Math Explorer

MATH & ASTRONOMY

Relatively Einstein!

scaling the solar system

LIGHT YEARS !!!

Vol. 2.3 December 1999

Math Explorer

Contents

Math Explorer

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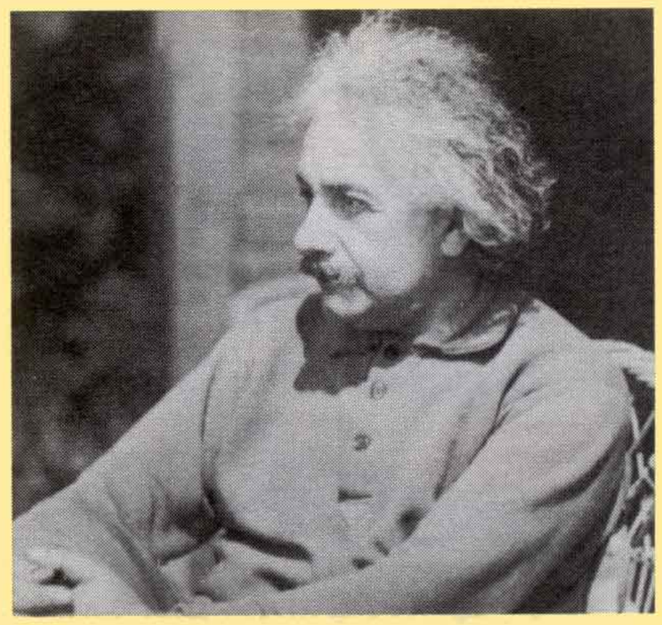
SWT

Albert Einstein



German-born Albert Einstein (1879--1969) was one of the greatest scientists of this century. Einstein changed the fundamental way we look at the universe, thereby expanding our horizon to pursue the yet unknown. As a child, he was amazed at how a magnetic compass worked and felt the wonder and power of unseen forces.

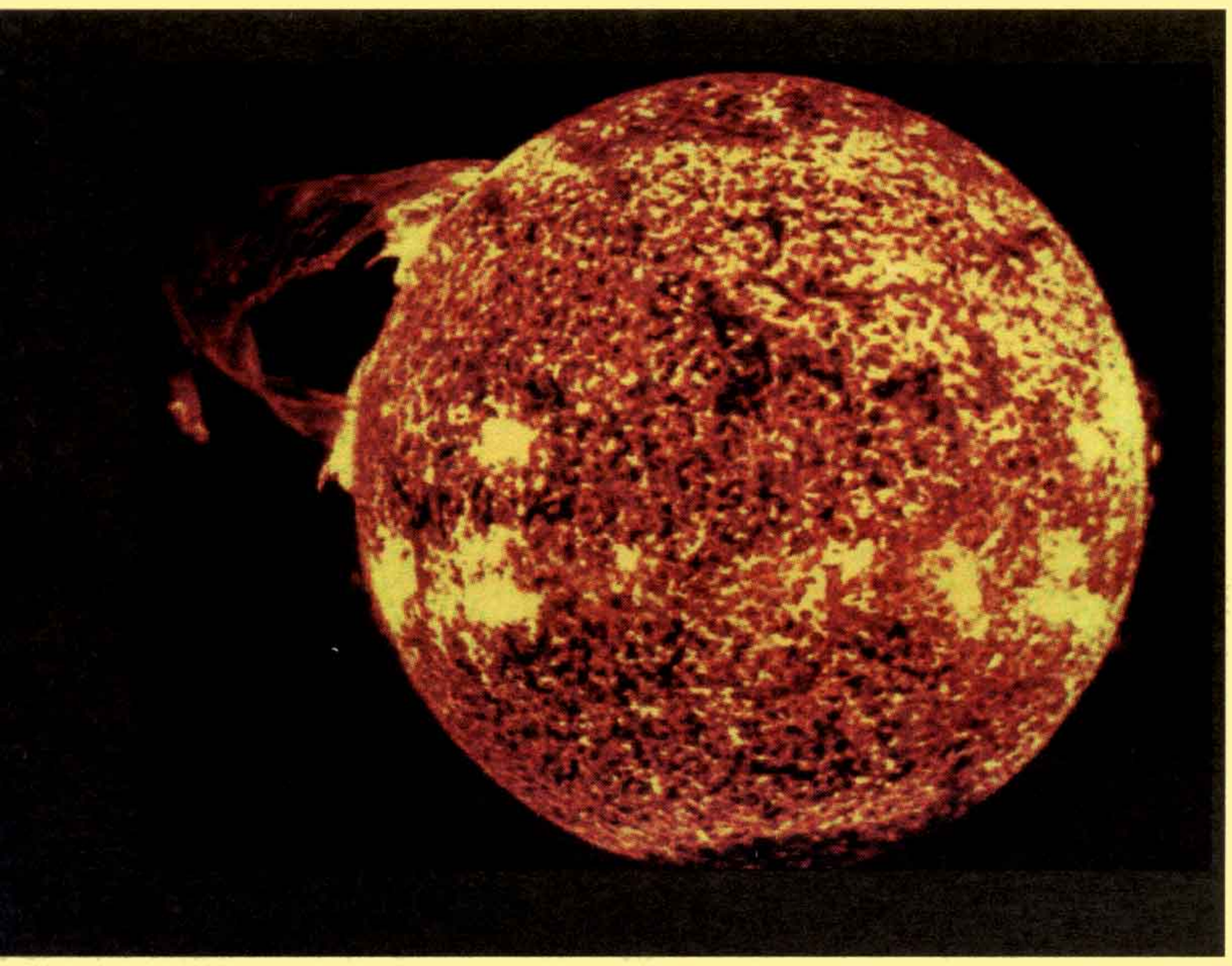
Formal schooling was a difficult experience for Einstein. He did not want to just memorize information. Instead, he found his relatives and his parent's friends would share scientific and mathematical knowledge with him. This form of education stimulated Einstein's keen and curious mind to learn more and to ask questions. After teaching for a few years, he took a job in a Swiss patent office. It was there, while he worked at the patent office in the early 1900's, that Einstein published several extraordinary articles on theoretical physics.

