Math Explorer

MATH & NATURE

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Fibonacci who?

What's the angle of the Golden Spiral?

Those space-saving Hexagons!!!

Math Explorer

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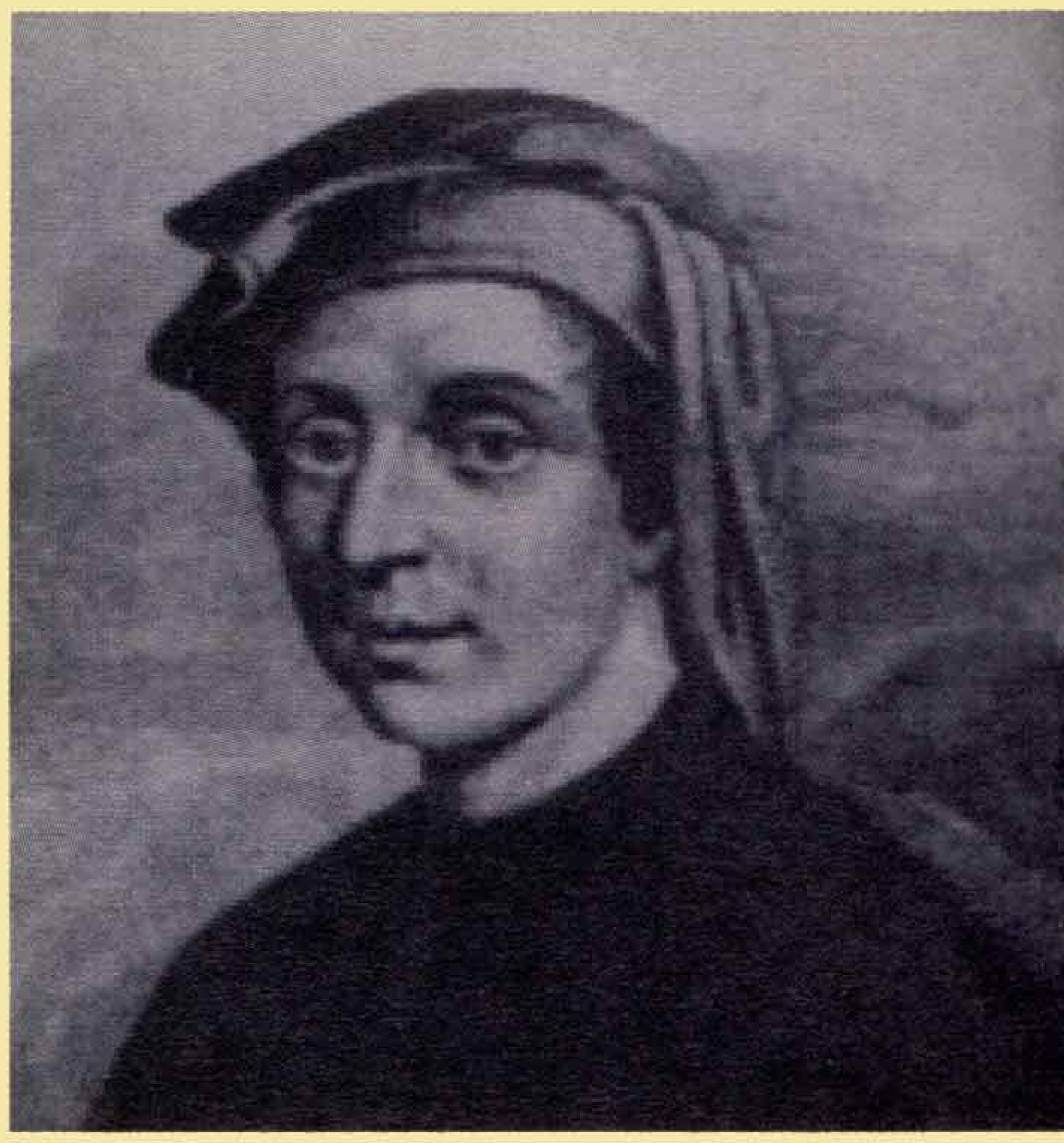
SWT

Leonardo Pisano Fibonacci



One of the greatest mathematicians of the Middle Ages was Leonardo of Pisa, known today as Fibonacci. He lived from 1170-1240, during which time construction of the Leaning Tower of Pisa began. Fibonacci travelled extensively throughout Europe, Africa, and Asia with his father who was an Italian diplomat. In his travels, Fibonacci was exposed to many ideas about the arithmetic, algebra, and geometry used in other countries.

