Math Explorer

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The Postage Stamp Problem

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Math Explorer

Contents

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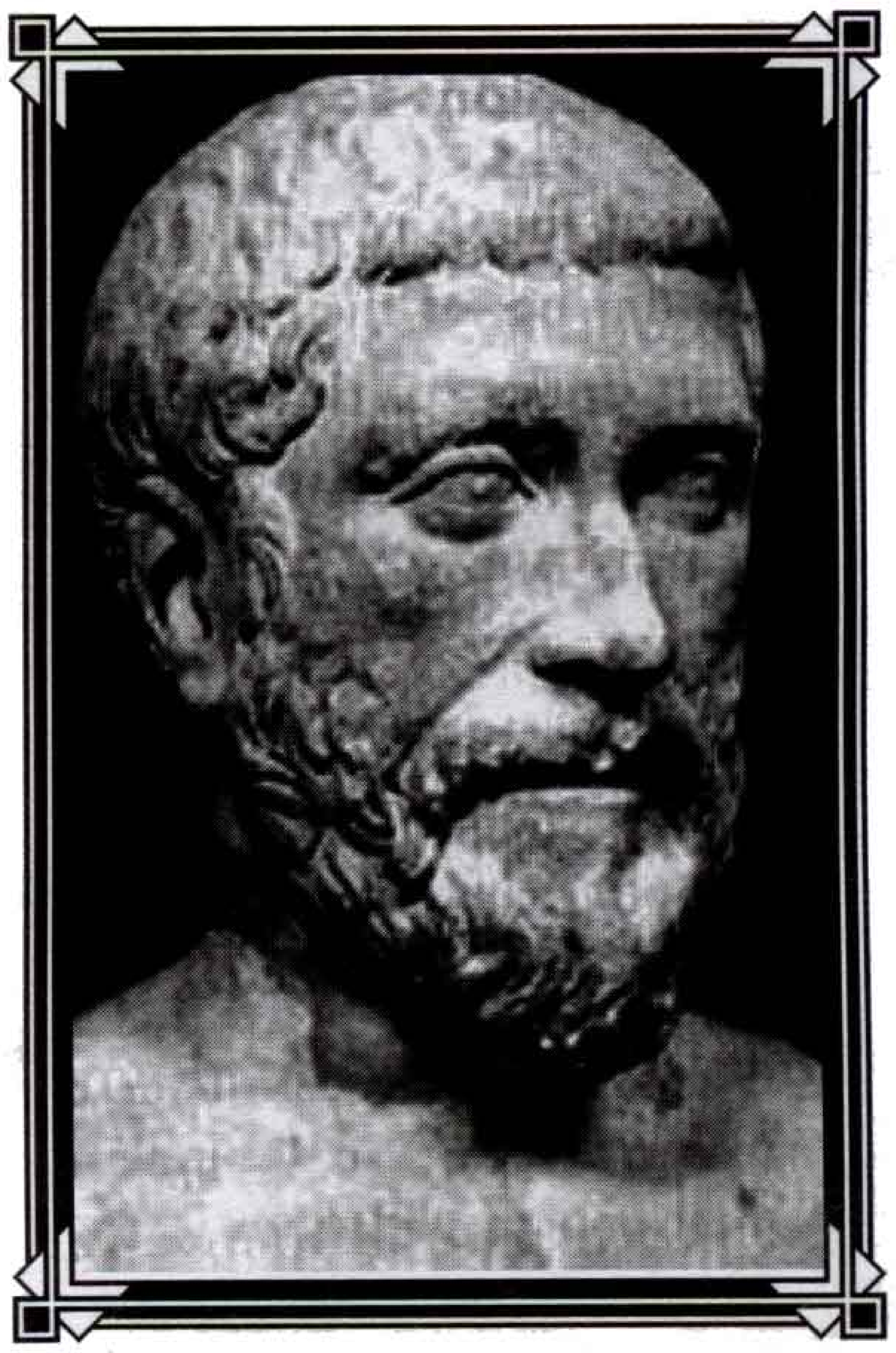
Pythagoras

