As I walk on the straight sections, the arrow's direction doesn't change, but at each corner the arrow changes direction by exactly the angle of the circular sector at that corner. Therefore the sum of all the angles at the corners must add up to the total amount of turning done by the arrow. This says that those circular sectors do join together exactly to make one full circle of radius x.

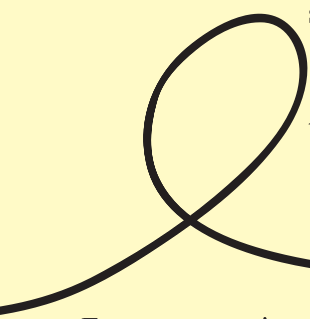
Consequently, the distance Phido walks equals the sum of the lengths of the straight segments and the lengths of those arcs. That equals my distance (from the straight segments) plus the circumference of that full circle of radius x, where x is the length of the leash. That is, Phido walks $2\pi \cdot x$ yards farther than I do. Whoa! That is the same result as before! The answers for a polygonal path around the park are the same as for a circular path.

Care has to be taken with paths that aren't nearly round. If the path zig-zags (with some left turns and some right turns) then Phido might trace some circular arcs backwards, and those arcs would have to be counted as "negative distances". Let's avoid those cases and stick to paths with only left turns.

What about smooth paths, like ellipses and more general ovals? Replace the given smooth path with a nearly polygonal one built from a large number of very short, straight steps (with a slight left turn at the end of each one). For that path, and leash length x, Phido walks $2\pi \cdot x$ yards farther than I do. Since we can make a polygonal path extremely close to the given smooth path, that formula holds for the smooth path as well.

This really is amazing: the difference in distances doesn't depend on the shape of the path! The answer depends only on the leash length x, and on the fact that I walk around once, making only left turns as I travel.

What if Phido stays on my left as I walk? In that case, his path is inside mine and his distance should be shorter than mine. How much shorter will it be? Let's reverse the roles and think of Phido walking around the park with me always x yards to his right. Then the discussion above (switching Phido and me) shows that my path is $2\pi \cdot x$ yards longer than his. That is, Phido's path is exactly $2\pi \cdot x$ yards shorter than mine.



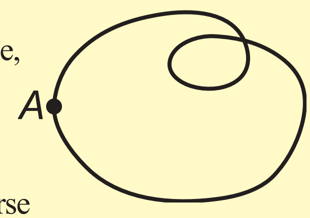
For you negative number fans, we can re-think this situation. Having Phido x yards to my left is the same having him $-x$ yards to my right. Also, Phido's path is $2\pi \cdot x$ yards shorter, which is that same as $-2\pi \cdot x$ longer, than mine. That results in the same formula as before: For my path as above, if Phido stays x yards to my right then he travels $2\pi \cdot x$ yards farther than I do, even if x is negative!

Our formula can be generalized even further. For instance, if I walk only part of the way around the park, how much farther does Phido travel? If I go halfway around I hope Phido travels $\pi \cdot x$ yards farther. But what does "halfway" mean here-is it half the distance, or half the angle turned?

Here are a few questions that come to mind when thinking about my faithful pooch:

1. The inner lane of a running track around a football field is one-quarter mile in length. The next lane is one yard further out. If the finish lines are the same, how should the starting line on the outer lane be adjusted to make the two lanes have equal length?

2. If I walk around the park twice, how much farther does Phido travel? How about n times around? What happens if I walk once around the park in the reverse direction? (Is that -1 times around?) What if I walk along the path pictured here, starting and ending at point A?