In 1202, Fibonacci wrote the book *Liber Abaci*, or "Book of Calculating," which included many of the concepts and techniques that he had learned in his travels. He went to school in North Africa and learned mathematics in Arabic. Fibonacci's contributions include introducing the Hindu-Arabic number system we use today to Western Europe.

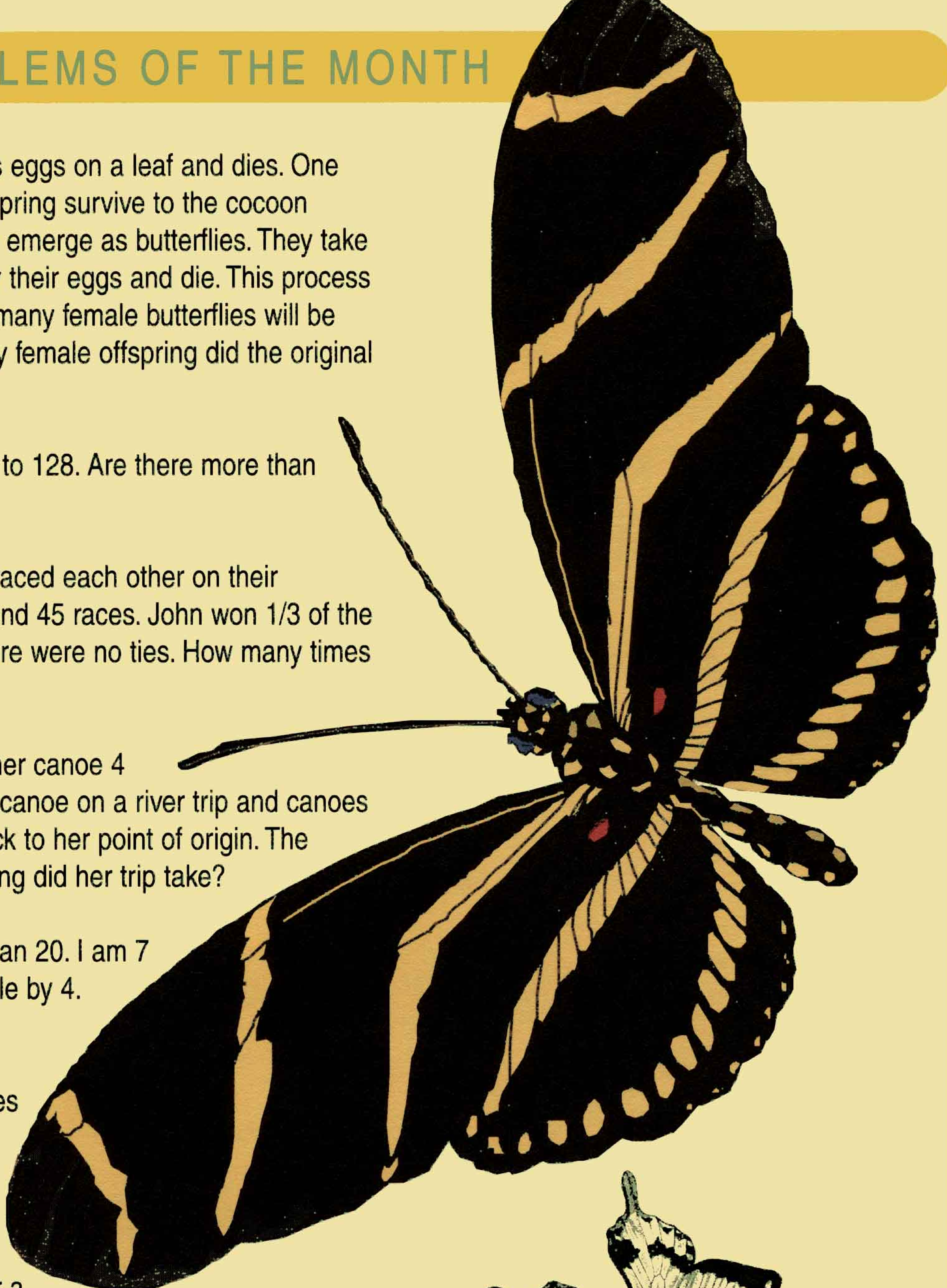
Fibonacci's mathematical skills were much admired in his days, and in 1225, the Roman Emperor Frederic II came to Pisa with a group of mathematicians to test Fibonacci's mathematical ability. One of the questions they asked Fibonacci was this: Find a fraction that is a square of another fraction (for example, $4/9 = (2/3)^2$) and that remains a square of a fraction when decreased or decreased by 5. Fibonacci found that the number 1681/144 (or $(41/12)^2$) works. Can you check that this really works?

