Math Explorer

MATH & ART

Vol. 2.5 February 2000

da Vinci designs!

Where has the Hexa-gone?

A Star is born!!!

Math Explorer

Contents

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SWT

Leonardo da Vinci

Leonardo da Vinci, regarded as one of the greatest artists and scientists of the Renaissance period, was born near Vinci, Italy on April 15, 1452. Throughout his lifetime, Leonardo worked as an architect, an engineer, an inventor, a painter and even a musician. Leonardo began his career as a painter in the city of Florence, where he apprenticed in a very prestigious artist's workshop. After thirteen years he moved to Milan and worked as a painter in a duke's court. He began his works **The Last Supper** and **Madonna of the Rocks** while in Milan.

One of the world's most famous and well recognized paintings, the **Mona Lisa,** which is displayed at the Louvre in Paris, France, was painted by Leonardo. Among the numerous sketches and manuscripts left by Leonardo are studies of machines, weapons, buildings and many detailed drawings of the human anatomy. He made plans for towns, worked on hydraulic engineering projects and studied optics as well. He had a keen scientific mind and when he performed experiments he carefully recorded his observations.

Leonardo used mathematical concepts such as the golden rectangle in his work. **The Annunciation**, one of his paintings, incorporates the golden ratio. He also used mathematical principles to convey movement, symmetry and order. Leonardo was interested in using proportion (the relative sizes of objects to each other) to control the way a piece would look to the eye. The proportions, he believed, could unconsciously create beauty. Leonardo was fascinated by order and rationality.

His abilities in mathematical design and mechanics and his talents in art were not separate areas in his life. An eye for design and aesthetics (what appeals to the eye) and a knack for

(continued on page 7)

by Laura Chavkin, who attends Yale University, and Hiroko Warshauer, who teaches mathematics at Southwest Texas State University.

PROBLEMS OF THE MONTH

Send us your solutions! Every month, we will publish the best solutions on our website: www.mathexplorer.com. If we print your solutions, we will send you and your teacher free *Math Explorer* pens!

Drawing Stars--First read the article on pages 4-5 for an explanation of problems 1-5.

1. The stars {5/2} and {5/3} look the same, but they can be distinguished by the directions they are drawn, clockwise or counterclockwise. But the {5/6} is identical with {5/1}, and {5/7} is identical with {5/2}. How about {5/5} and {5/0}? What is the general rule here?

2. Which stars are connected? How many separate pieces does the star {n/d} have? For example, {6/2} is made from 2 smaller stars so it has 2 pieces, {6/3} has 3 pieces. {6/4} has 2 pieces, and {5/0} has 5 pieces. What is the pattern?