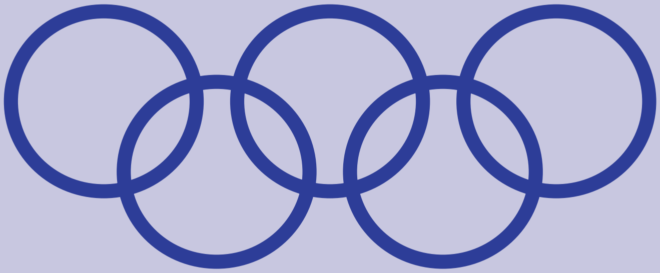


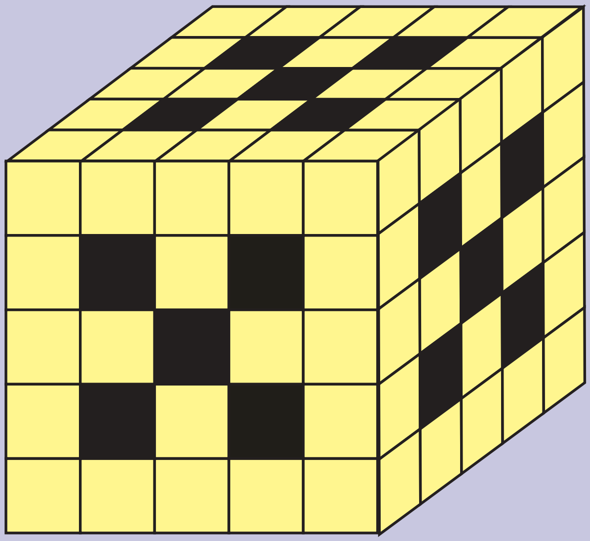
Daniel Shapiro is a professor of Mathematics at Ohio State University. He is also director of the Ross Mathematics program and a frequent contributor to Math Explorer.

Puzzle Page

Math Explorers: We want to print your work! Send your original math games, puzzles, problems, and activities to:
Texas Mathworks, 601 University Dr., San Marcos, TX 78666

In the diagram below, fill each of the nine regions with a different non-zero digit, so that the sum of all the digits in each circle is the same. What is the maximum value of this sum?





A 5 by 5 cube is formed using 1 by 1 by 1 cubes. A number of the smaller cubes are removed by punching out the 15 designated columns from front to back, top to bottom, and side to side. What is the number of smaller cubes remaining?

Word Search

Forwards or backwards, up, slanted, or down.
Where can the words in this puzzle be found?

Segment
Sector
Polygonal
Path
Radius
Arrangement
Geometry
Configuration
Puzzle

Y	N	M	I	L	A	S	D	Z	S	Q	W	E	R
U	T	O	R	Q	E	R	T	U	Y	I	O	P	S
O	P	F	I	G	J	U	I	E	W	Q	W	A	E
D	A	T	G	T	M	D	U	I	O	P	S	A	C
F	T	S	D	Q	A	B	S	E	G	M	E	N	T
E	H	Y	U	R	O	R	P	Z	X	C	V	N	O
W	T	R	W	M	O	V	U	O	L	I	O	J	R
I	A	W	A	R	R	A	N	G	E	M	E	N	T
Q	B	V	D	F	G	W	A	L	I	Q	R	T	Y
Y	R	T	E	M	O	E	G	P	O	F	L	J	K
Q	G	Q	A	O	U	Y	T	R	W	E	N	H	G
O	P	O	L	Y	G	O	N	A	L	D	W	O	N
H	D	U	L	J	O	P	U	Z	Z	L	E	K	C

S
S H S
S H T H S
S H T A T H S
S H T A M A T H S
S H T A T H S
S H T H S
S H S
S

How many different paths are there that begin with “M” and end with “S” to spell the word “MATHS” in this picture?

Bulletin Board

Discover Engineering Online

Did you know an audio or data track is recorded onto a CD-R (Compact Disc - Recordable) in a spiral pattern? Discs are written from the inside of the disc outward. On a CD-R you can verify this by looking at the disc after you've written to it. When full, the spiral track makes 22,188 revolutions around the CD, with roughly 600 track revolutions per millimeter as you move outward. If you "unwound" the spiral, it would be about 3.5 miles long! For more interesting information about careers in engineering, games and other resources check out www.discoverengineering.org/home.asp

Congratulations Siemens Westinghouse Winners!

The Mathworks Honors Summer Math Camp team of Araceli Fernandez, Yiduo “David” Wang and Hannah Chung was a National Finalist for the 2004 Siemens Westinghouse Competition. Their project in graph theory was mentored by Texas State University-San Marcos mathematics faculty member Weizhen Gu.