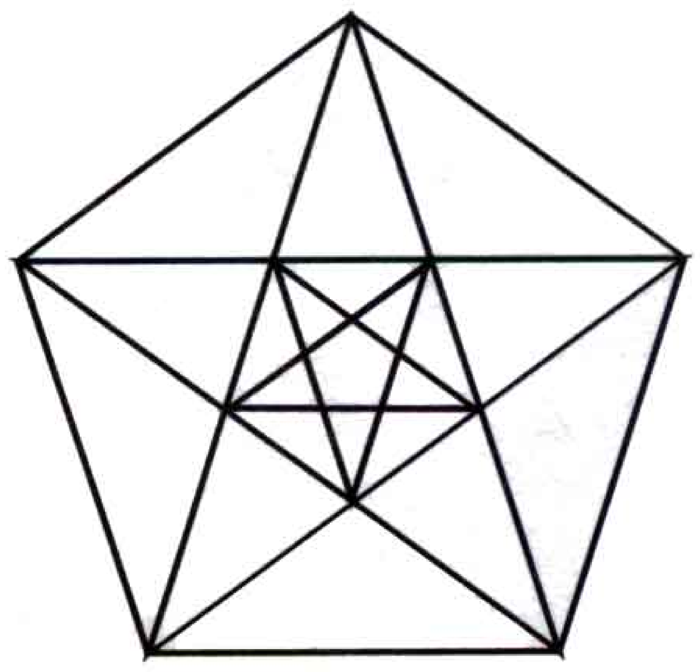
by Hiroko Warshauer



The famous Pythagorean theorem, a topic in this issue of *Math Explorer*, is named after the Greek mathematician and philosopher Pythagoras. Pythagoras of Samos lived from about 580 BC to 500 BC. He founded a school of mathematics and philosophy that used algebra to study properties of numbers.

The Pythagorean school made many contributions to geometry, including proving the Pythagorean Theorem and proving that the sum of the interior angles in a triangle equals 180 degrees. This school also discovered the existence of "irrational" numbers, numbers that cannot be expressed as a fraction with an integer in the numerator and denominator.

The star pentagon, or pentagram was the badge of the Pythagorean school. The sequence of stars and pentagons can continue nesting infinitely. A picture of a pentagram is next to this paragraph.



To find out more about Pythagoras and the contributions he made to mathematics as well as to astronomy, look up Pythagoras in the references below. You'll find out more about Pythagoras and as well as other mathematicians.

[H] Heath, Sir Thomas, *A History of Greek Mathematics*, Volume I, Dover, NY, NY, 1981.

[Ho] Hollingdale, Stuart, *Makers of Mathematics*, Penguin Books, NY, NY, 1989.

[B] Bell, E. T., *Men of Mathematics*, Simon and Schuster, NY, NY, 1965.

PROBLEMS OF THE MONTH

Directions: Send your solutions to *Math Explorer* ! We will publish the best solutions each month, and send a free *Math Explorer* pen to you and your math teacher if we print your solution.

1. In the country of Mathland, the post office sells only 3-cent and 4-cent stamps. How many different ways can you buy stamps for a letter that costs 12 cents to send? 14 cents? 16 cents? 18 cents? 24 cents? Fill out the tables below with your answers.

12 cents	

14 cents	

16 cents	

2. **(The Postage Stamp Problem)** A letter can hold at most 4 stamps. If you have available only 1-cent, 3-cent, and 5-cent stamps, what is the smallest amount of postage that you cannot put on a letter?

3. In a knock-out tennis tournament, a competitor plays a new match only if she has won her last. If such a tournament has twenty entrants, how many matches must be played to finish the tournament?

4. Before her grandchildren came to visit, Grandma was baking their favorite cookies. She counted the cookies and thought, "To give each of them 5 cookies, I would have to bake 3 more. But if I give each only 4 cookies, I can eat 3 of these cookies myself." How many grandchildren must Grandma have had?