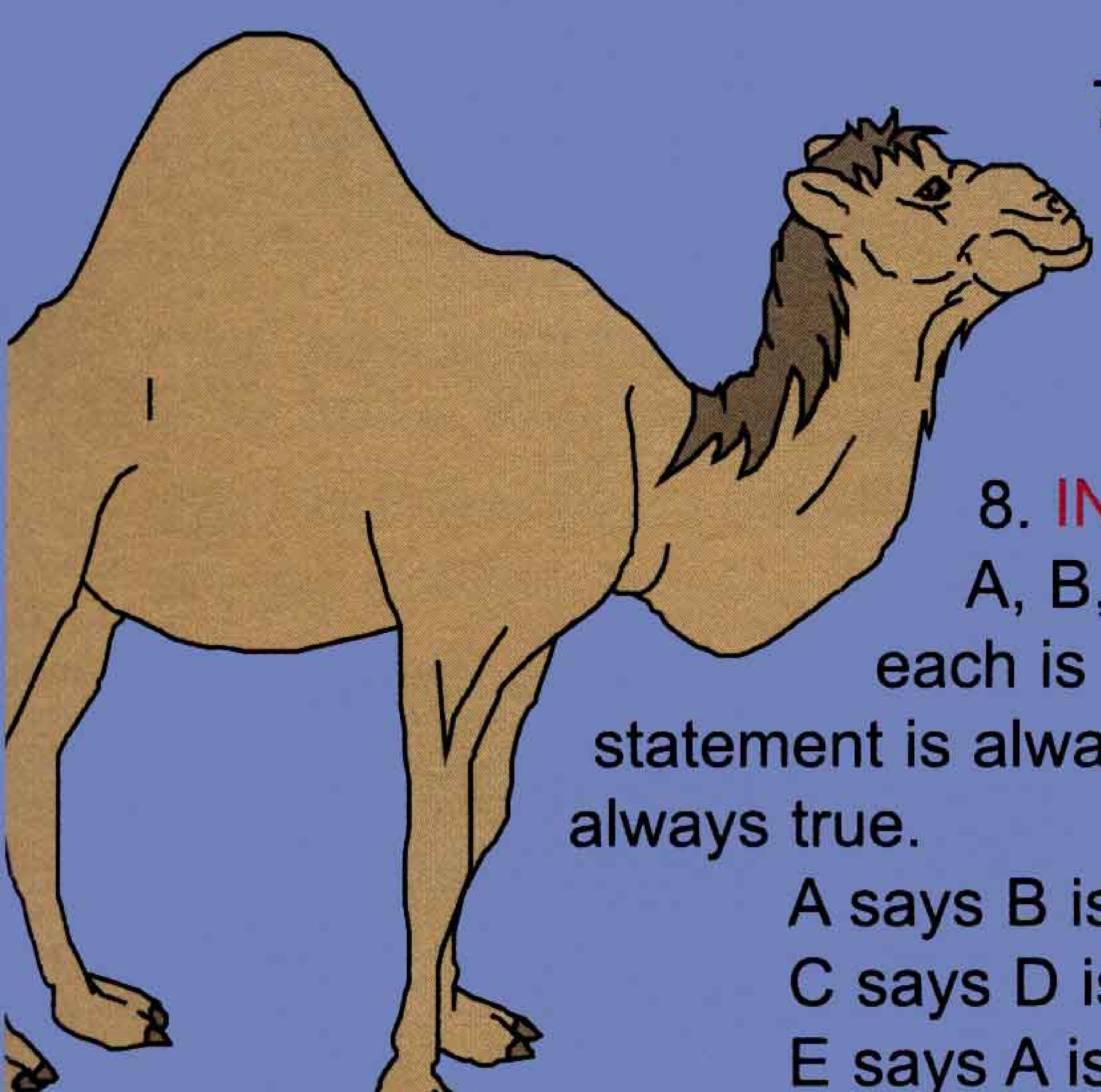
3. How is the star {n/d} related to the fraction n/d? "Reducing the fraction to lowest terms" seems to be related with drawing the associated star. For example, the fractions 6/2 and 3/1 are equal. How are the stars {6/2} and {3/1} related?

4. Suppose a line is drawn through the center of the n-star {n/d}. How many sides of the star does that line cross? (Avoid passing through a corner of the star.)

