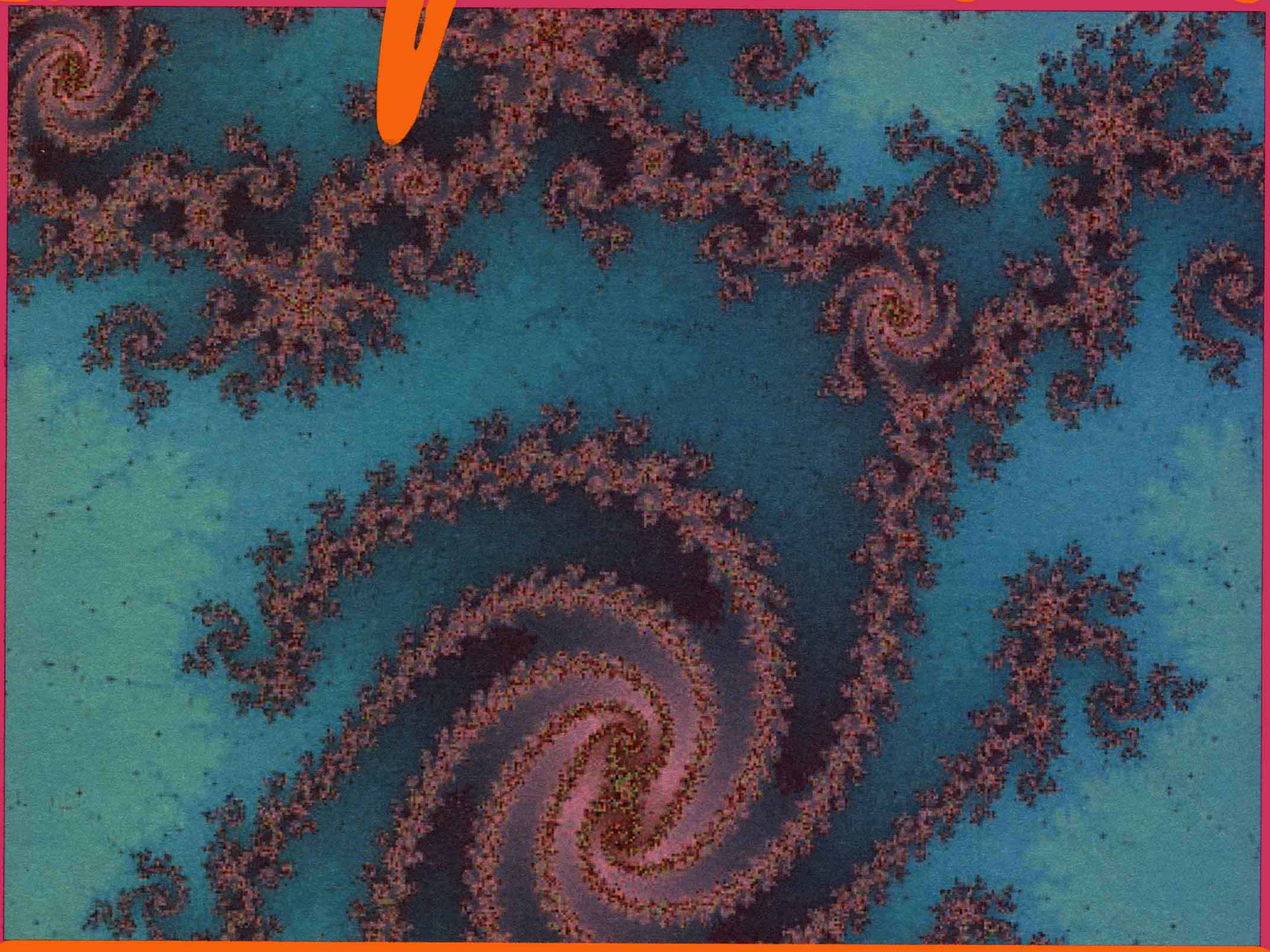
Math Explorer



FRACTAL MATH

Patterns in Chaos?

What has Mandelbrot Wrought?

Infinite Iterations...

Vol. 4.1 Fall 2001

Math Explorer

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Math Explorer
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SWT

Benoit Mandelbrot

Joyce Fischer

Benoit Mandelbrot was born November 20, 1924 in Warsaw, Poland. His first glimpse of mathematics came through two of his uncles while he was still a young boy. In 1936, his family immigrated to France, where one of his uncles tried to interest him in theoretical mathematics. Instead, Mandelbrot rebelled against this field, as he preferred to work with visual mathematics, especially geometry, and to apply his results to real world problems.