Fibonacci is perhaps best remembered for an interesting sequence of numbers he introduced in *Liber Abaci*. We'll explore some of the fascinating features of these Fibonacci numbers in this issue.

Source: wahoo.esuz.K12.ne.us/MS/MathWebsites

by Hiroko K. Warshauer, who teaches mathematics at Southwest Texas State University. She enjoys music and the arts, as well as working with students on math puzzles, problems, and activities.

PROBLEMS OF THE MONTH



1. On January 1, a mature butterfly lays eggs on a leaf and dies. One month passes, and 3 of her female offspring survive to the cocoon stage. Another month passes, and they emerge as butterflies. They take one month to mature, and then they lay their eggs and die. This process continues for the rest of the year. How many female butterflies will be alive at the end of that year? How many female offspring did the original butterfly produce in that year?

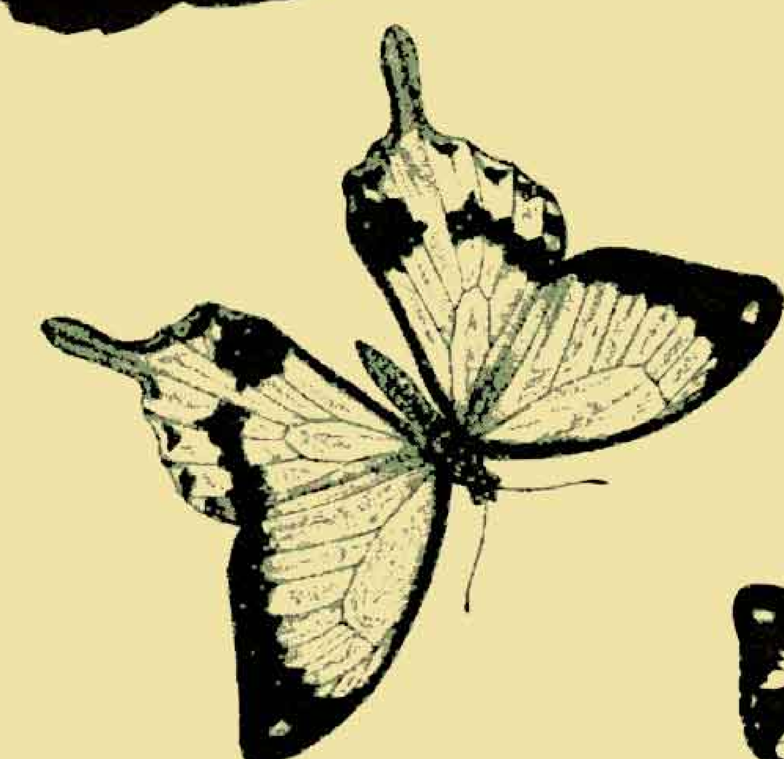
2. Find two prime numbers that add up to 128. Are there more than one pair of primes that work?

3. This summer, Juan, Jack, and John raced each other on their bicycles. They completed between 25 and 45 races. John won 1/3 of the time, and Jack won 1/4 of the time. There were no ties. How many times did they race?

4. In still water, Samantha can paddle her canoe 4 miles per hour. One day, she takes her canoe on a river trip and canoes 2 miles upstream, and then 2 miles back to her point of origin. The river flows at one mile per hour. How long did her trip take?