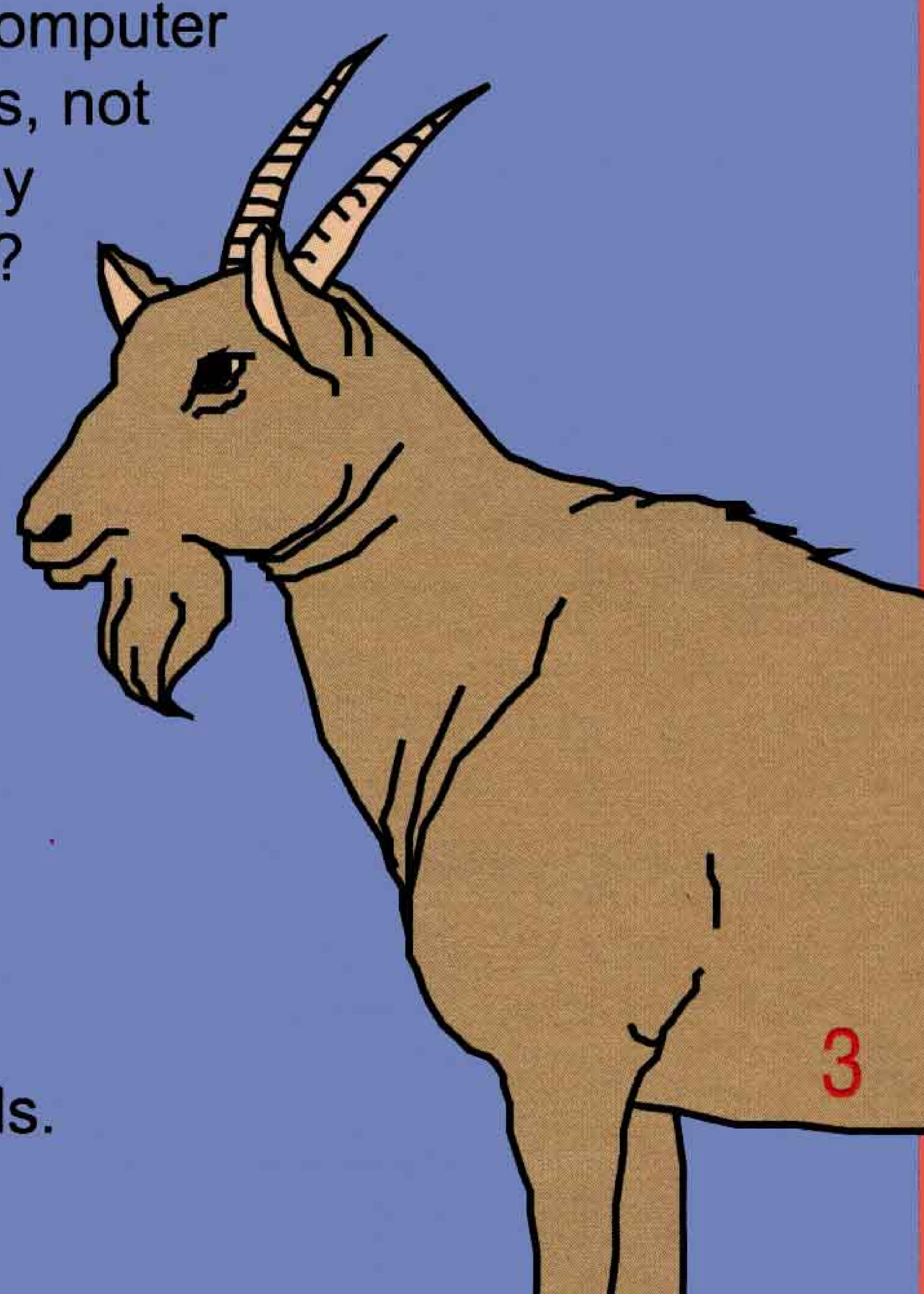
5. How many connected n-stars are there for a given number n? For example, there is one 1-star (a dot). There is one connected 2-star (a segment), there are two connected 3-stars (triangles traced in opposite directions), and there are two connected 4-stars (squares traced in opposite directions). If we write $\phi(n)$ to represent the number of connected stars, then:

$\phi(1) = 1$ $\phi(4) = 2$ $\phi(7) = 6$
$\phi(2) = 1$ $\phi(5) = 4$ $\phi(8) = 4$
$\phi(3) = 2$ $\phi(6) = 2$ $\phi(9) = 6$.

6. A candy-bar and a soda cost 80 cents. A candy-bar and an ice cream cost 85 cents. A soda and an ice cream cost 87 cents. How much does each cost?



7. You decide to make your 7-letter computer password consist of 4 A's and 3 B's, not necessarily in that order. How many different passwords can you make?



8. INGENUITY: (The Camels and Goats)
A, B, C, D and E play a game in which each is either a camel or a goat. A camel's statement is always false and a goat's statement is always true.

A says B is not a goat.
C says D is a camel.
E says A is not a camel.
B says C is not a goat.
D says that E and A are different kinds of animals.