Einstein's paper on the **Special Theory of Relativity** states that measurements of time and space change relative to the observer. This went counter to Isaac Newton's long held theory that the laws of physics are the same for any frame of reference. Another article, the **Quantum Theory of Light** states that light exists in photons or tiny particles. This work forms the foundation in modern physics—we live in a quantum universe which is built out of tiny chunks of energy and matter.

The famous equation, $\mathbf{E = mc^2}$, was incorporated in Einstein's paper on relativity. E stands for energy, m for mass and c for the speed of light. He found that if an object emits a certain amount of energy, then its mass must decrease proportionately. So mass is a direct measure of energy. For most of the rest of his life, Einstein worked on a **Unified Field Theory**. He wanted to show that electromagnetism and gravity could all be put into a mathematical framework from which the physical concepts could be explained.

by Hiroko K. Warshauer, who teaches mathematics at Southwest Texas State University. She enjoys music and the arts, as well as working with students on math puzzles, problems, and activities.

PROBLEMS OF THE MONTH



1. A binary sequence of length 4 is a number with only 4 digits and the digits can be only 0's or 1's. How many binary sequences of length 4 are there? How many binary sequences of length 5 are there?

2. A rectangular array of M x N dots has 2700 dots total, of which 262 are in the interior. Find M and N. Assume M < N.

3. How many people need to be in a room to guarantee that there are at least four people who were born on the same month?