In the 1940s, he was accepted at the prestigious Ecole Normale Superiéure in Paris, entering as the number one student in the class, but his love of geometry and his fear and hatred for the controlling mathematics organization known as Bourbaki caused him to leave after only one day. He transferred to the Ecole Polytechnic in 1944, and after graduation moved to the United States, where after a visit to the California Institute of Technology he moved to Princeton's Institute for Advanced Study (with John von Neumann as his sponsor).

In 1958, he became a Research Fellow and a Research Professor at IBM's internationally known Watson Laboratories. The freedom, support, and computer graphics technology provided by IBM in the late 1960s encouraged him to establish models that were eventually applied to a variety of practical problems. His breakthrough came in 1979-1980 with the famous discovery now known as the Mandelbrot Set.

Mandelbrot's non-traditional reasoning method gave him the freedom to explore topics in a curious, uninhibited, non-conventional manner. His unique way of visualizing mathematics combined with computer-assisted graphics created a new field, which he named Fractal Geometry. Fractals, in turn, formed the underlying structure for a new interdisciplinary science, Chaos.

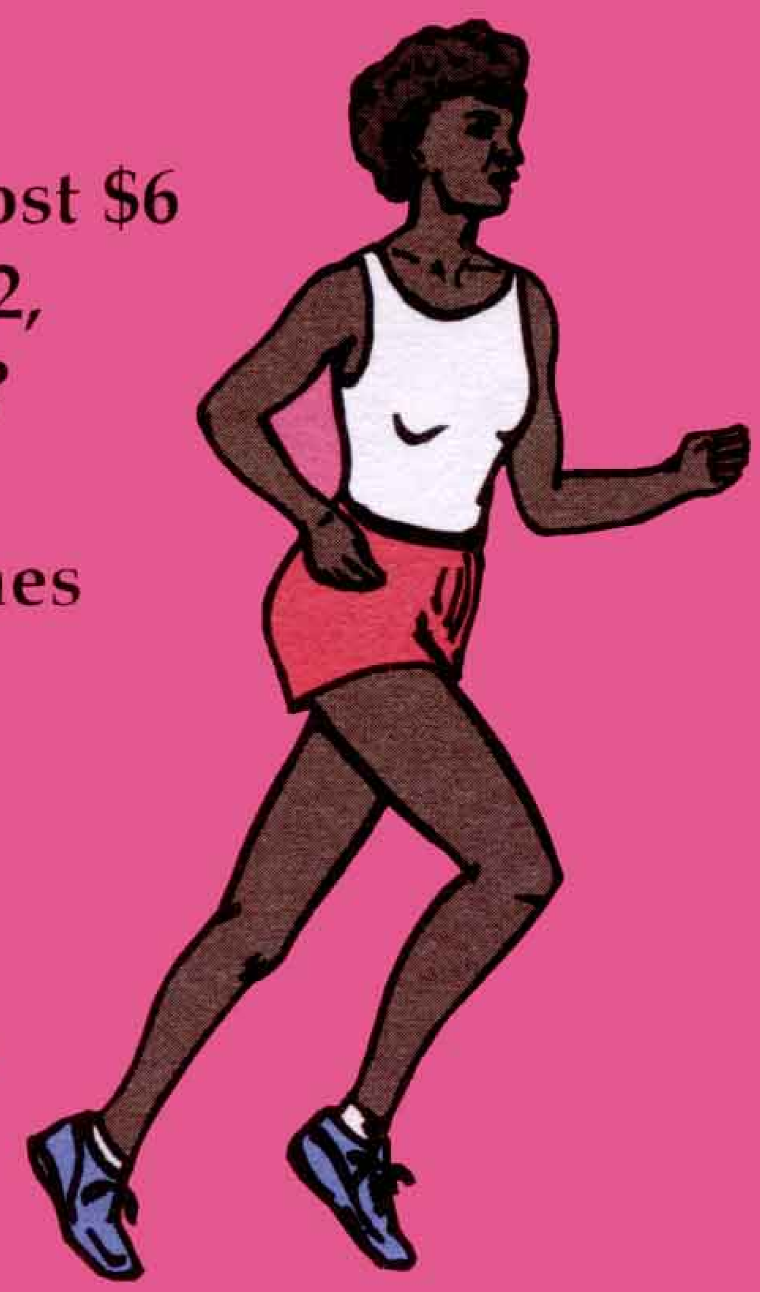
Just a few of the honors and awards Mandelbrot has received include the Barnard Medal for Meritorious Service to Science (1985), the Franklin Medal (1986), and the Wolf Prize for Physics (1993). He currently splits his time between the Yale University Mathematics Department and IBM.