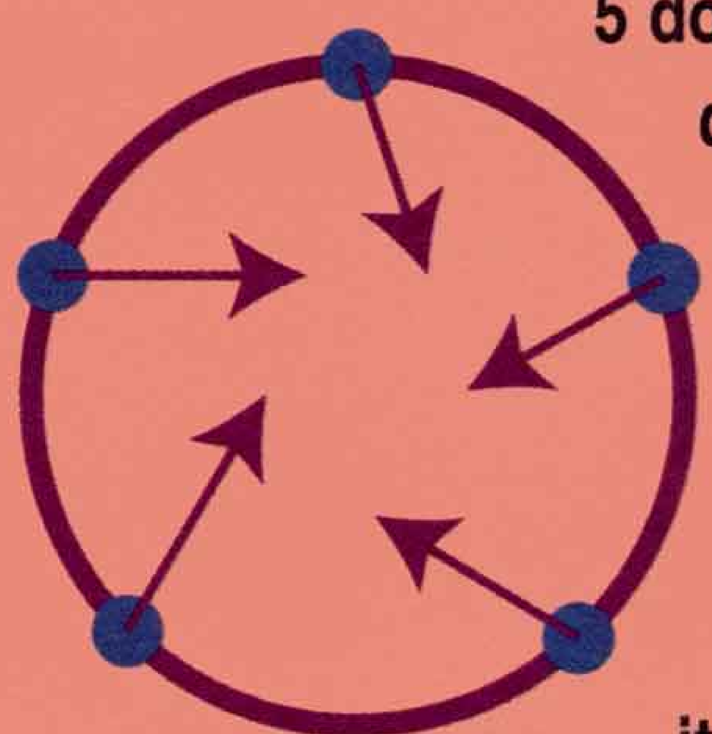
What kind of animals are A, B, C, D, and E?

DRAWN

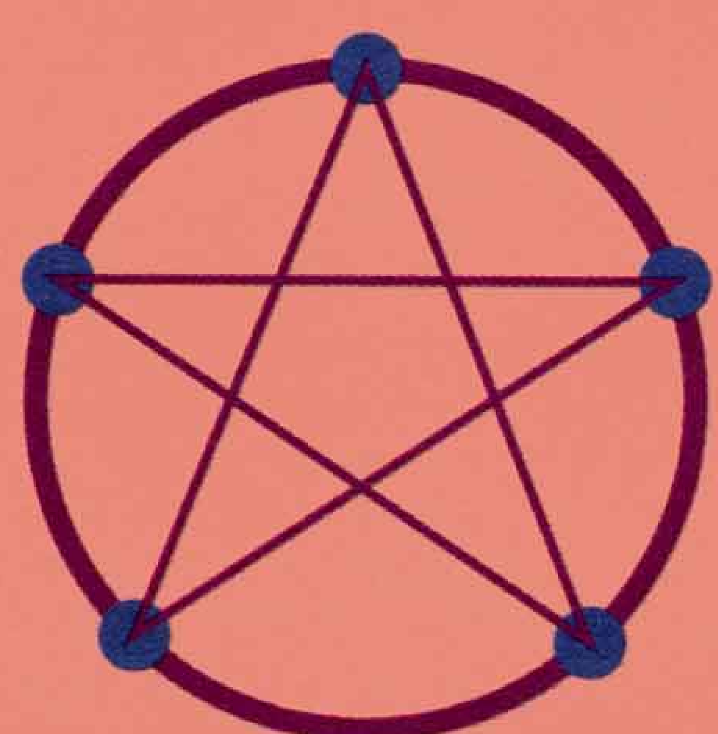
by Daniel B. Shapiro

The usual 5-pointed star is a wonderful  figure. It was the mystic symbol, called a pentagram, for the "Pythagorean" cult of ancient Greece. Members investigated many of its mathematical and magical properties. The 6-pointed star, called a hexagram or Star of David, also has a long history as a religious symbol. What other stars can you draw?