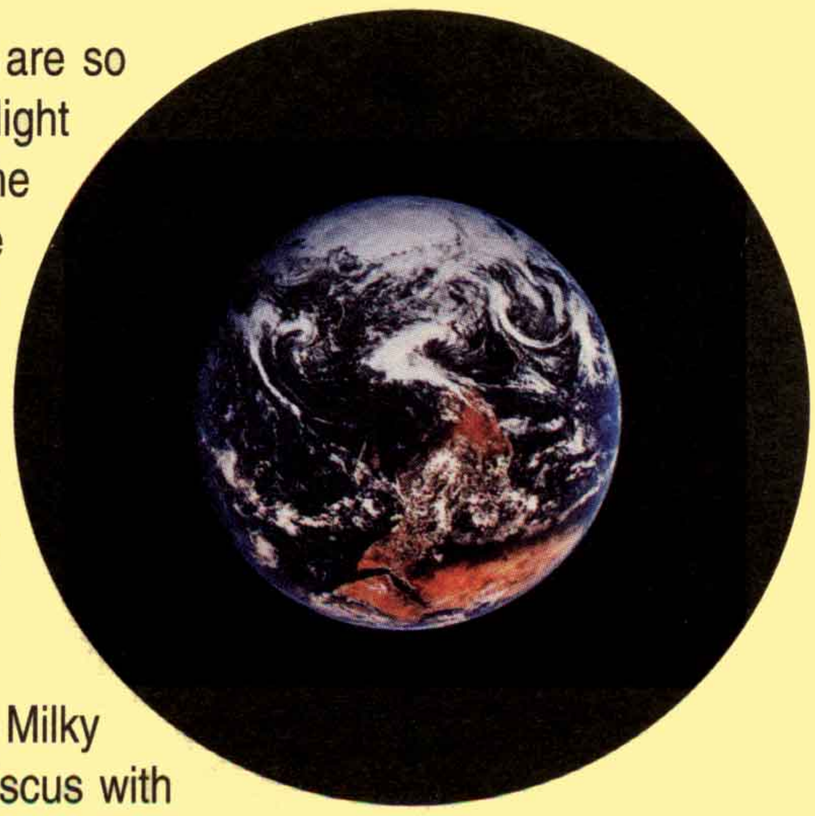
4. Three boys and three girls go to a movie together. They find a row with 6 seats together. How many seating arrangements are possible if no boys can sit beside each other?

5. In the sequence 8, 5, 2, 1, 4, 7, 6, 9, 3 the numbers 1, 4, 7, and 9 occur in increasing order and the numbers 8, 7, 6 and 3 occur in decreasing order. Arrange the numbers 1 to 16 so that no five of them occur in increasing or decreasing order.

6. How many 4-digit numbers have four different digits occuring in increasing order?

7. What is the 1000th digit to the right of the decimal place in 1/111?

8. If you roll two standard 6 sided dice, what is the probability that the two numbers showing will be different?



9. Distances between stars are so large, we measure them in light years. One light year is the distance light travels in one year at the rate of 186,000 miles per second. Find the distance to the nearest star from earth in light years and in miles. (Use books or an internet search.)

10. Our galaxy is called the Milky Way and is shaped like a discus with the middle thicker than the edges. Find the distance across the Milky Way in light years. Find the distance to the nearest neighboring galaxy. What is its name?

Send us your solutions! Every month, we will publish the best solutions on our website: www.mathexplorer.com. If we print your solutions, we will send you and your teacher free *Math Explorer* pens!

Modeling the Earth's

by Kevin Jones

One of the most amazing things about the solar system (our Sun with its planets, asteroids, and comets) is how much space there is among the planets. Sometimes we make scale models of objects in order to study them. This means that all of the measurements are proportionally larger by the same amount, which is like either enlarging or reducing a picture or a recipe.

Think of the following ingredients for brownies: 1.5 cups flour, 2 cups sugar, 1 cup shortening, 10 tablespoons cocoa, 1/4 teaspoon salt, 4 eggs, and 2 teaspoons of vanilla. We could put the ingredients in the table below in the first column Now, if you wanted to double the recipe, you'd simply multiply the ingredients by two. To halve the recipe, you'd multiply the original recipe by 1/2 (or you could divide by two). We say we want the ingredients to have the same proportion. Notice that there is always twice as much sugar as shortening.