5. Which number is greater, 19/98 or 1919/9898?

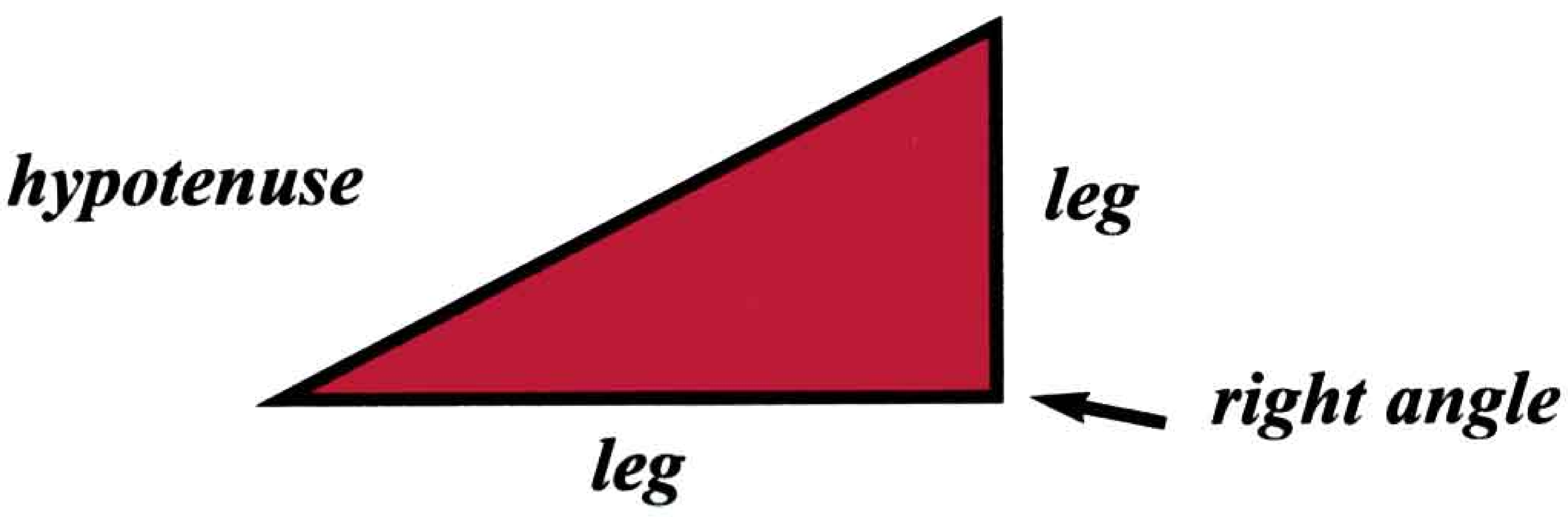
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

6. INGENUITY There are twenty lockers in a row, numbered 1 to 20. Twenty students pass by these unlocked lockers. The first student opens every locker. The second closes every even-numbered locker. The third changes every locker with number divisible by 3, opening those which are closed and closing those which are open. The fourth changes every locker with number divisible by 4, and so on up to the twentieth student, who changes only locker #20. At the end of this process, which lockers are open and which are closed?