5. I am a positive whole number less than 20. I am 7 more than a prime number. I am divisible by 4. Who am I?

6. A farmer sells a pig for $90. He makes a 50% profit over the price that he had paid when he bought the pig. He also sold a sheep for $90 that same day. This sale amounts to a 50% loss. In the end, did the farmer make a profit or a loss with these two transactions?



7. What is the last digit of the number 9^{100}? What is the last digit of the number 2^{100}? Hint: Look for pattern.

8. In a large glass-enclosed garden, a scientist has 200 exotic butterflies. Each week, half of them die from old age or disease. The scientist, however, releases 40 new butterflies into the garden each week. How many weeks will it take for the number of butterflies to get below 100? Below 85? Below 50?

9. INGENUITY. Some mathematicians think a word is any string of letters with no space between them. What is the next word in the sequence below?

a, ab, aba, abaab, abaababa, ________________

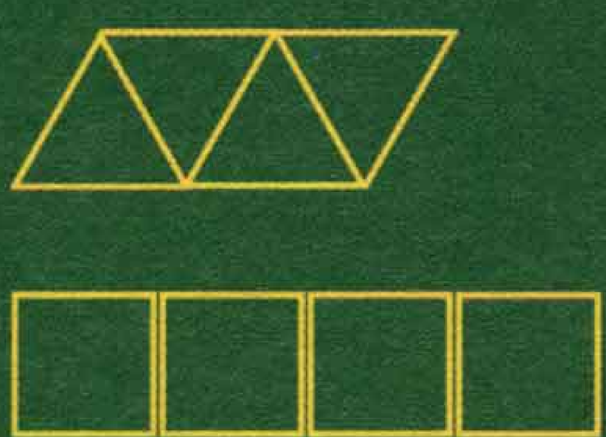
Send us your solutions! Every month, we will publish the best solutions on our website: www.mathexplorer.com. If we print your solutions, we will send you and your teacher free *Math Explorer* pens!

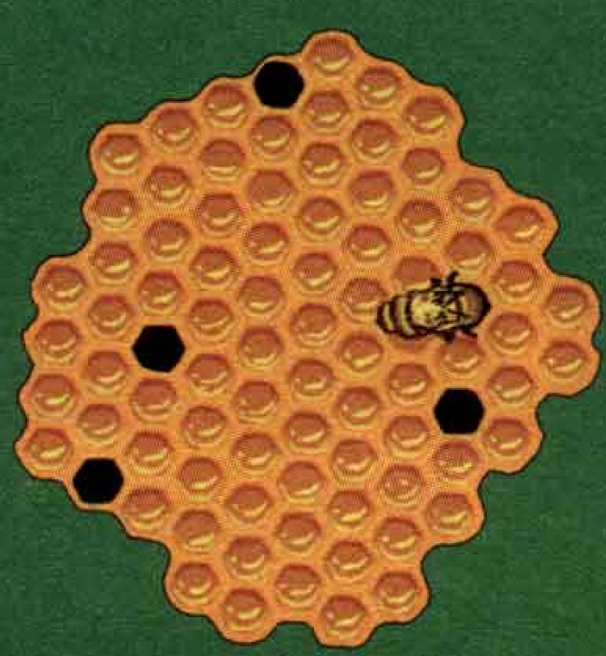
NATURAL CO

"Nature is the most fertile source of mathematical discoveries."   Jean Baptise Joseph Fourier

Patterns in Nature

Have you ever noticed the cells in a honeybee's hive? Each cell has a geometry that is hexagonal in shape. A **hexagon** is a six-sided polygon; the basic shape of a snowflake is hexagonal. But in the case of the honeybee's hive, we observe that the neat arrangement of cells are packed so that they don't waste any space. Hexagons in this arrangement are more efficient than triangles or squares, for example. Hexagons allow for less wax to be used to make each cell. This kind of efficiency in nature, which occurs with distinct patterns, has been closely analyzed by both scientists and mathematicians.