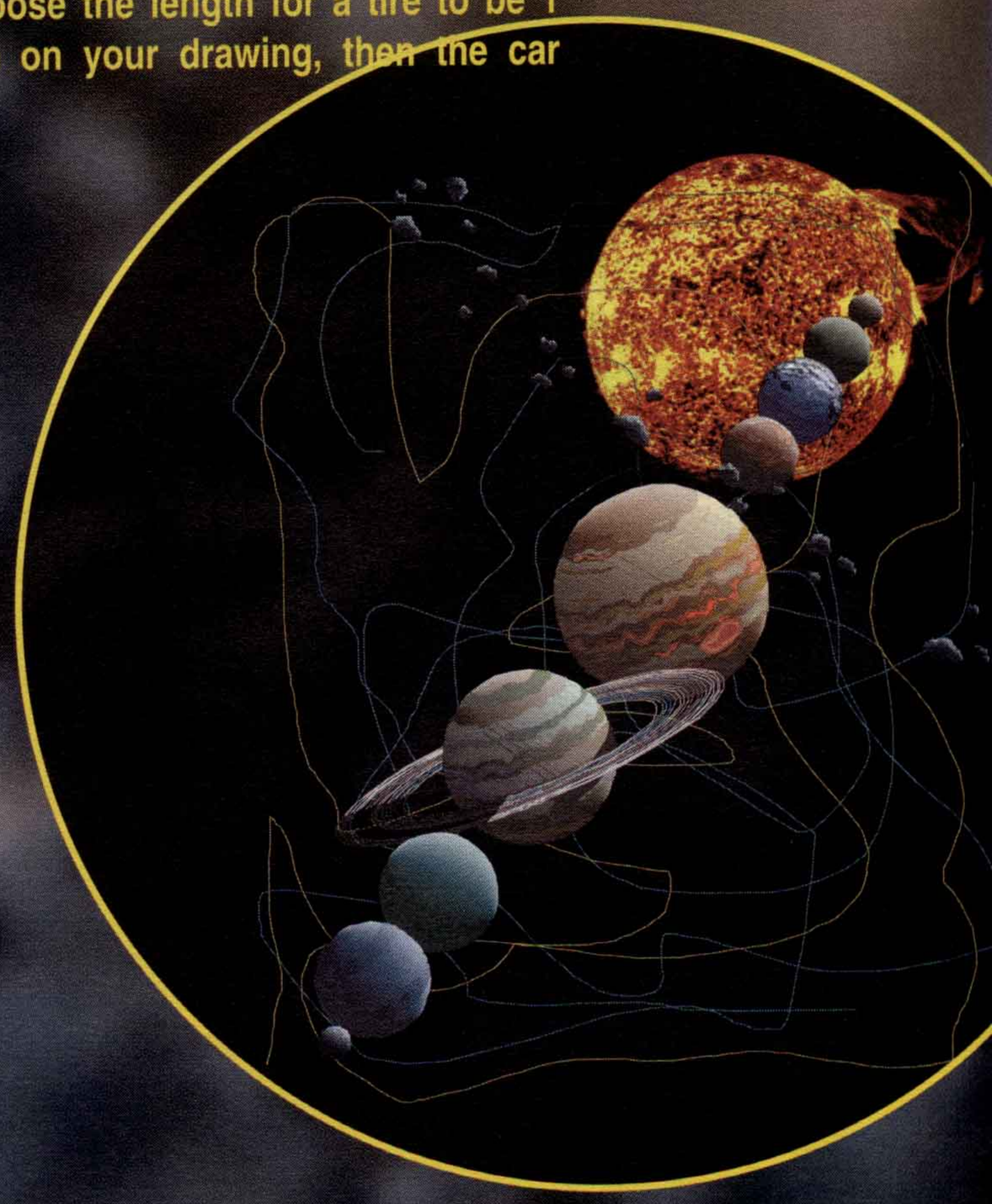
STANDARD RECIPE	DOUBLE RECIPE	HALF RECIPE
1.5 cups flour	3 cups flour	3/4 cups flour
2 cups sugar	4 cups sugar	1 cup sugar
1 cup shortening	2 cup shortening	1/2 cup shortening
10 tablespoons cocoa	20 tablespoons cocoa	5 tablespoons cocoa
1/4 teaspoon salt	1/2 teaspoon salt	1/8 teaspoon salt
4 eggs	8 eggs	2 eggs
2 teaspoons vanilla	4 teaspoons vanilla	1 teaspoons vanilla

Suppose you want to make a drawing of a car that accurately records what the car looks like. The car is 11 feet long, the top is 4.5 feet above the ground, and the wheels are 2 feet in diameter. If we wanted to make a model car 11 inches long, a top 4.5 inches off the ground, and tires 2 inches in diameter, then the model would be in correct proportion to the real car and look "realistic" even though it is much smaller—what we call a scale model.