The Pythagorean Theorem

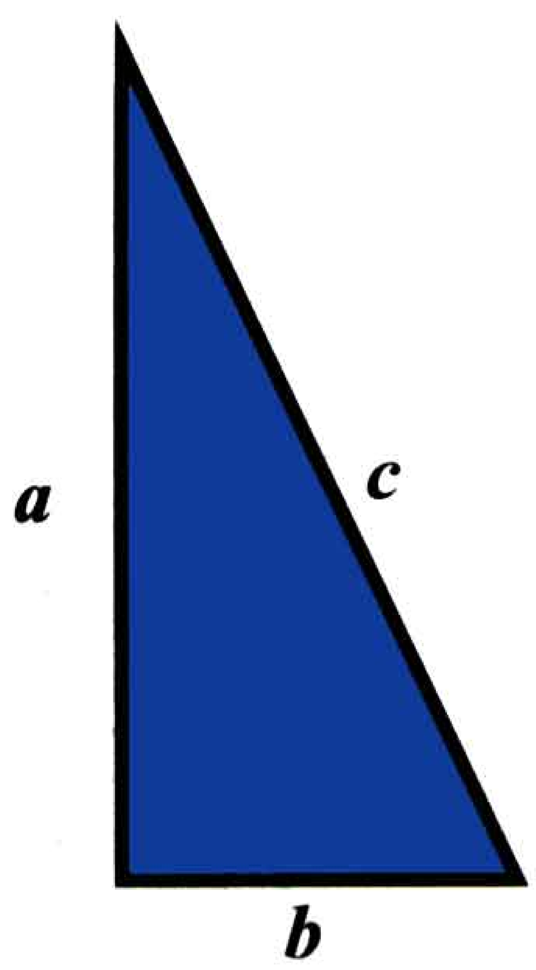
Anne Sung

The Pythagorean Theorem is a statement about a very important kind of triangle, the right triangle. Any triangle with a ***right angle*** (one shaped like a square corner) is a right triangle. The longest side of a right triangle is always the side opposite the right angle; this longest side is called the ***hypotenuse*** of the triangle. The other two sides of the triangle are shorter and meet at the right angle of the triangle; these sides are called the ***legs*** of the triangle.



Now that you know a little about right triangles, here's what the Pythagorean Theorem says:

For a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

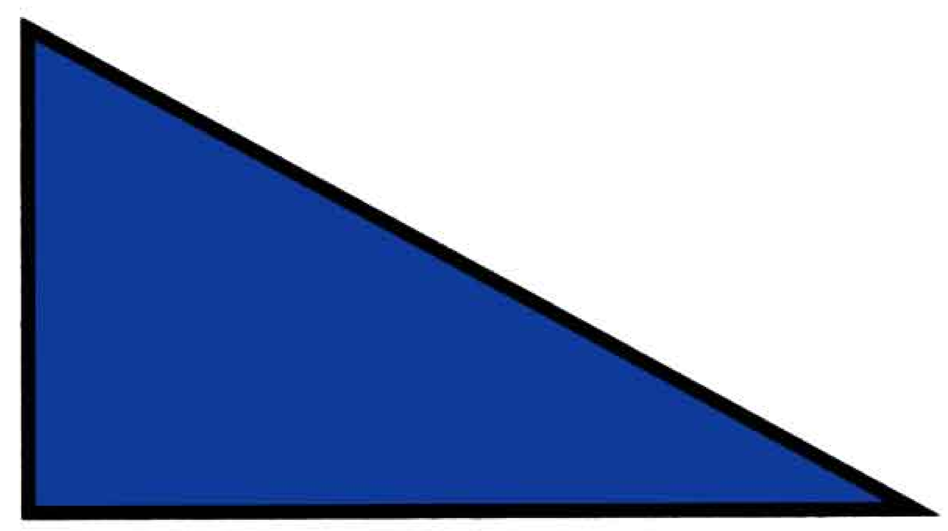


This theorem becomes easier to understand when we introduce some variables. Suppose we know that the triangle drawn to the left is a right triangle, but we don't know exactly what the lengths of its sides are. Since we don't know what numbers represent the lengths of the sides, we'll call the lengths of the sides by variable names, ***a*** and ***b*** for the lengths of the legs, and ***c*** for the length of the hypotenuse. Using the algebraic expression we practiced before, we can now state the Pythagorean Theorem this way:

$$a^2 + b^2 = c^2 .$$

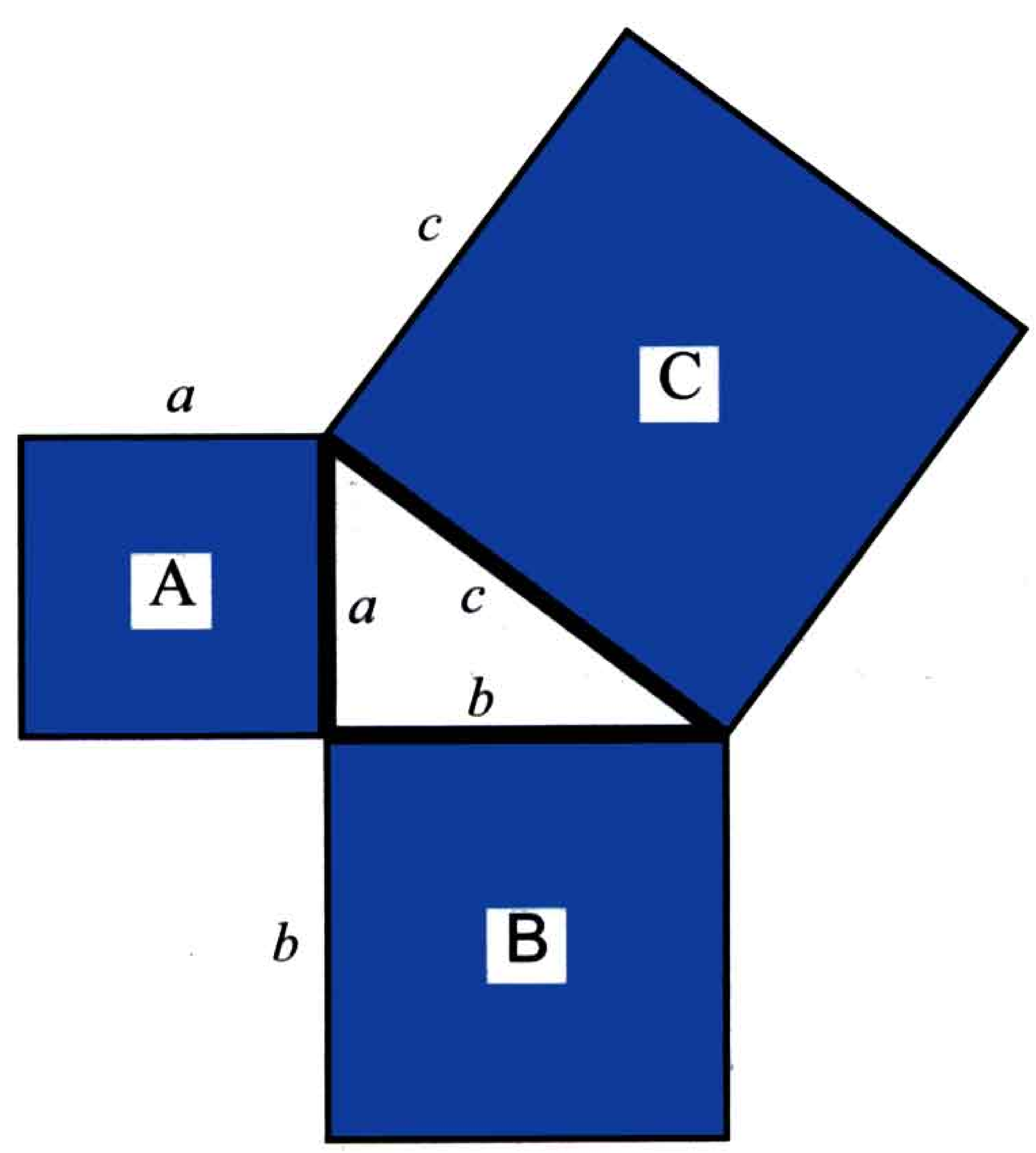
Remember that this is only true if the triangle is a right triangle and if ***c*** is the length of the hypotenuse and ***a*** and ***b*** the lengths of the two legs.

Check for yourself that the Pythagorean Theorem is true. Measure the legs and hypotenuse of this triangle, and see if the Pythagorean Theorem holds. Try this with your own right triangle. Does this always work?



