Math Explorer

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Premier Issue

Math Explorer

Contents

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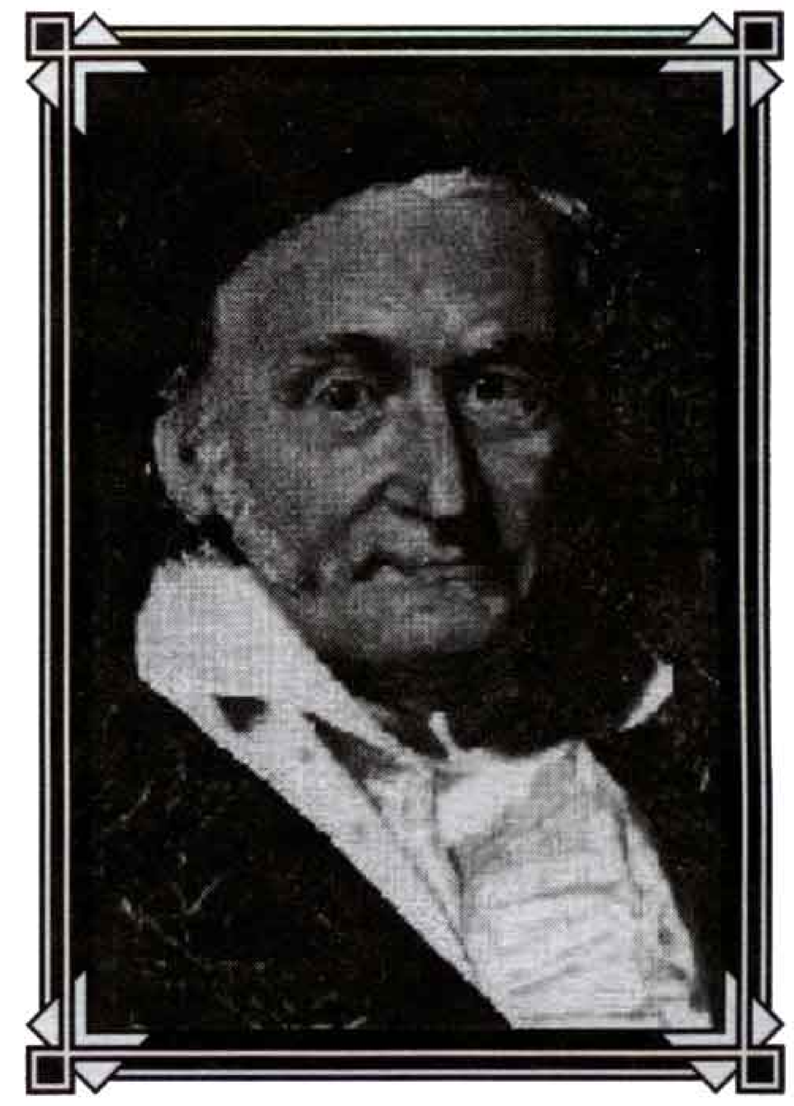
The Way Little Gauss Did It

This article is written by the internationally-known problem writers Sándor Róka, a math professor at the Bessenyei Gyorgy Teachers' Training College in Nyiregyhaza, Hungary, and Tivadar Divéki, a math teacher at the Grace Church School in New York City, NY .



The most famous mathematician wonder-child was Gauss (Carl Friedrich Gauss, 1777-1855, German mathematician). People called him Princeps mathematicorum, "The Prince of Mathematicians." He said about himself that he learned to count before he learned to speak. This is the story of how his talent was discovered.

One day a small town schoolteacher was very busy and desperately wanted a free period, so he assigned his students the problem of adding all the numbers from 1 to 100. He thought that it would keep the class busy for awhile. To his great surprise, in a few minutes little Gauss announced that the answer was 5,050. At first the teacher thought that Gauss was just fooling around, so he asked him, "How did you get your answer?"

Gauss responded, "1+100, 2+99, 3+98, and so forth, up to 50+51; all the sums are equal to 101. There are 50 such sums, so the total sum is 50 × 101 = 5,050."

The teacher realized that this was probably the most important event that had happened in his life, and he began to work with Gauss. Gauss respected his teacher and remained on good terms with him for the rest of his life.