Mathworks Celebrates 15 years!

Texas Mathworks is a center dedicated to raising the level of mathematics abilities in all students by integrating innovative math curriculum, special programs and teacher training. To learn more about Mathworks, visit **www.txstate.edu/mathworks**

Summer Math Reading

Looking ahead to summer vacation? Here are a few books that might interest you:

“She Does Math! Real-Life Problems from Women on the Job” by Marla Parker, editor

“From Zero to Infinity” Fourth Edition by Constance Reid.

“In Code A Mathematical Journey” by Sarah Flannery.

“Mathematicians are People, too: Stories from the Lives of Great Mathematicians, Volumes 1 and 2” published by AIMS Education Foundation.

Math Joke

What did zero say to eight?

Nice belt.

Order Form (Please type or print)

Send to:

Name: ____________ Number of subscriptions: ______

Address: ____________ Method of Payment:

City, State: ________ Zip: ______ __Check (payable to Texas Mathworks)

School (optional): ____________ __Purchase Order number: ____________

Phone: ( ) ________ E-mail: ________ __Master Card __Visa

Amount of Payment: $ ______ Card Number: ____-____-____-____ Exp. date: ___/___

Cardholder's Signature ____________

Type of Subscription: Select one. All subscriptions include postage and handling for one year, 4 issues.

__Individual ($8) __Group ($6, min. 25 subscriptions) __School ($4, min. 100 subscriptions)

Yes! I want to subscribe.

Math Explorer magazine (aimed at grades 4-8) is published four times a year. An annual subscription is $8.00 for individuals, $6.00 for group purchases of 25 or more, and $4.00 for school purchases of 100 or more. For subscriptions, fill out the order form above or contact *Math Explorer* at the address, phone, or e-mail on page 2.

Cube Math

The Rubik's Cube has six faces with a different color on each face. Each face has nine tiles of the same color.

We can rotate any face to mix up the colors on the other faces. Notice, however, that the face opposite the face being rotated does not change at all. One question we could ask is: how many different arrangements can we produce? With careful counting, we can compute the answer by first examining the possible arrangements of the corner pieces, and then looking at how we can arrange the edge pieces.

The total comes out to equal 43,252,003,274,489,856,000. Wow! That is a lot of possible arrangements. This is because the total number of possible arrangements is the product of the number of arrangements of the corner pieces multiplied by the number of arrangements of the edge pieces. Counting this total number is a little tricky.

As a simple example of how tricky counting can be, can you count the number of ways of placing the letters A, B, C, D on a square? There are 4 places to put the A. After we place the A, there are 3 places to put the B. Then 2 places for the C, and finally the D must go into the last place. We then find that there are 4 x3 x 2 x 1 ways to place all 4 numbers. Do you see why?

Here are some of the ways:

A	B
C	D

B	A
C	D

C	B
A	D

B	D
C	A

Can you draw all 24 arrangements?

The challenge in our Rubik's Cube is to consider the 8 corner pieces to determine the number of possible positions for them. Each corner piece has 3 different sides facing out, so that will have to be taken into account. There are also the 12 edge pieces to consider and the possible positions for them. A design feature in the Cube makes it impossible to switch just 2 edge pieces, so that too will have to be taken into account.

With over 43 quintillion possible arrangements, the Rubik's Cube can certainly be a very challenging puzzle to solve!

Dear Math Explorers,

Longer days in summer mean more time to be outdoors. In our main article, our author looks at taking Phido, his dog, on a walk. Do you suppose both Phido and our author walk the same distance? Find out what determines the difference in the distance walked.

Summer is a great time for recreational math. A challenging puzzle, the Rubik's Cube, is featured in this issue along with a biography about its creator, Erno Rubik.

Thank you for joining us in another year of math explorations. Don't forget to send in your subscription renewals early for the fall issue!

Have a wonderful and safe summer!

Sincerely,

Hiroko K. Warshauer

Hiroko K. Warshauer, executive editor