Reference: Gleick, James. ***Chaos: Making a New Science***.

1. I have a number. I add 4. I divide my answer by 2. I multiply that answer by 3, then add 7. Now I have 43. What was my initial number?

2. Jason had a 300 batting average for his first 20 times at bat in the season, and a 400 batting average for his next 30 times [batting average = (# of hits) / (# of times at bat), without the decimal point]. What is his batting average for the first 50 times combined?

3. Max gave his 20 guests at the banquet a choice of a chicken dinner or a beef dinner. Each chicken dinner cost $6 and each beef dinner cost $7.50. If the total bill was $132, how many guests had chicken and how many had beef?



4. One of the sides of a rectangle with a 30-inch perimeter is 4 inches long. Find the area of the rectangle.

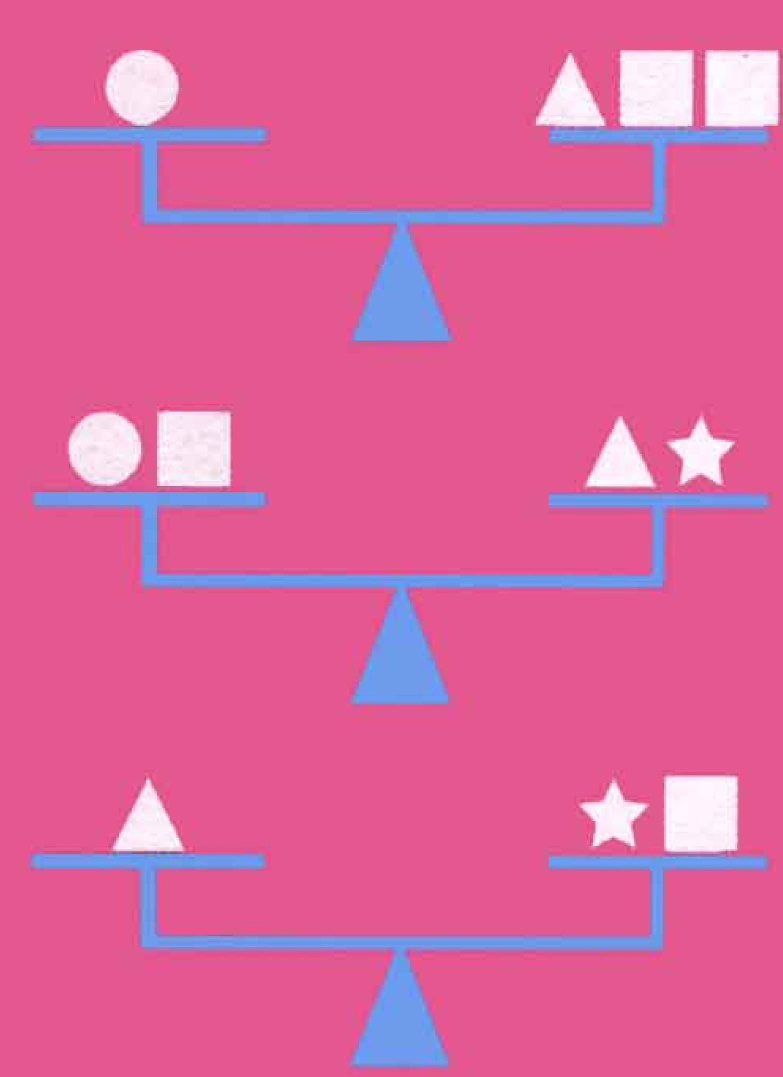
5. Mary goes for a run around the neighborhood every other day. Susan goes for a run every third day. John runs every Wednesday and Sunday. All three went running this Sunday. How many days will pass before all three run on the same day again?

6. How many 3-digit numbers do not have 3 different digits? (The number 667 is an example.)

7. Find the sum: 1 + 2 – 3 + 4 + 5 – 6 + 7 + 8 – 9 + ... + 97 + 98 – 99

8. Jill was expecting 4 friends to visit, so her mom baked some cookies. She planned to divide the cookies evenly among the 5 girls. Jane called to cancel. Mom noticed that the cookies would not divide evenly among the 4 girls remaining. She solved this problem by eating two herself. Then Jasmine also cancelled, so Mom ate another cookieto give each ot the remaining three girls an equal number of cookies. How many cookies did Mom bake?