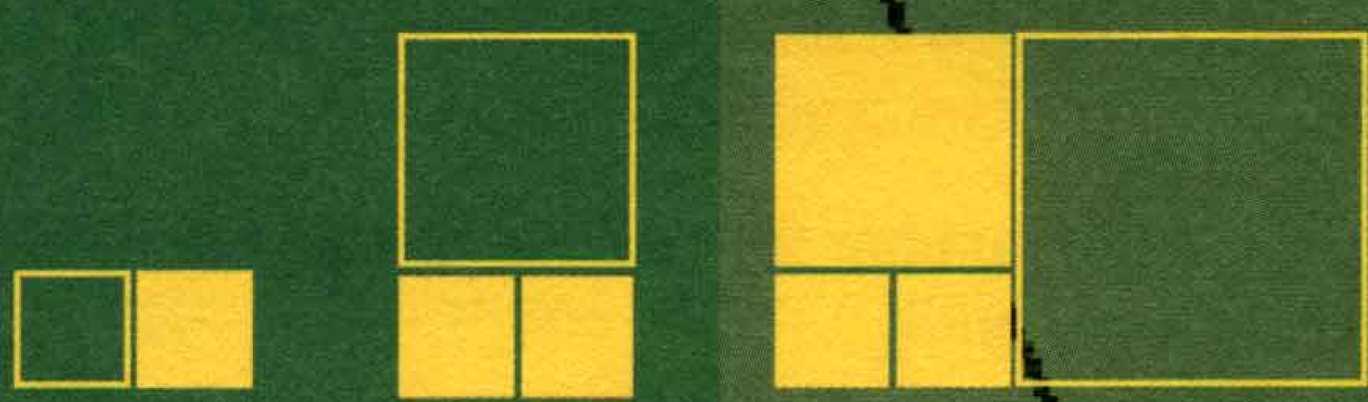


The interior of a chambered nautilus shell forms another geometric shape called a spiral. A **spiral** is a curve that winds about a fixed point from which it moves farther and farther away. The shell of a common snail also forms a spiral or a helix as it grows. Interestingly enough, botanists (scientists who specialize in the study of plants) have observed that the new cells of plants grow in a spiral form. The spiral can be described mathematically, however, with a sequence of numbers called the Fibonacci sequence. In 1993, this spiral pattern was studied by two French mathematicians, Douady and Couder who, in studying the efficient growth pattern of leaves and seeds, found that the spiral corresponds to a single angle. This very special angle is called phi, or the golden angle—it measures 137.5 degrees.

Fibonacci Numbers

What are the Fibonacci numbers? This is a case where this unique sequence of numbers is best explained visually, if we look at the incremental growth pattern of squares and rectangles. Follow the easy steps below to get the Fibonacci sequence.

Begin by drawing two squares of unit length 1 side by side. Place a square of dimension 2 by 2 above the 1 by 1 squares. To the right of this construction, draw a 3 by 3 square.