Sometimes it's helpful to pick one distinctive feature and use it as the basis of measurement. Let's put the numbers above in a table, and then divide by the diameter of the tire:

	actual measurement	new unts = actual measurement / 2 feet
length of car	11 feet	5.5
height of car	4.5 feet	2.25
diameter of tire	2 feet	1

We could call our new units "tire diameters" or "t.d.'s" for short: the height is 2.25 t.d.'s, and the length is 5.5 t.d.'s. This means that: if you choose the length for a tire to be 1 inch wide on your drawing, then the car



Neighborhoods

height must be 2.25 inches high, and the length is 5.5 inches. If you choose for the tires to be 4 inches wide on the drawing, then the car height must be 2.25 x 4 = 9 inches high, and the length must be 5.5 x 4 = 22 inches long.

Model 1: Creating a scale model of the near-Earth Environment

In the table you will find some actual measurements of Earth and its neighbors. Let's create a new unit of measurement

Object	Actual size (miles)	E.D.'s = Actual/8000	Distance (if 1 E.D. = 1 foot = 30 cm
Earth diameter	8000	1	
Upper reaches of atmosphere	100		
Space shuttle orbit height	200		
Height of satellite	18,000		

called Earth Diameters, or E.D.'S for short, by dividing all the measurements by the diameter of the Earth (close to 8000 miles). You'll probably want to use a calculator and round the number of "Earth Diameters" to the nearest tenth.

After you finish the table of E.D's, find a standard globe where the diameter of the Earth is 1 foot (about 30 centimeters). The first three distances in the table are measurements associated with the immediate Earth environment. Try to measure how high the atmosphere of the

Earth extends from the globe—then do the same for the Space Shuttle. You may be surprised how close they are to Earth. Now imagine there is a satellite above the equator. For example, if you watch the weather on TV they almost always show the "satellite map" of the country.

Get out a globe and measure these distances. Are you surprised about anything? People are often astounded by how thin the atmosphere is and how close the Space Shuttle skims above the Earth.

Now let's measure the Moon and Sun in E.D.'s. Fill out the table below:

Object	Actual size (miles)	E.D.'s = Actual/8000	Distance (if 1 E.D. = 1 foot = 30 cm
Moon diameter	2100		
Earth/Moon distance (average)	240,000		
Diameter of Sun	830,000		
Earth/Sun distance (average)	93,000,000		

Get a globe of the Earth and find a ball that would be about the correct size for the Moon -- maybe a baseball. Place the "Moon" the correct (scaled) distance from the Earth. When you stand at the Earth globe and look toward the baseball Moon, the model Moon covers the same area as the actual Moon as seen from the actual Earth. Do you believe it? Find a time when the Moon is almost full -- it will rise in the east around sundown -- and have a friend hold the baseball 30 feet away where you can readily compare the apparent size of the baseball and the actual Moon. Are they about the same?