Can you use Gauss' method to help solve some of the problems of the month? Think of some problems of your own that use Gauss' method.

TRIANGULAR NUMBERS

Directions

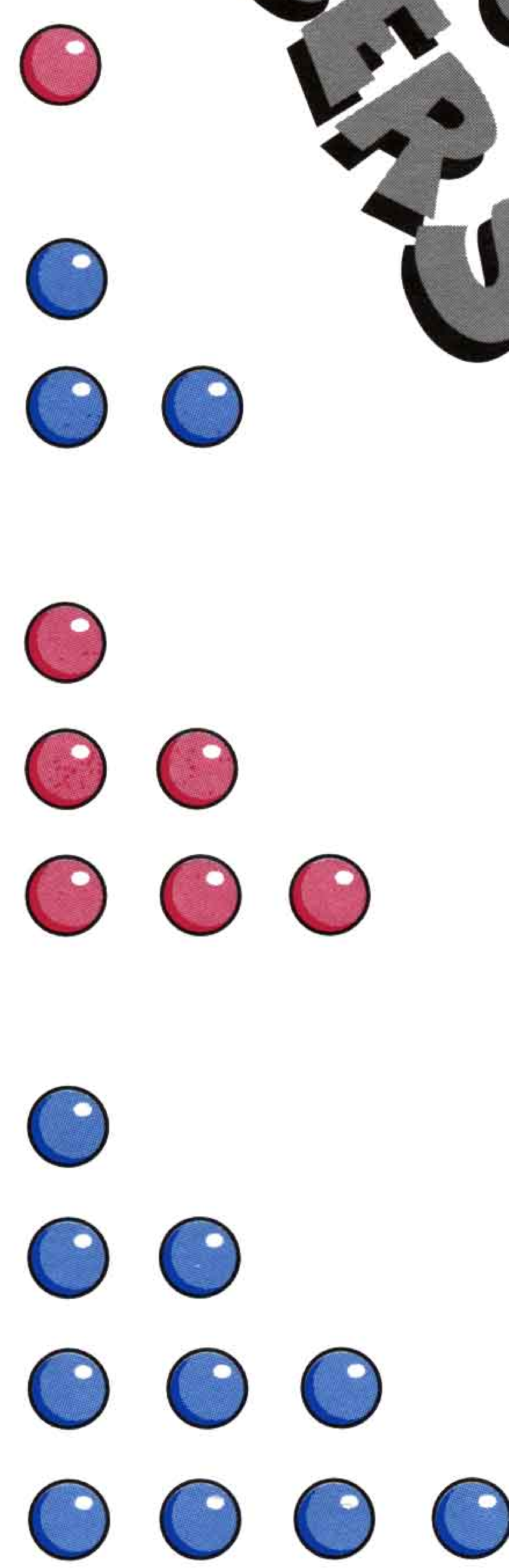
Send your solutions to Math Explorer! We will publish the best solutions each month. Note: The problems are not sequential and you should feel free to skip around.

1: What is the sum 1+2? What is 1+2+3? What is 1+2+3+4? Fill in the table below with the "triangular" sums. Do you see a pattern?

Number of Terms	2	3	4	5	6	7	8	9	10	20
Sum	3									

2: How much is 1+3? 1+3+5? 1+3+5+7?... What is the sum of the first 10 odd numbers. Fill in the table with the sums to find a pattern. Why does it work?

Number of Terms	2	3	4	5	6	7	8	9	10	20
Sum	4									



3: Divide the numbers 1, 2, 3, ..., 100 into two groups of 50 numbers so that the sum of the numbers in both groups is the same.

4: Divide the numbers 1, 2, 3, ..., 100 into five groups of 20 numbers so that the sum of the numbers in every group is the same.

5: What is the value of the following sum?

$$\left(\frac{1}{19}+\frac{2}{19}+\ldots+\frac{18}{19}\right)+\left(\frac{1}{20}+\frac{2}{20}+\ldots+\frac{19}{20}\right)+\left(\frac{1}{21}+\frac{2}{21}+\ldots+\frac{20}{21}\right)+\left(\frac{1}{22}+\frac{2}{22}+\ldots+\frac{21}{22}\right)$$

6: Is it possible to pick the + and - signs so that we would get a true equality in this equation?
$\pm 1 \pm 2 \pm 3 \pm \ldots \pm 20 \pm 21 = 1$

7: What is the smallest positive value of the following sum?
$\pm 1 \pm 2 \pm 3 \pm \ldots \pm 1998$

8: Ingenuity Suppose a Math Explorer lassos the moon with a circle of rope. How much rope would have to be fed into the noose to make it float one foot off the surface all around the moon?