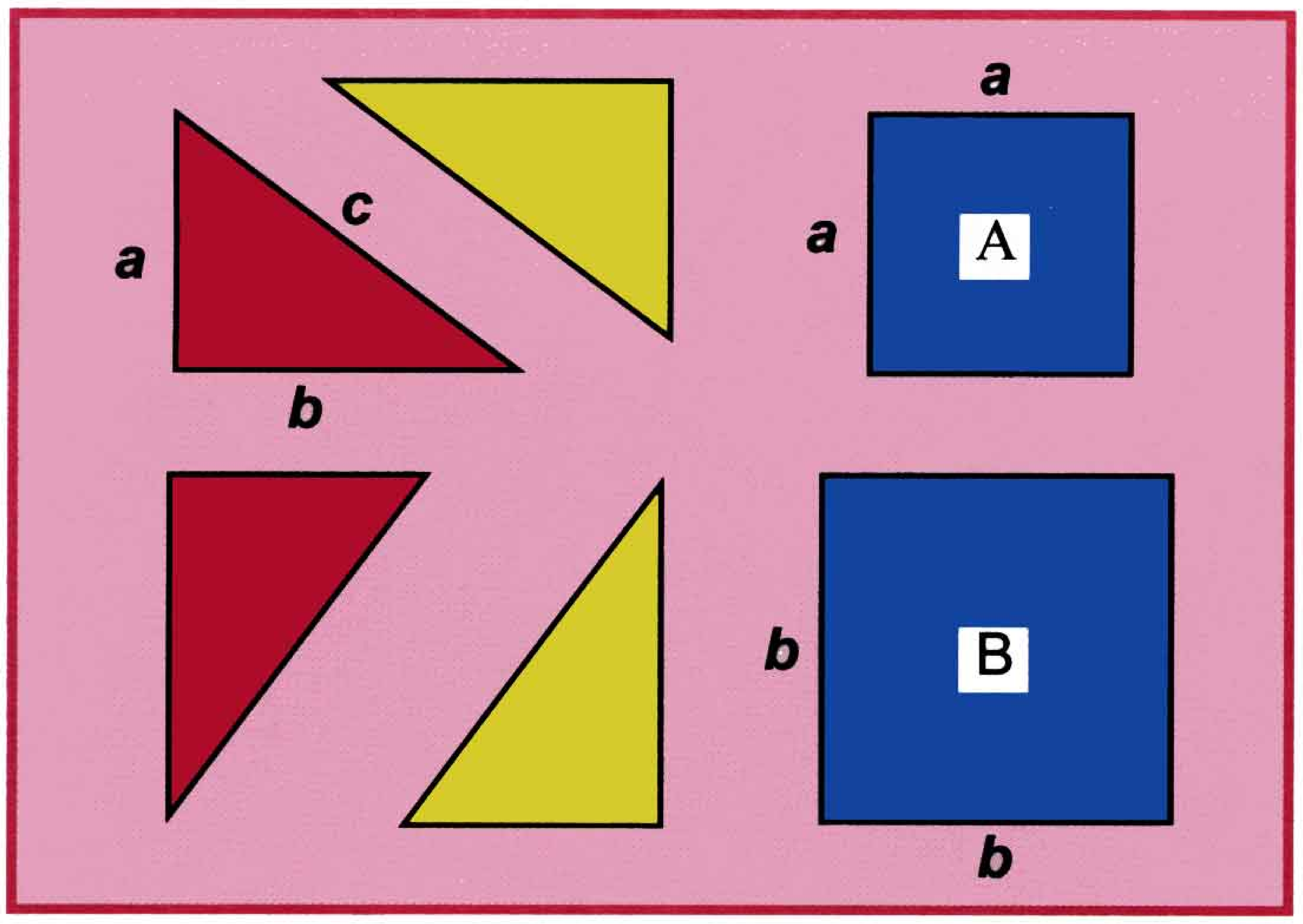
Is the reverse of the Pythagorean Theorem true? If you have three sticks and the square of one's length is the sum of the squares of the others' lengths, will they form a right triangle when you put them end to end? Try it!

The picture below is one way we can use area to visualize what this theorem says.