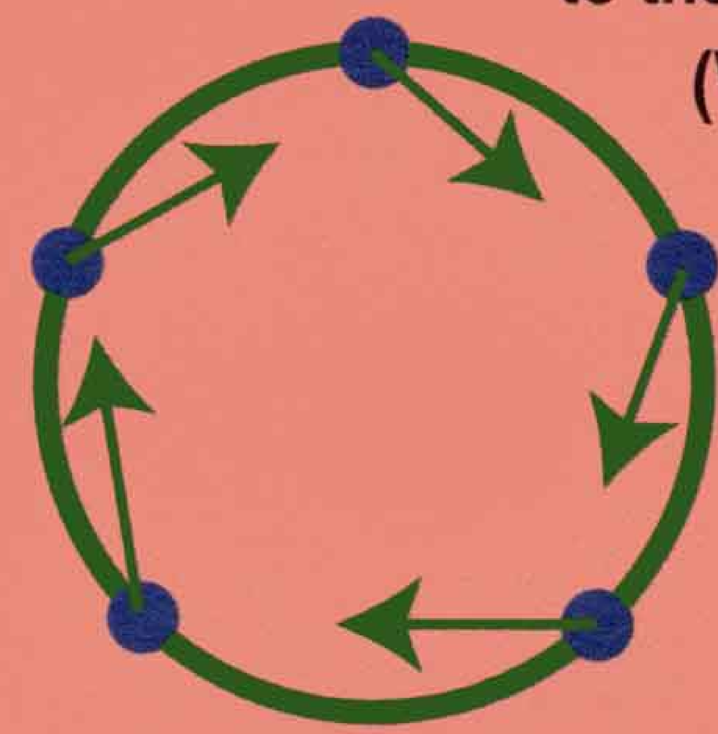
Let's look at the pentagram more closely. Start with 5 dots, equally spaced around a circle. At each dot we place an imaginary jumping spider which leaves a straight web-trail wherever it jumps. Suppose each spider jumps directly to the second dot to its left, that is to the dot which is two steps away clockwise around the circle. Here is what that traced figure might look like:

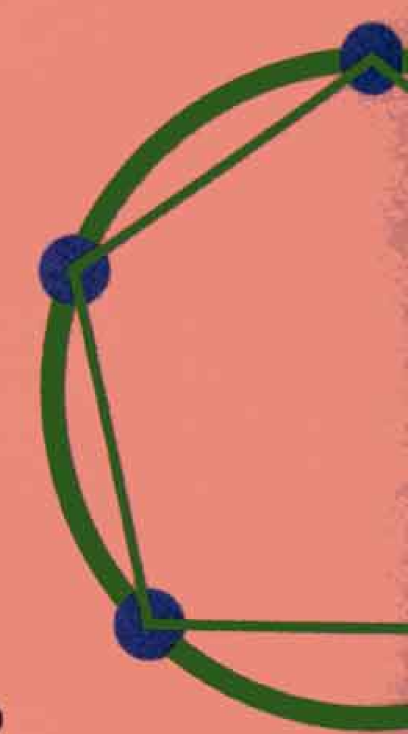


The typical way to create this star on paper is to draw one segment after another, without lifting the pencil from the paper.

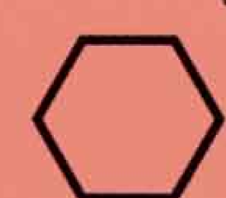


Now use the same 5 dots, but let the spiders jump to the dot which is just *one* step away. (We use the word "step" to mean the motion from one dot to the next one clockwise around the circle.) This produces a different figure, the regular pentagon:





This "star" can also be drawn one segment after another, without lifting the pencil from the paper. What if you use those 5 dots, but now each spider jumps to the dot which is 3 steps away? What figure do you get? What happens when they jump 4 steps each time? How about 5 steps?

Let's try 6 dots now, rather than 5. We can have each of the 6 spiders jump to the next dot clockwise (1 step). The resulting figure is a regular  hexagon. If each spider jumps to the dot 2 steps away, we get the hexagram. This one cannot be drawn without lifting the pencil, since it is made of 2  overlapping triangles. If each of the 6 spiders jumps to the dot which is 3 steps away we get an "asterisk". It is made of 3  separate line segments.

After pondering these examples, you might have guessed that we would go on to discuss more general stars. For whole numbers n and d let's construct the "n-sided star with step size d". More briefly, let's call it an "n star with d steps." This is built using n equally spaced dots on a circle and n

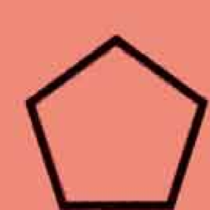
G STARS

spiders, each jumping to the dot which is d steps away clockwise around the circle. To avoid such a long description of this figure we'll refer to it by the compact symbol:

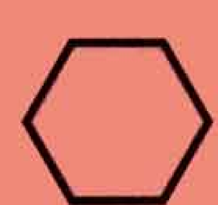
{ n / d }

We already mentioned some 5-stars and 6-stars. A {5/2} is a 5-star with 2 steps, which is just a pentagram.