Hint: Use the fact that the circumference, C, of a circle is 2π times the length of its radius, r. ($C = 2\pi r$)

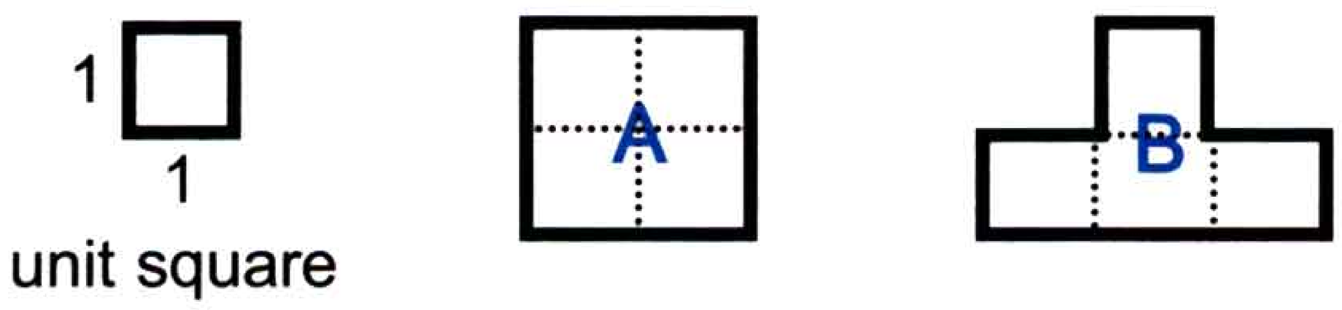
The Proof is in the Picture

By Anne Sung

Math is the language of science. People use math to explain how and why things work. This article has three parts. In part 1, we describe what area is and how to find the area of different shapes. In part 2, we introduce the alphabet of mathematics, variables, which enables us to write down general mathematical ideas. In part 3, we use variables and pictures to explain mathematical formulas.

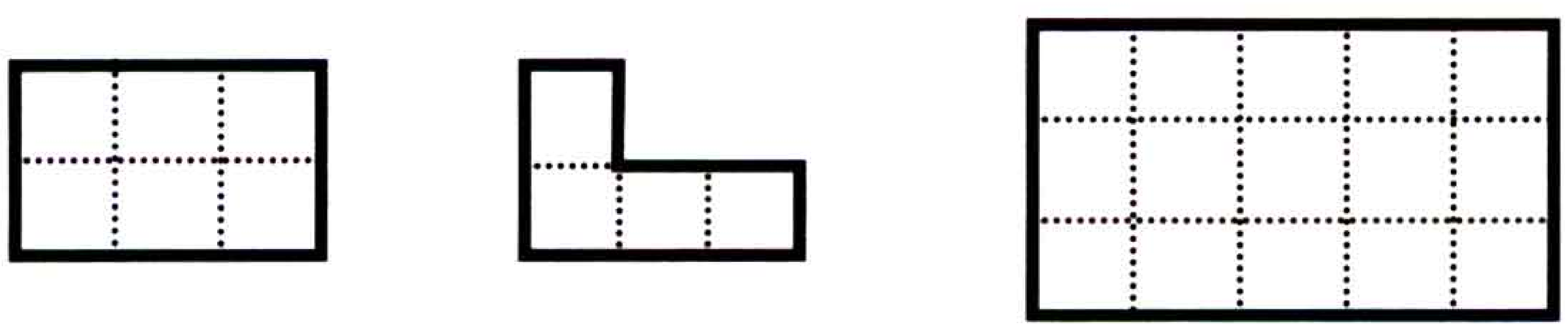
Part 1: Area

Area is a way of measuring how much space different shapes take up on the page. A unit square is a square with sides of length 1. *One way to determine the area of a shape is to count the number of unit squares in the shape.* The number of unit squares is the area of the shape. Shapes A and B below have the same area, 4, because they are each built from four unit squares.