9. The number 6 is called perfect, since the sum of its smaller divisors gives 6. (1 + 2 + 3 = 6). Find the next 2 perfect numbers.



Ingenuity: Suppose that we have weights that satisfy the following conditions: One ● will balance one ▲ and two ■. One ● and one ■ will balance one ▲ and one ★. One ▲ will balance one ★ and one ■. These conditions are illustrated by the balances to the right. How many ■ will balance a ●?

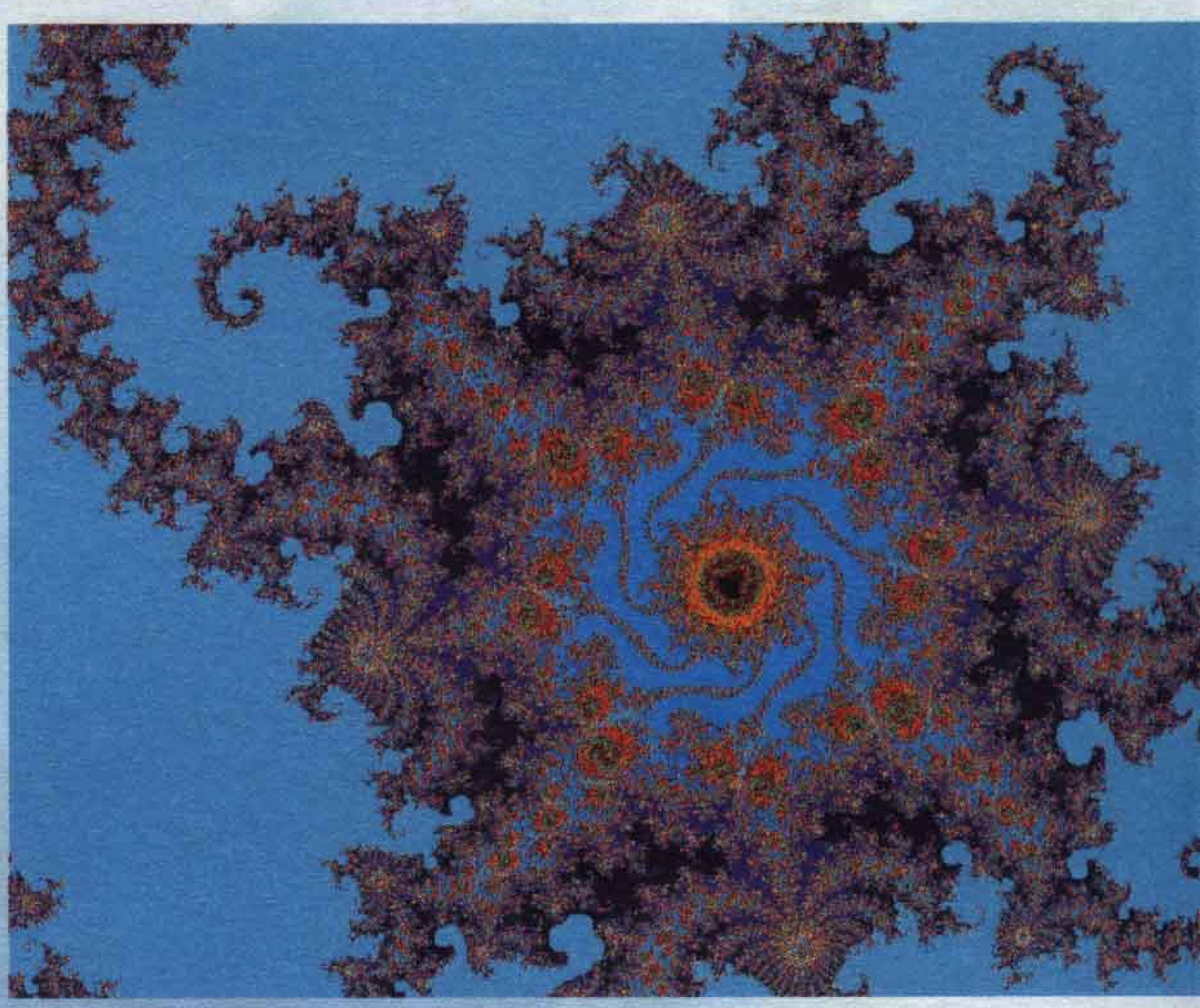
Answers: 1) 20 2) 360 3) 8 beef 12 chicken 4) 44 sq. inches 5) 24 days 6) 252 7) 1584 8) 30 9) 28, 496 Ingenuity) 6

Fractals: A Picture of Chaos

by Joyce Fischer

Is it possible that something can appear to be totally erratic and yet contain hidden patterns? For years scientists and mathematicians believed that there were two basic types of systems: ones that were predictable and ones that were random and unpredictable (chaotic). In 1980, Benoit Mandelbrot discovered that some systems, which were believed to be complex and random, were actually patterned and predictable. He called his discovery "fractals" after the Latin word fractus because of their fractional dimension and fragmented, irregular appearance.