Similarly a {5/1} is a pentagon,



a {6/1} is a hexagon,



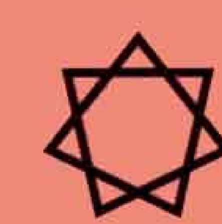
and a {6/2} is a hexagram.

Remember that this {6/2} is formed from 2 overlapping {3/1}'s, which are the triangles

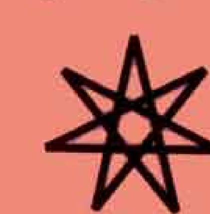
Also the asterisk {6/3} is built from 3 overlapping {2/1}'s, which are the segments

What about "degenerate" cases like {5/0}? That 5-star has 5 dots but no segments (the spiders stay in place without jumping), resulting in a picture that doesn't look much like traditional star. It is built from 5 separate pieces, each one a single dot (which would be simply 5 copies of the 1-star {1/0}).

Let's move up to the 7-stars. The {7/1} is a regular heptagon, but the {7/2} is more interesting. It is constructed by placing 7 dots around a circle and drawing all the 2-step segments. Once you see what it looks like you can practice drawing it freehand, without lifting the pencil from the paper.

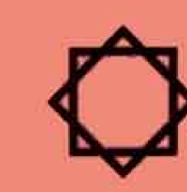


Some people think that a {7/3} looks nice, but others find it too pointy.



Some artistic students decorate their notebooks and papers with these stars. (But others think that sort of thing is very annoying. There's no accounting for tastes!) What would the stars {7/4}, {7/5}, and {7/6} look like? How about {7/0} and {7/7}?

The 8-stars provide more examples of "non-connected" behavior. For example, an {8/2} is built from 2 overlapping squares



or {4/1}'s and an {8/4} consists of 4 overlapping line segments or {2/1}'s.

More interesting to draw is the {8/3}, which seems somewhere between the two 7-stars. Stars with more sides become harder to draw freehand. Practice drawing 9-sided stars with different steps. Start with 1-step.

See problems 1-5 on page 3. They might inspire you to give this subject some more thought.

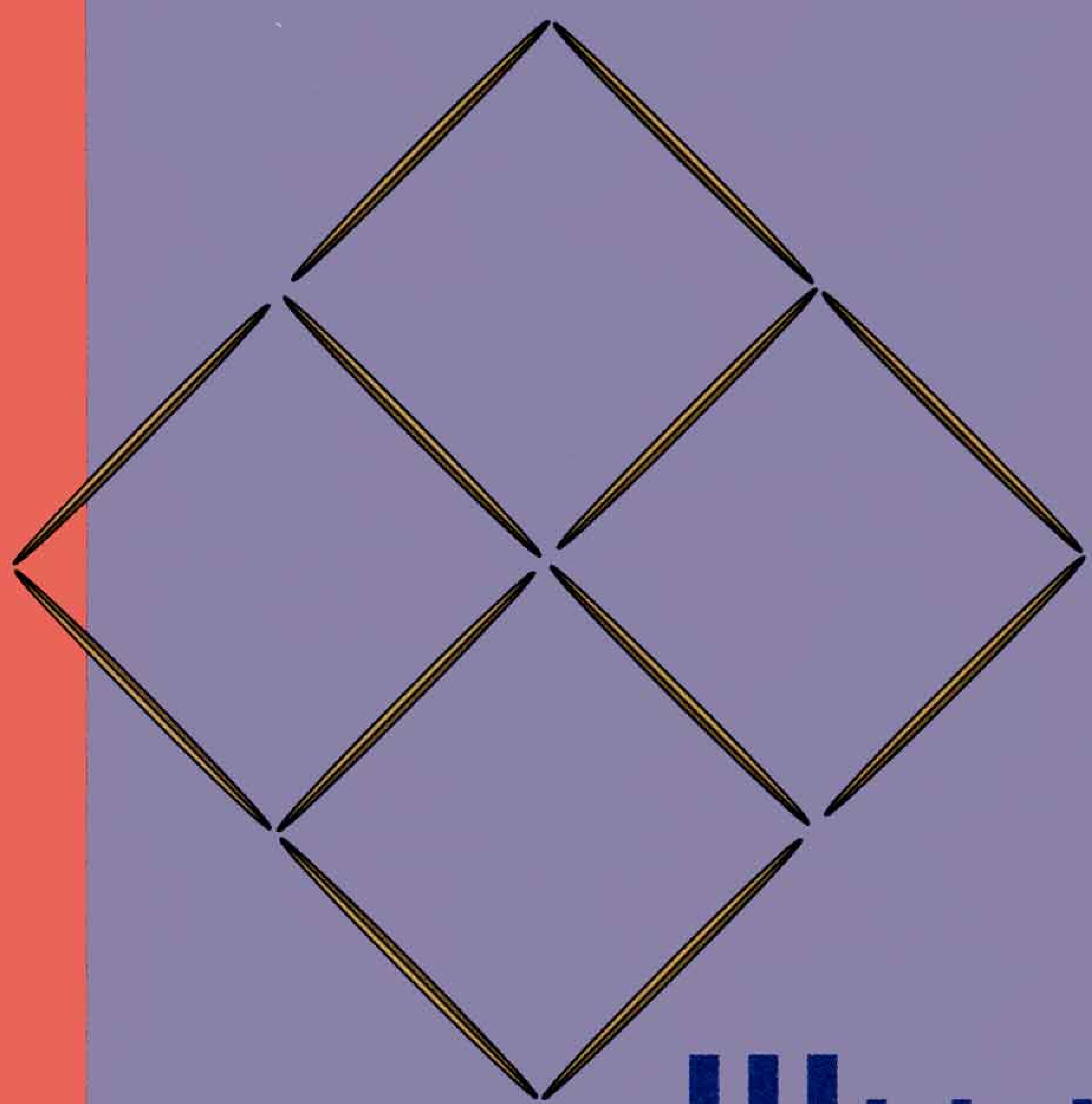
The author, Dan Shapiro, teaches mathematics at Ohio State University.