area = ___   area = ___   area = ___

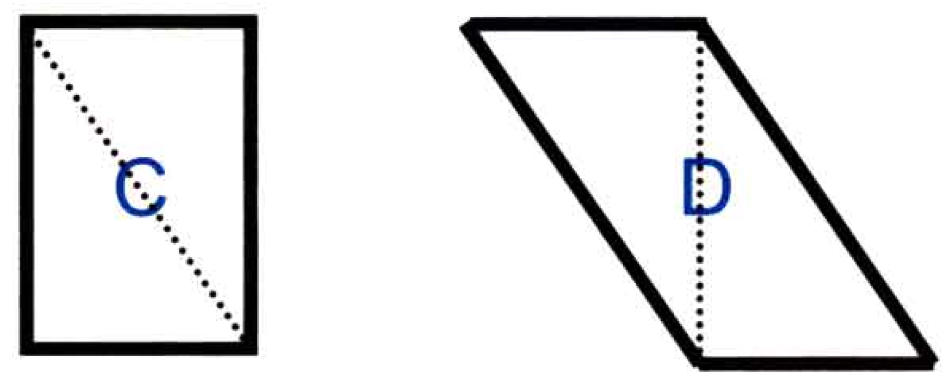
What are the areas of the other shapes below?



area = ___   area = ___   area = ___

Notice that *multiplying the width times the length of a rectangle gives the area of the rectangle.* Perhaps you already know why this is true. If a rectangle is drawn lying on its side, then its length is the number of columns and its width is the number of rows. Multiplying the width times the length gives the number of unit squares in the entire rectangle.

We can decide if two shapes have the same area by checking if they can be built from the same building blocks. Shapes C and D below are each divided into two triangular building blocks. The triangular blocks from one shape can be used to build the other, so the two shapes have the same area.

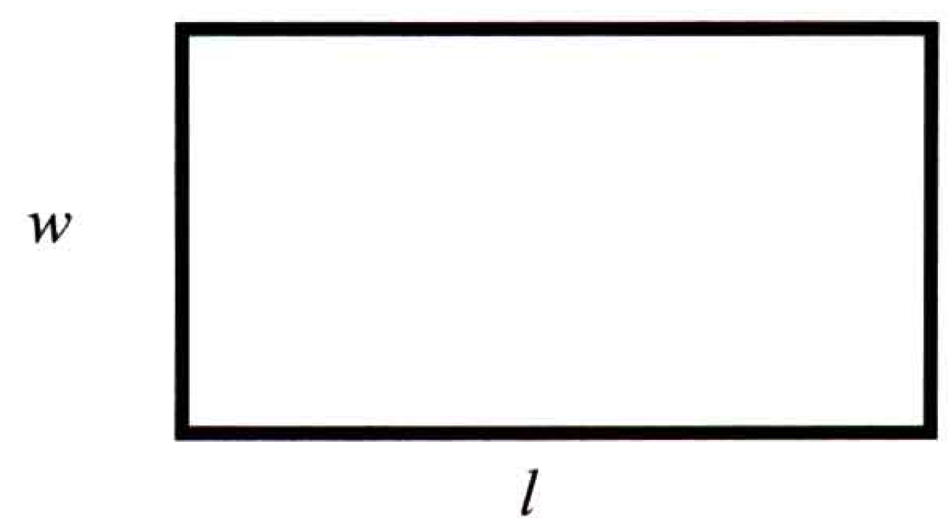


Can you arrange shape E below to make shape F? If you do this, then you will have shown that E and F have the same area.

Part 2: Algebraic Notation

In math, we can use letters to represent numbers in much the same way that we use words in English to represent objects. The word "chair" is English for something we sit in. The variable "x" could be used in math for the "number of chairs in a room." When a letter is used this way, it is called a variable.