Fractals describe patterns in diverse phenomena throughout the physical world where none were formerly detected. For example, in the growth of some flowers, broccoli and mammalian cells, in the economic behavior of prices on the stock exchange, in satellite views of a coastline, and in the complex behavior of turbulence. Fractals share three properties: *Self-Similarity*, *Iteration* (Feedback), and *Fractional Dimension*.



Self-Similarity is a picture within a picture, basically meaning that an object somehow contains a miniature of itself. Scale is a number which describes the relation of the size of one item compared to another. So, if you look exactly like a smaller-scale version of your father, then he has created a self-similar image—you. If your dad is six feet tall and you are three feet tall, then he is 2 times as tall as you or you are 1/2 as tall as him. In fractals, miniatures contain even smaller copies of themselves at all levels of magnification. Some real life examples are patterns in ferns, clouds, and veins (e.g., the human body).

Iteration occurs when you repeat a process over time. A real life example of iteration would occur if you decided to practice hitting golf balls. Usually the way this is done is to purchase a bucket of balls first, place one ball at a time on the tee, and hit each ball as far as you can. You repeat this process until the bucket is empty. Mathematically speaking, this form of iteration involves putting numbers into an equation, thus producing a sequence which can then be examined for patterns. For instance, we can substitute whole numbers into the equation $y = 2x + 1$ like this:

$y = 2(0) + 1 = 1$
$y = 2(1) + 1 = 3$
$y = 2(2) + 1 = 5$
$y = 2(3) + 1 = 7$, and so on.

We then try to see the pattern in the sequence of answers: each number is 2 more than the previous one.

With fractals the process for determining a sequence is similar with three important changes: the equation that you begin with looks different, the iterative process involves feedback, and you can only choose the initial conditions. Very slight variations in these initial conditions can lead to huge differences in the results. For instance, let's iterate a simple equation like $y = x^2 + c$. Remember, x^2 means to multiply x times x, and c represents the initial condition.

Let's choose $c = 1$ and start at $x = 0$. For fractals we use a special process called feedback iteration. With this method, each time you obtain an answer, you substitute that answer for your new x like this:

$y = 0^2 + 1 = 1$
$y = 1^2 + 1 = 2$
$y = 2^2 + 1 = 5$
$y = 5^2 + 1 = 26$.

As you can see, the numbers are getting larger and larger, so the sequence approaches infinity. Let's vary the initial condition slightly. We'll keep $x = 0$ but now let $c = 0$. So,

$y = 0^2 + 0 = 0$
$y = 0^2 + 0 = 0$, and so on.

This is an easy pattern: the only result is 0.

This time let's change c to -1. Then,

$y = 0^2 + (-1) = -1$
$y = (-1)^2 + (-1) = 0$
$y = 0^2 + (-1) = -1$
$y = (-1)^2 + (-1) = 0$.

For these conditions two values emerge, 0 and -1. As you can see, when we substitute whole numbers for the initial x value, use the feedback iteration process, and change only the initial value for c, the answers vary dramatically.

Dimension in space requires three numbers: one that represents longitude, one that represents latitude, and one that represents altitude. Mandelbrot looked at dimensions in a unique way, declaring that dimension was dependent on the observer's viewpoint. For instance, consider a ball formed by wrapping a continuous piece of yarn around itself. From across the room, the ball will look three dimensional (like a sphere) but from outer space, it will look like a point. This example represents one of the two types of fractals, the connected continuous type.

Fractals represent a geometric pattern that is the underlying structure for the "Chaos" of Nature. They give us a clear picture of what seems to be random irregular complexities in our natural world. These irregularities reveal consistent patterns, regardless of the scale used, when careful attention to dimension is considered.

Reference:
http://math.bu.edu/INDIVIDUAL/bob/index.html

Joyce Fischer teaches in the Department of Mathematics at Southwest Texas State University. Her research concerns the relationship between logical reasoning and students' learning of mathematical concepts.