Puzzle Page

Math Explorers:

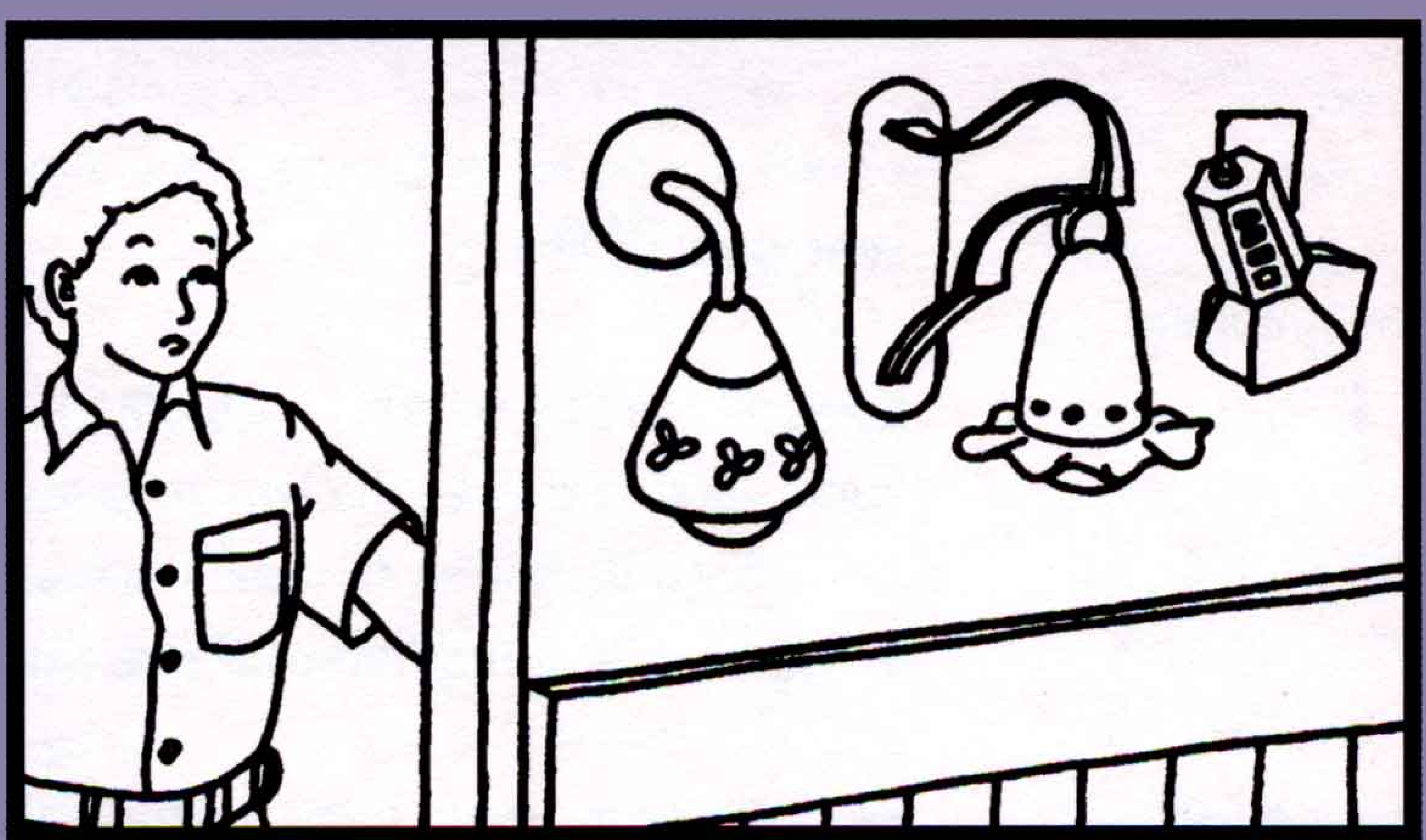
We want to print your work! Send us original math games, puzzles, problems, and activities. If we print them, we'll send you and your math teacher free *Math Explorer* pens.