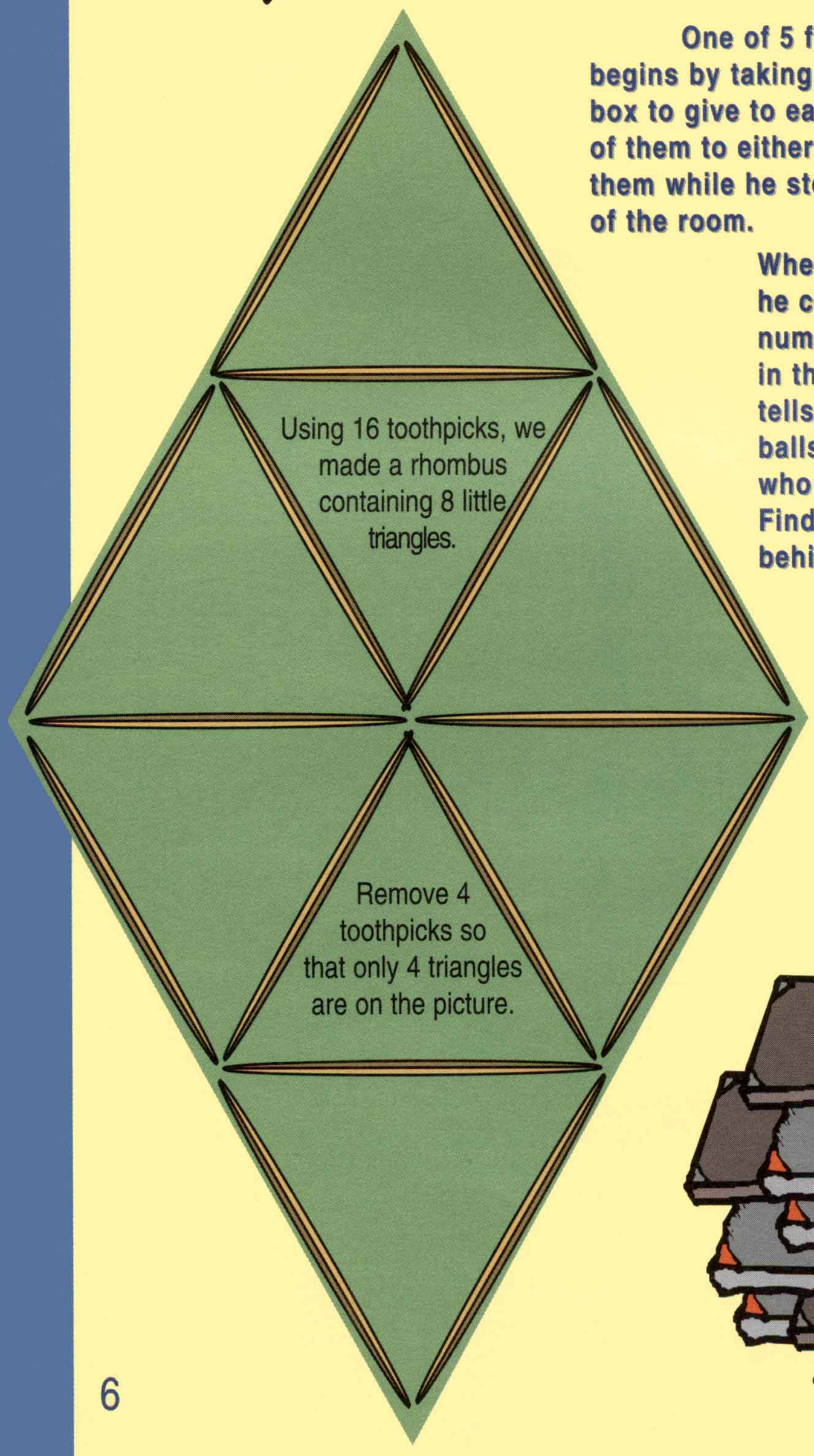
(continued on page 7)

Puzzle Page

Math Explorers:

We want to print your work! Send us original math games, puzzles, problems, and activities. If we print them, we'll send you and your math teacher free *Math Explorer* pens.