Puzzle Page

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Word Search

Forwards or backwards, up, slanted, or down.
Where can the words in this puzzle be found?

Iteration
Fractal
Chaos
Similarity
Patterns
Sequence
Infinite
Dimension
Mandelbrot
Feedback

N	O	I	S	N	E	M	I	D	D	R	O	O	E
O	M	H	T	E	S	E	Q	U	E	N	C	E	A
N	D	I	G	E	T	E	W	U	I	O	T	E	F
F	O	B	A	V	R	R	A	G	O	I	X	R	U
E	S	E	Q	U	E	A	C	E	A	S	A	Y	N
E	T	F	E	T	Y	E	T	E	E	C	R	T	D
D	I	D	B	C	A	R	B	I	T	V	B	I	M
B	N	E	A	N	S	T	A	A	O	I	E	R	B
A	F	E	X	P	O	N	L	N	T	N	G	A	R
C	I	T	O	R	B	L	E	D	N	A	M	L	R
K	N	G	C	O	Q	E	I	O	T	B	A	I	K
Q	I	P	A	T	T	E	R	N	S	I	A	M	C
U	T	V	I	T	U	C	S	O	A	H	C	I	S
J	E	E	L	O	O	B	I	E	R	O	C	S	P

One container holds 8 pints and a second container holds 11 pints. Using only these two containers, how would you leave twice as much water in one container as the other?

Of 7 coins, 5 are identical and 2 counterfeits are slightly heavier. Using a balance of 2 pans, how many weighings are needed to tell which are the counterfeit coins?

$$\begin{array}{r} XYZ \\ +\ AB \\ \hline CDEF \end{array}$$

Each letter represents a different digit. What are they?

$$\begin{array}{r} XYZ \\ -\ AB \\ \hline BGA \end{array}$$

Bulletin Board

Texas Math Team Goes to Hong Kong

It was the first time Alyssa Ibarra, age 12, had ever been to Hong Kong, more than 8000 miles away from Texas. Joining Alyssa were three other twelve-year olds, Jennifer Schmerling, Yan Zou and Robert MacNguyen. These four young Texans comprised the San Marcos, Texas team at the 5th Primary Mathematics World Contest held in Hong Kong July 16-21, 2001. Among 40 teams from throughout the world, this was the only US team. The team received awards and recognition including first runner up in several divisions.

Max Warshauer, director of the Southwest Texas State University Mathworks and Sam Baegthe, coach for the Texas ARML team, conducted the team preparation and coaching during a two-week math camp held in June. Hiroko K. Warshauer, SWT math faculty, served as team leader and Laritza Diaz, a McAllen middle school math teacher, was deputy team leader at the contest. SWT Mathworks provided funding for the team travel.

For more information about the competition and camp call 512-245-3439

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Port Lavaca	Harlington	Zapata
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"7 days without math makes one weak."

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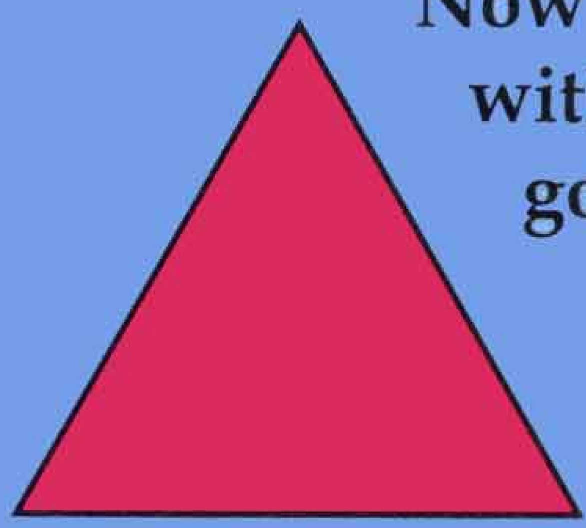
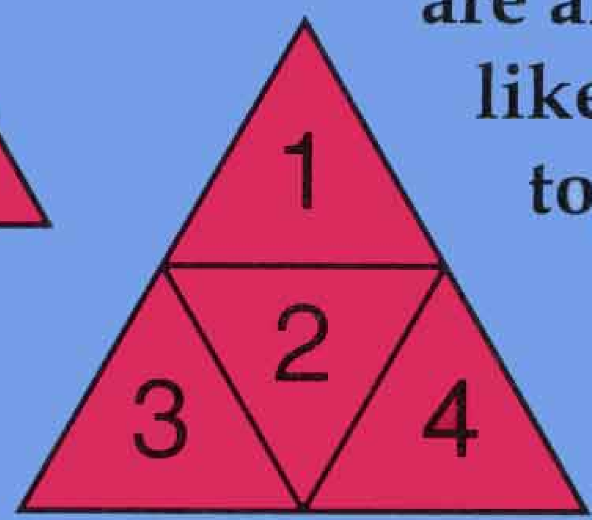
MATH ODYSSEY