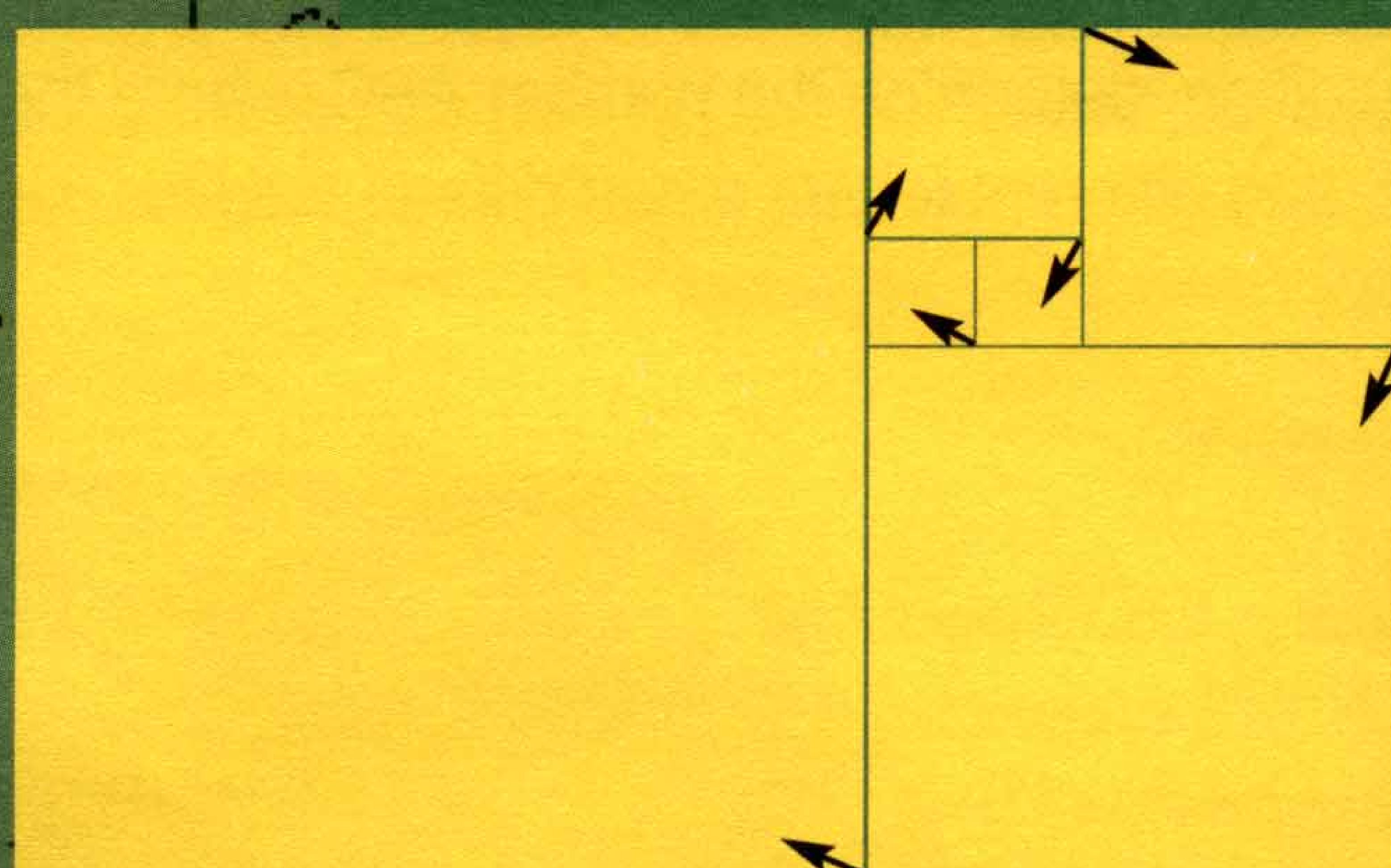


Now draw a 5 by 5 square beneath the whole arrangement. To its left, draw a square 8 by 8 square. As you can see, this construction process can be continued infinitely in a clockwise direction. And if you connect the opposite vertices of all the

ONNECTIONS

by Hiroko K. Warshauer

squares that you have drawn, the golden spiral is revealed.



An important observation to make is that each new square in this construction process has sides of length equal to the sum of the lengths of the previous two squares. With this in mind, we can determine the size of the next square by adding the last two that you have drawn. So, because 5+8=13, the next square has a side of length 13. We can generate some more dimensions: 8+13=21; 13+21=34; etc. This gets us to the sequence of numbers:

1, 1, 2, 3, 5, 8, 13, 21, 34 ....

This sequence of numbers is called the Fibonacci sequence, named after the famous Italian mathematician, Leonardo Pisano Fibonacci. The Fibonacci sequence is an infinite sequence (it continues without end).

Now that you have mastered the principle of this sequence, what is the twentieth Fibonacci number? What about the 100th Fibonacci number? You'd be right in saying that it would be the 98th and 99th numbers added, but exactly what is that number?

Fibonacci Numbers in Nature

Fibonacci first wrote about the interesting sequence of numbers when he posed a problem about the number of rabbits that result in the breeding process. The sequence of numbers he observed were later called the Fibonacci numbers. Since then, similar observations have been made in nature. A sunflower, for example, contains small disc flowers at its center and large ray flowers at the periphery. The disc flowers open from the outside towards the inside and form a spiral pattern that follows the Fibonacci sequence. Similarly, the pinecone's scales grow in various spiraling arrangements that also follow this sequence. Fibonacci numbers also appear as the number of petals on flowers: An iris has three petals; a wild geranium blossom has five; the delphinium flower has eight; and the corn marigold has thirteen. Amazingly, daisies have been found to have 34, 55, or even 89 petals.