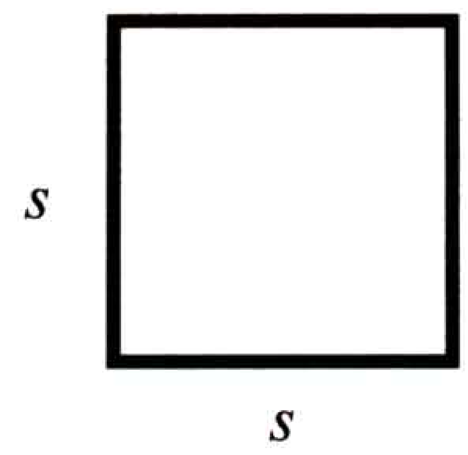
Variables help us explain ideas more clearly. For example, suppose you know that the shape below is a rectangle, but you don't know exactly what its length and width are. You could use the variable l to represent its length and the variable w to represent its width. A picture of the rectangle is below:



area = $w \times l = wl$

You can then say, "The area of this rectangle is "$w \times l$." We are now using variables to express the mathematical idea that "the area of a rectangle is the product of its width and length." *The multiplication sign is usually not written when two variables are multiplied together.* So you could simply say, "The area of this rectangle is wl."

If you use s to represent the length of a square's sides, how could you write the area of the square below?



area = _____

Multiplying a number times itself is called squaring the number. For example, "4 squared" means "4 × 4." Instead of writing "4 × 4" we write "4^2". Why do you think we call this "squaring a number?"

When we write 4^2 we call 4 the base and 2 the exponent. The exponent tells us how many times to multiply the base. For example, if the exponent is 3, then we have $4^3 = 4 \times 4 \times 4 = 64$ and we say "4 cubed equals 64". Can you find 2^3 ? What is the base and what is the exponent?

Part 3: Using variables to give picture proofs

Remember the saying "A picture is worth a thousand words?" Pictures can sometimes help show why a property of numbers is true. Here's an example, known as the *distributive property*:

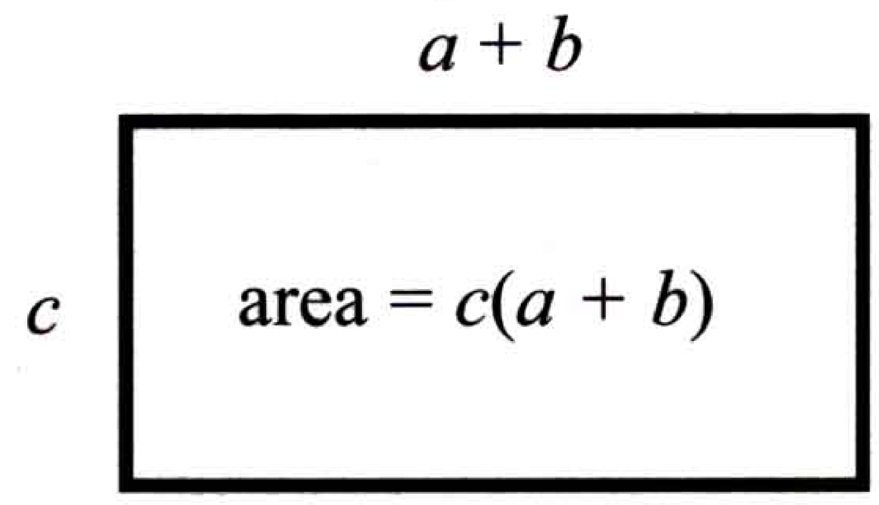
Pick any three numbers, a, b, and c. Then

(∗) $c(a + b) = ca + cb.$

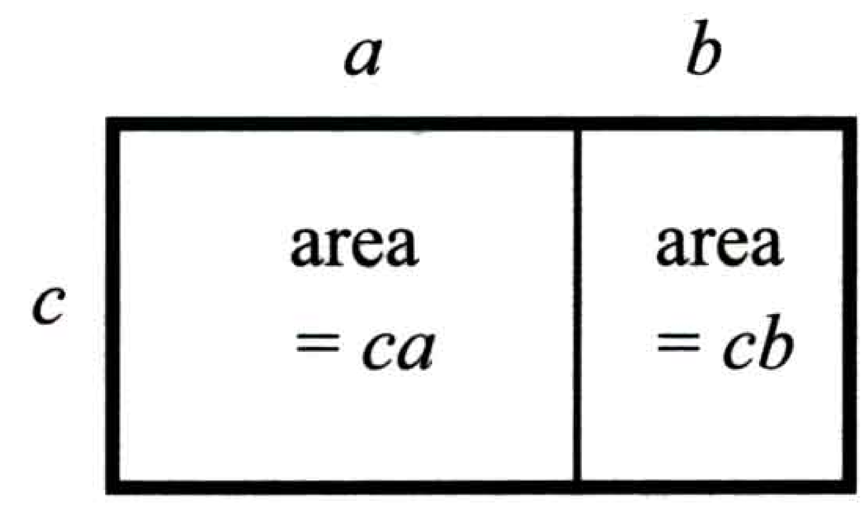
On the left hand side of this equation, to compute $c(a + b)$, we first add a and b, and then multiply the result by c. We always do the operation in parenthesis first, which is why we add before we multiply on the left hand side.