An Exploration of Feedback Iteration and Self-Similarity

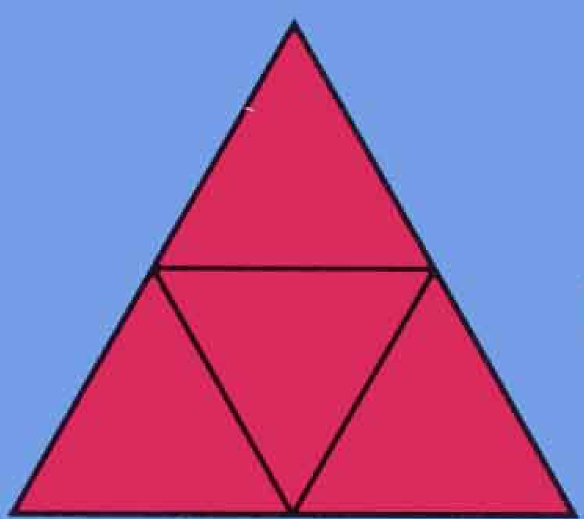
by Joyce Fischer



Now let's explore a geometric version of the feedback iteration process combined with the self-similarity property. We will begin with an equilateral triangle. The goal is to subdivide the original triangle into 4 smaller equilateral triangles that are all the same size. Each triangle that you draw should look exactly like the original one, except smaller. All 4 of the new triangles together should be exactly equal in size to the first one.

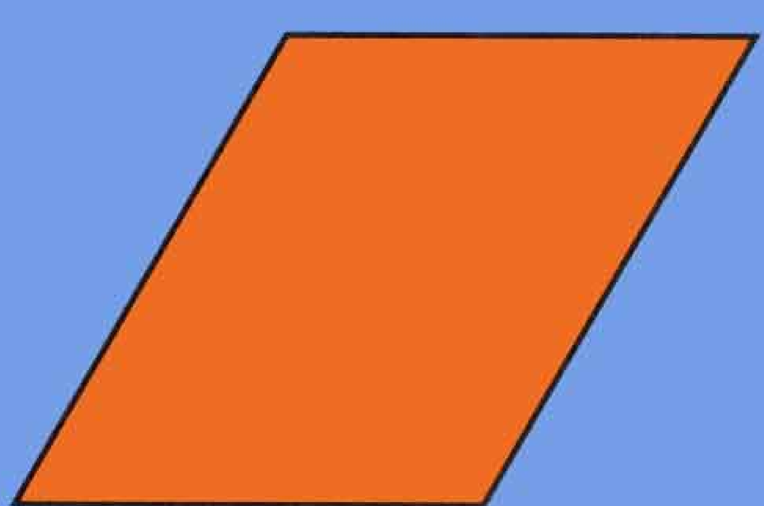


Notice that each new triangle is self-similar to the original triangle.

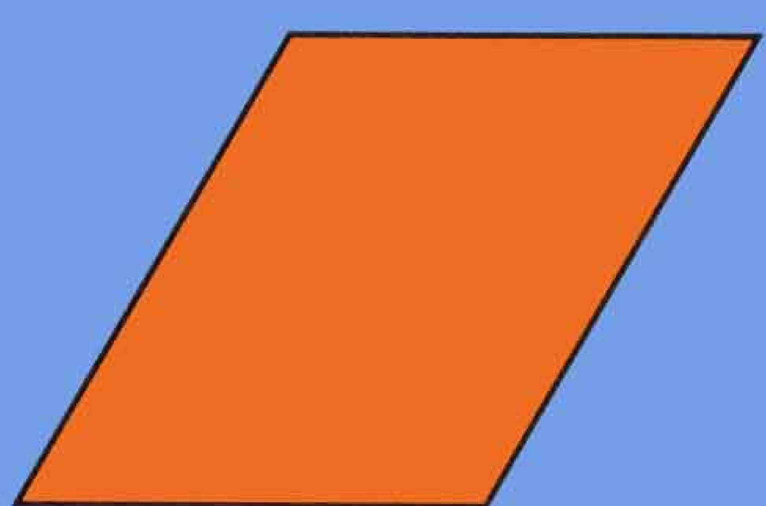


Using the figure to the left, can you use feedback iteration and self-similarity to subdivide each of the new equilateral triangles into 4 equilateral triangles? What do you know about the 16 new triangles you have created? Would it be possible to subdivide each of the newest triangles into 4 equilateral triangles? How long could you continue this process? Do you think this process could be done on an isosceles triangle?