Can you find other places where the Fibonacci sequence appears?

Puzzle Page

Math Explorers:

We want to print your work! Send us original math games, puzzles, problems, and activities. If we print them, we'll send you and your math teacher free *Math Explorer* pens.

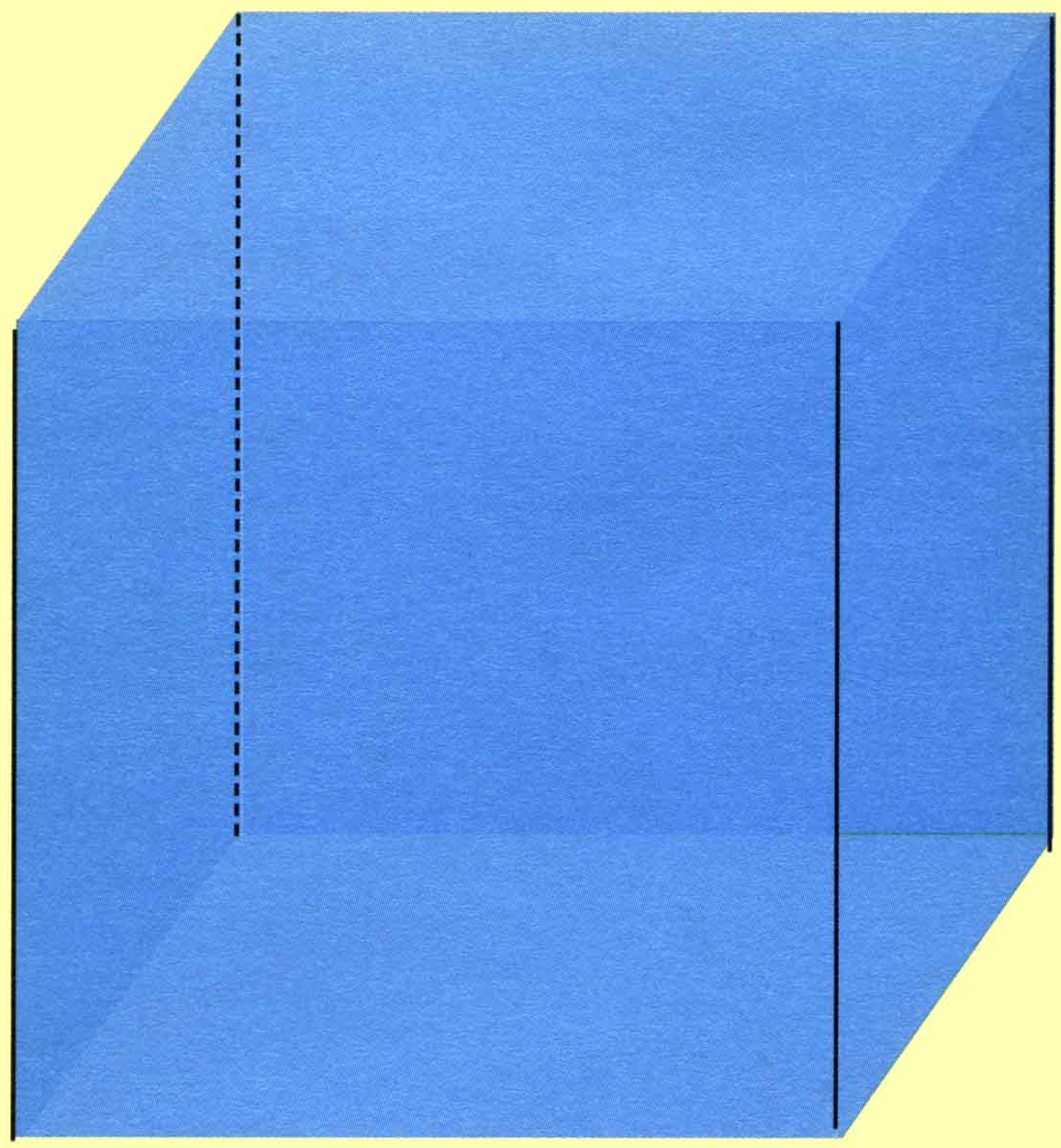
Adding Alphabetically

$$\begin{array}{r} AAAA \\ BBBB \\ +\underline{CCCC} \\ BAAAC \end{array}$$

From this addition, determine the number ABC, where the same letters mean same digits, and different letters mean different digits

Cubic Slice

Cut the cube into 8 identical parts with 3 planes. Do the same with 4 planes.