On the right hand side of the equation, to compute $ca + cb$, we first multiply to compute ca and cb, then we add the results together. We say that multiplication takes precedence over addition, since if there are no parentheses, we multiply before adding. The equation (∗) says that we get the same result either way. This is called the distributive law.

We can draw a picture that shows why the distributive property is true. We first draw a rectangle with sides of length $(a + b)$ and width c. Since the area of a rectangle is the product of its length and width, the area of this rectangle must be $c \times (a + b) = c(a + b)$, the expression on the left side of the distributive property.

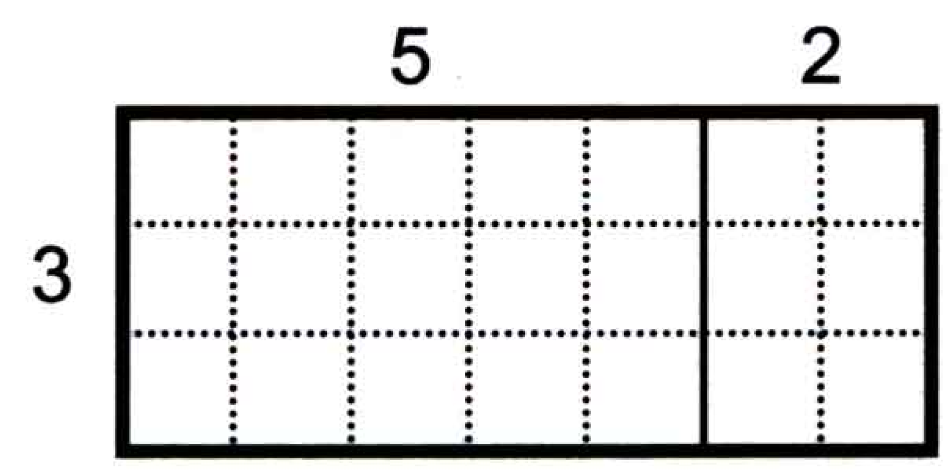


Another method for calculating the area of this rectangle is to divide the side of length $(a + b)$ into two parts, of length a and length b. This will split the rectangle into two pieces as shown below:



The area of these two rectangles, ac and bc, combine to give the area of the whole rectangle, so we have that $(a + b)c = ac + bc$. Next month we will see how pictures and area can be used to prove the Pythagorean Theorem, a famous theorem in geometry.

As an example, consider what the distributive law says if $a = 5$, $b = 2$, and $c = 3$. The picture is:



area = ___

We can check that the distributive law works:

$$c(a + b) = 3 \times (5 + 2) = 3 \times 7 = 21$$

$$ca + cb = 3 \times 5 + 3 \times 2 = 15 + 6 = 21$$

Try putting other numbers in for a, b, and c. Do the two procedures always give the same result?

Challenge: What picture can you use to show that for any numbers a, b, c, and d:

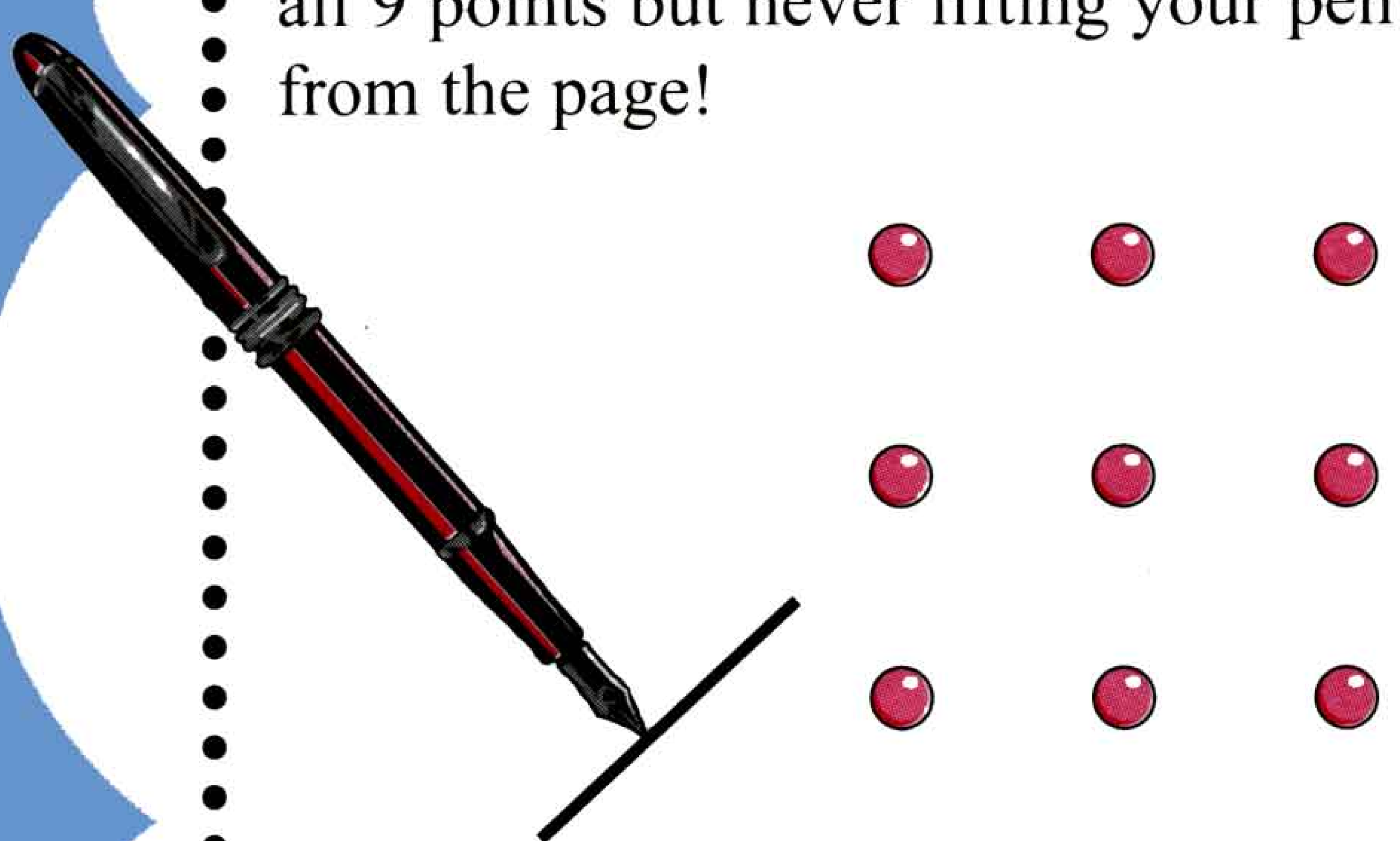
$$(a + b)(c + d) = ac + ad + bc + bd?$$

Puzzle Page

Math Explorers,
We want to print your work! Send us your math problems, puzzles, games, and activities. If we print them, we'll send you and your math teacher free ***Math Explorer*** *pens.*

The Dot Connection

Draw four straight lines, passing through all 9 points but never lifting your pen from the page!