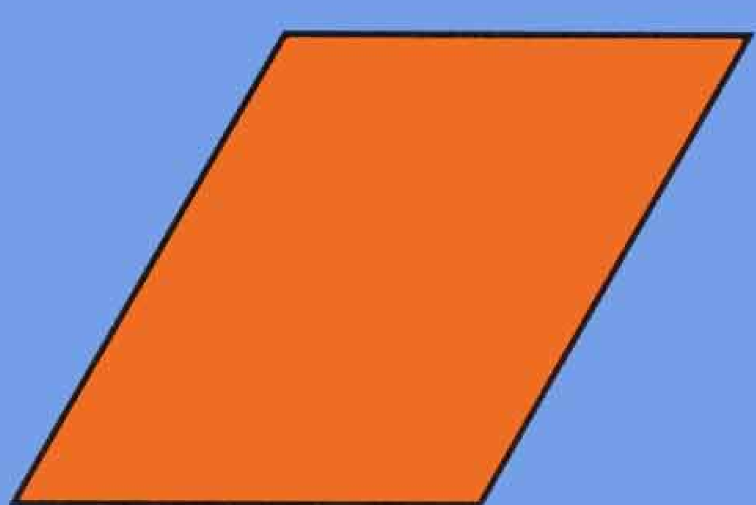
Let's try another one. Can you do at least 2 iterations on this shape?



Iteration 1



Iteration 2

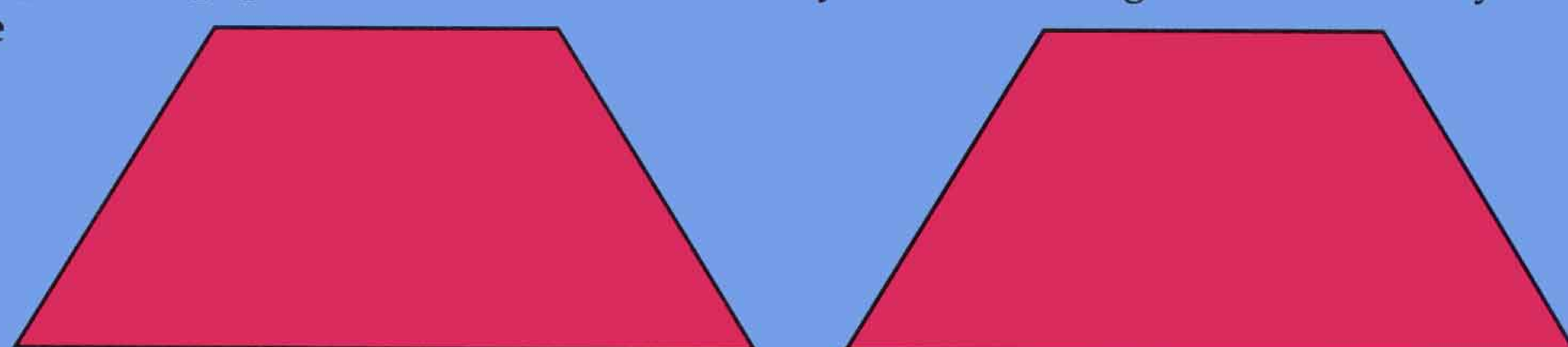


Great job!!

Next we will try to do a harder example. Can you do at least 2 iterations on the shape below?

Hint: you may have to trace this shape on a piece of paper, cut out 4 of them and try to fit them together so that they form a shape that looks exactly like the one you started with, only bigger.

Good luck!!



Reference: math.rice.edu/~lanius/fractals/WHY/

Welcome to an exciting new year of math exploration. This first issue of Math Explorer for the 2001-2002 school year contains an article on fractal geometry and the biography of Benoit Mandelbrot, the person who pioneered the study. Just the pictures alone are beautiful, but the mathematics behind it makes the topic all the more intriguing. We hope this will only be an introduction to a topic that you may wish to explore further.

Of course, the magazine also contains challenging math problems, fun puzzles, Math Odyssey and more. Please write to us with your math activities at school and comments at any time.

Sincerely,

Hiroko K. Warshauer

Hiroko K. Warshauer, editor