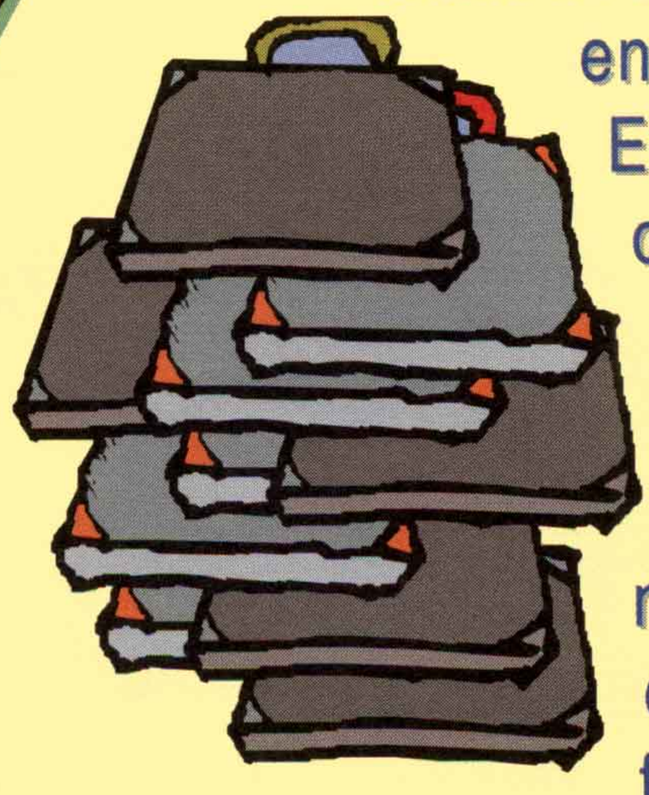
Now you see it . . . now you don't!



One of 5 friends shows a trick to the others: He begins by taking some number of identical balls from a box to give to each of his four friends. Then he asks each of them to either put the balls back in the box or keep them while he steps outside of the room.

When he returns, he counts the number of balls in the box, and tells who put the balls back and who kept them. Find the secret behind this trick.

10 suitcases arrived at a store with 10 keys to them in an envelope. Every key opens only one suitcase. What is the least number of trials needed to open every suitcase for sure?



Bulletin Board

Thanks to Our Sponsors

Southwestern Bell Communications (SBC) Foundation sponsored the Rio Grande Valley Summer Math Program, which was coordinated by Adelina Alaniz from Mission, Texas. Participating school districts included McAllen, Donna, Progreso, and Mission. Welcome to our new partners! And a special thanks to SBC which made the program possible!

Modeling Earth (cont'd)

Notice from the chart how large the Sun is compared to the Earth. You should have found that the correct scale model for the Sun is just over 100 feet in diameter if the Earth is 1 foot in diameter, and that it is over 2 miles away (there are 5280 feet in a mile). If you had a 100-foot ball and placed it just over two miles away, it would appear to be the same size as the baseball at 30 feet away. The Moon can just barely cover the surface of the Sun, and when it does this we say there is a total solar eclipse.

Model 2: Visualizing the solar system

If you want to know how the Earth compares to the other planets in the Solar System, measure their diameters in E.D.'s using the table below. Measure the distances from the Sun in E.D.'s as well, even though you'll get large numbers. By the way: did you know that these values (except for Pluto's diameter) were known well before satellites ever went to space? Careful measurements combined with mathematics allowed astronomers to figure out the values.

Object	Actual diameter	Earth diameters	Distance from Sun in miles	Earth Diameters
Sun	830,000 miles			
Mercury	3000 miles		35,000,000	
Venus	7500 miles		66,000,000	
Earth	7900 miles		93,000,000	
Mars	4200 miles		140,000,000	
Jupiter	88,000 miles		480,000,000	
Saturn	73,000 miles		880,000,000	
Uranus	31,000 miles		1,800,000,000	
Neptune	30,000 miles		2,800,000,000	
Pluto	1400 miles		3,600,000,000	

In order to try to make out a scale model of the Solar System, assume the Earth is a tiny ball of clay only 1 mm across (check it out -- that's pretty small!). How large are the planets on this scale? How far away are they from the Sun? You might want to go with some friends, make out model planets and Sun out of clay (even mighty Jupiter is just over a centimeter in diameter), go to a football field, and mark off the relative distances from the Sun. You may be surprised how far apart the planets are!

Math Explorer

Welcome to *Math Explorer*!

From the time of the Babylonians and earlier, mathematics and astronomy have been linked closely together. We at *Math Explorer* hope you will enjoy reading about some of the connections in this issue. Write to us about other interesting connections that you discover!

Please continue to send us your ideas and solutions to puzzles and problems so that we can share them with our readers!

Sincerely,

Hiroko K. Warshauer

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