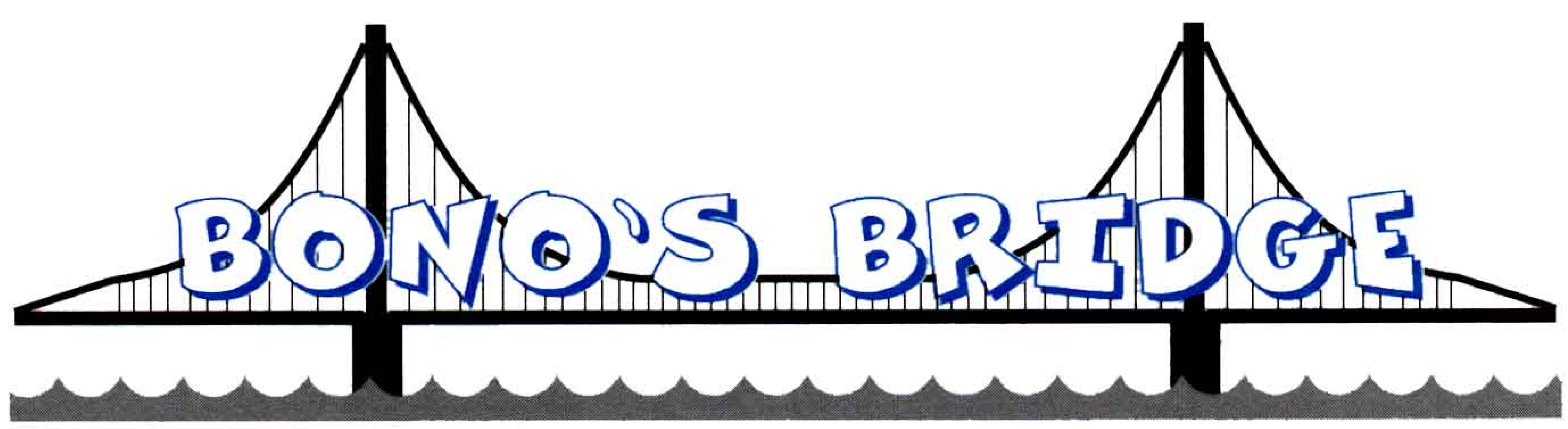


Jack and Jill

Jack: Time to fetch water again.
Jill: Not again! Remember what happened last time?
Jack: Don't complain, it's in the script. Anyway, I have a four-gallon pail here, and there is a seven-gallon pail beside the well.
Jill: Oh, that's lucky! We'll get just two gallons.
Jack: But there's no halfway marker on the four gallon pail to measure exactly two gallons!
Jill: Oh that doesn't matter. You can still get your two gallons using both of the buckets.

Tell Jack how he can use the two pails to fetch two gallons of water.

Bono's Bridge



Bono, The Edge, Adam, and Larry, the band members of "U2," must cross a bridge to get to their concert on time. Bono takes 1 minute to cross, The Edge 2 minutes, Adam 5 minutes, and Larry 10 minutes. The bridge can hold at most two people at a time. There is only one flashlight which must be used to cross the dark bridge, and when two people share the flashlight, they go at the rate of the slower person. How can all four band members cross the bridge in 17 minutes?

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Donna, Rio Grande City, Progreso, McAllen, Mission and Port Lavaca began Junior Summer Math Camps as part of the Southwest Texas (SWT)- South Texas Community College (STCC) consortium, sponsored by an Eisenhower grant.

Math Reader is another kids' magazine published by the SWT Math Institute for Talented Youth for elementary students. Check it out!

At this year's national **MATHCOUNTS** competition on May 15, the Wisconsin, Texas, and Massachusetts team placed first, second, and third respectively. See the **MATHCOUNTS** webpage at http://mathcounts.org.

SWT Junior Summer Math Camp and Teacher Institute

Pictured above are 86 students and 12 teachers who participated in the 1998 SWT Junior Summer Math Camp. Teachers began new Junior Summer Math Camps at Donna, Mission, Rio Grande City, McAllen, Progreso, and Port Lavaca under the auspices of Southwest Texas State University (SWT) and South Texas Community College (STCC) sponsored by a grant from the Eisenhower program. Over 125 students (grades 3-6) from the Rio Grande Valley participated and scored impressive results on the Orleans-Hanna algebra prognosis exam. In fact, many of these younger students scored above the national average for 8th graders!

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Math Notes

Dear Reader,

Welcome to our new magazine! Math Explorer is a magazine designed for intermediate students. I hope you'll have an exciting time exploring new math problems and sharing ideas with each other.

Math Notes is our Reader's Showcase. Write us with news from your school; about math events you've enjoyed; or with your own puzzles, activities, and problems. Please include:

• Your name • Your teacher's name • Any related pictures.

We'll publish as many letters as we can each month. I hope to hear from you soon.

Sincerely,

Max Warshauer

Max Warshauer