What Next?

Continue the following sequence using the same logical pattern.

Bulletin Board

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Mathcounts is a national competition for 7th and 8th grade students. Over 28, 000 students participate each year. See their webpage at http://www.mathcounts.org for more information or write to: Math Counts, 1420 King Street, Alexandria, VA 22314

SWT Junior Summer Math Camp News

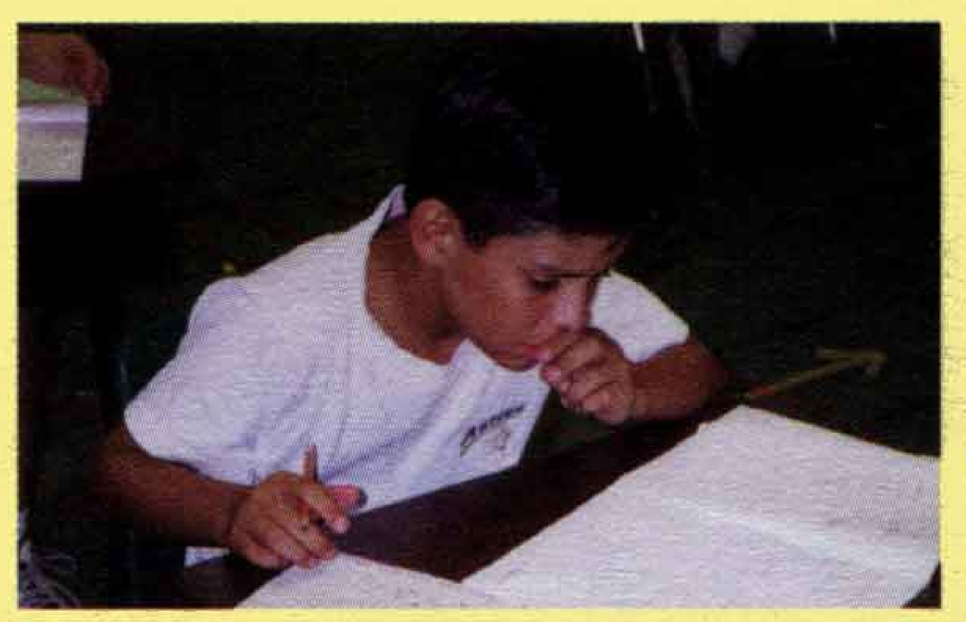
186 students and 8 teachers participated in the 1999 SWT Junior Summer Math Camp. This was sponsored by a grant from the Eisenhower program. Camps using the SWT model were held in Lockhart, Houston, Austin, Mission, Donna, Hidalgo, McAllen, Progreso and Port Lavaca.

Kudos

The Rockwell Fund, Inc. is helping to underwrite Math Explorer and Math Reader magazines. They also sponsor scholarships for students to attend the SWT Junior Summer Math Camp and SWT Honors Summer Math Camp! **Southwestern Bell Communications** gave a 4-year grant to help train teachers in mathematics.

Camp Snapshots!

Judy Carlin, camp instructor, assists Oscar Perez during the McAllen Jr. Summer Math Camp, 1999.



José Zamora reflects on a problem at the McAllen Jr. Summer Math Camp, 1999.

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Math Explorer

Welcome to *Math Explorer*!

We at *Math Explorer* are looking forward to sharing an exciting year with you. In this first issue we will explore mathematical ideas and connections to nature. Coming issues will examine the relationship between mathematics and other disciplines, such as architecture, astronomy, and sports. We have launched our new internet site www.mathexplorer.com. Visit us there for further discussion of articles and problems, including hints and solutions, as well as letters and solutions from fellow *Explorers*.

Join us for a full year of math exploration!

Sincerely,

Hiroko K. Warshauer

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