PICK UP STICKS



Using 12 toothpicks, we made a big square containing 4 little squares. Relocate 4 toothpicks so that only three squares are on the picture.

Which Switch?



Alfredo had to answer the following question during his school's math competition:

There are 3 lightbulbs on the wall in the room next to ours. Each bulb has its own switch right outside of the room next to the door. Before you enter the room you may touch or flip the switches as you like, but once you enter the room you have to tell which switch operates which lightbulb. How would you do it?

four 6's

Use the operations

$+ \quad - \quad \times \quad \div$

and parentheses to combine the four 6's and make each equation below true. For example, we can use four 6's to make 0 like this:

$(6 + 6) - (6 + 6) = 0$

6 6 6 6 = 1
6 6 6 6 = 2
6 6 6 6 = 3
6 6 6 6 = 4
6 6 6 6 = 5
6 6 6 6 = 6
6 6 6 6 = 7
6 6 6 6 = 8
6 6 6 6 = 30

Bulletin Board

Check it out!

You can get a free "America Goes Back to School" kit from the Department of Education by calling 1-800-433-7827.

Look for Science News online.
Ivars Peterson's MathTrek
http://www.sciencenews.org

Words of Wisdom

"...treat Nature by the sphere, the cylinder and the cone"
--Paul Cezanne

Did you know that?

Did you know that in a class of 23 kids there is a 50% chance that at least two children share the same birthday? Why?

Two mathematicians have found that there are 85 practical ways to tie a necktie. Can you tie a necktie?

Leonardo da Vinci (continued)

practical calculations worked alongside a curious scientific mind to improve all of his projects, artistic or mathematical. Leonardo once said, "Painting is a science and all sciences are based on mathematics. No human enquiry can be a science unless it pursues its path through mathematical exposition and demonstration." Imagination was required to make advances in geometry, devise new machines or to imagine Mona Lisa's enigmatic smile. Leonardo had no shortage of imagination!

http://www.enet.org/math/davinciphilos.html
http://www.enet.org/math/archrelation.html
http://www-history.dcs.st-and.ac.uk/~history/Mathematicians/Leonardo.html

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Math Explorer

Welcome to *Math Explorer*!

Beneath the surface of art, in the beauty of its design and composition or its use of color, lies a mathematical principle. For example, using the golden ratio is pleasing to the eye, proportion and perspective add depth and dimension to the picture and geometry is an important part of many works of art.

We hope you will enjoy the articles and problems in this issue. We'd like to hear how you used mathematics in your next art project!

Sincerely,

Hiroko K. Warshauer

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