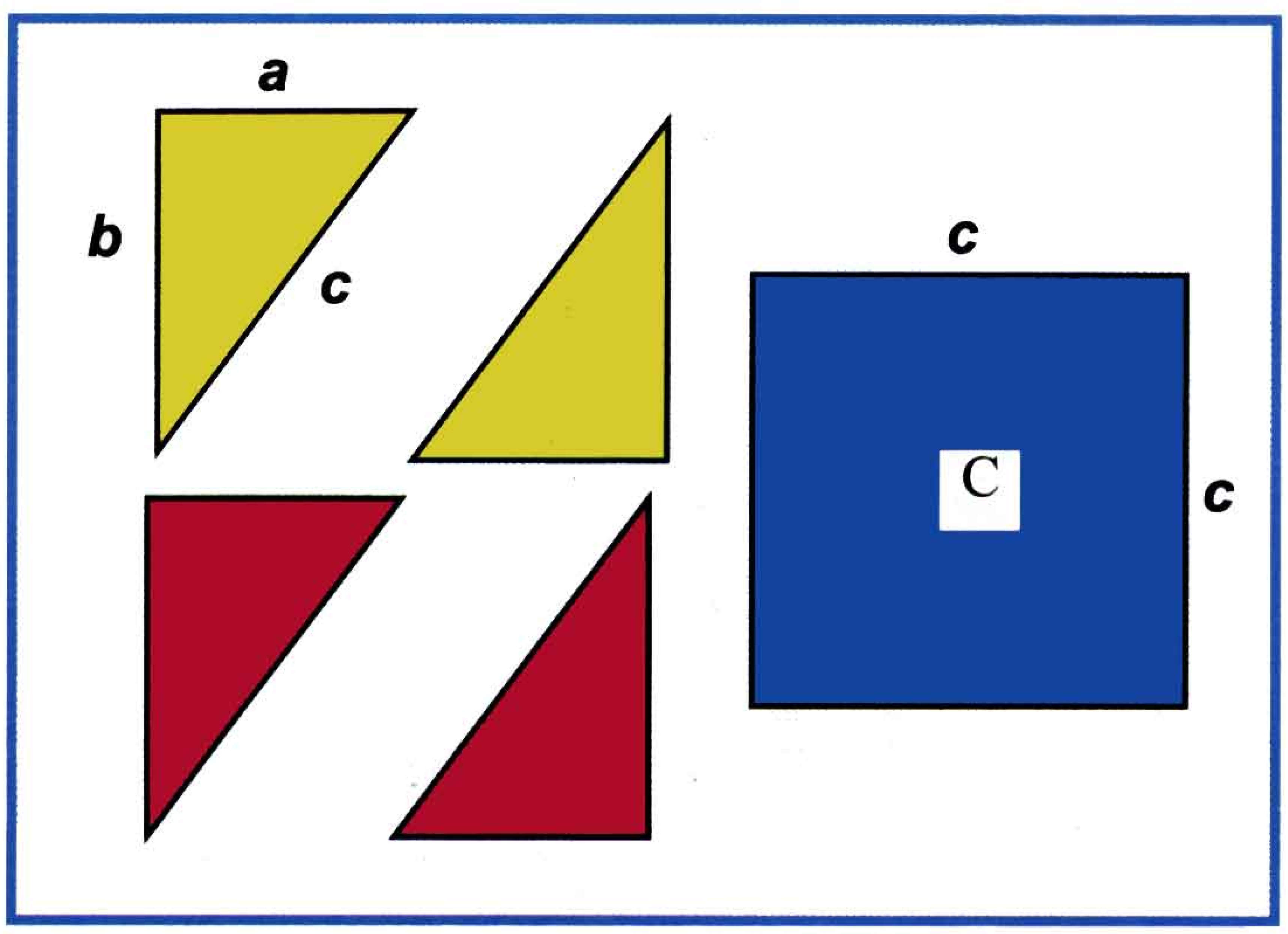


Taking the triangle above with sides of lengths ***a*****,** ***b*****,** and ***c*****,** squares are drawn against the sides of the triangle. Square A shares the side of the triangle with length ***a***. In last month's article, we saw that a square with sides of length ***a*** must have area a^2. Similarly, Square B shares the side of the triangle with length ***b*** and has area b^2, and Square C shares the side of the triangle with length ***c*** and has area c^2. The Pythagorean theorem tells us that combining the areas of squares A and B gives us exactly the area of square C, i.e. $a^2 + b^2 = c^2$.

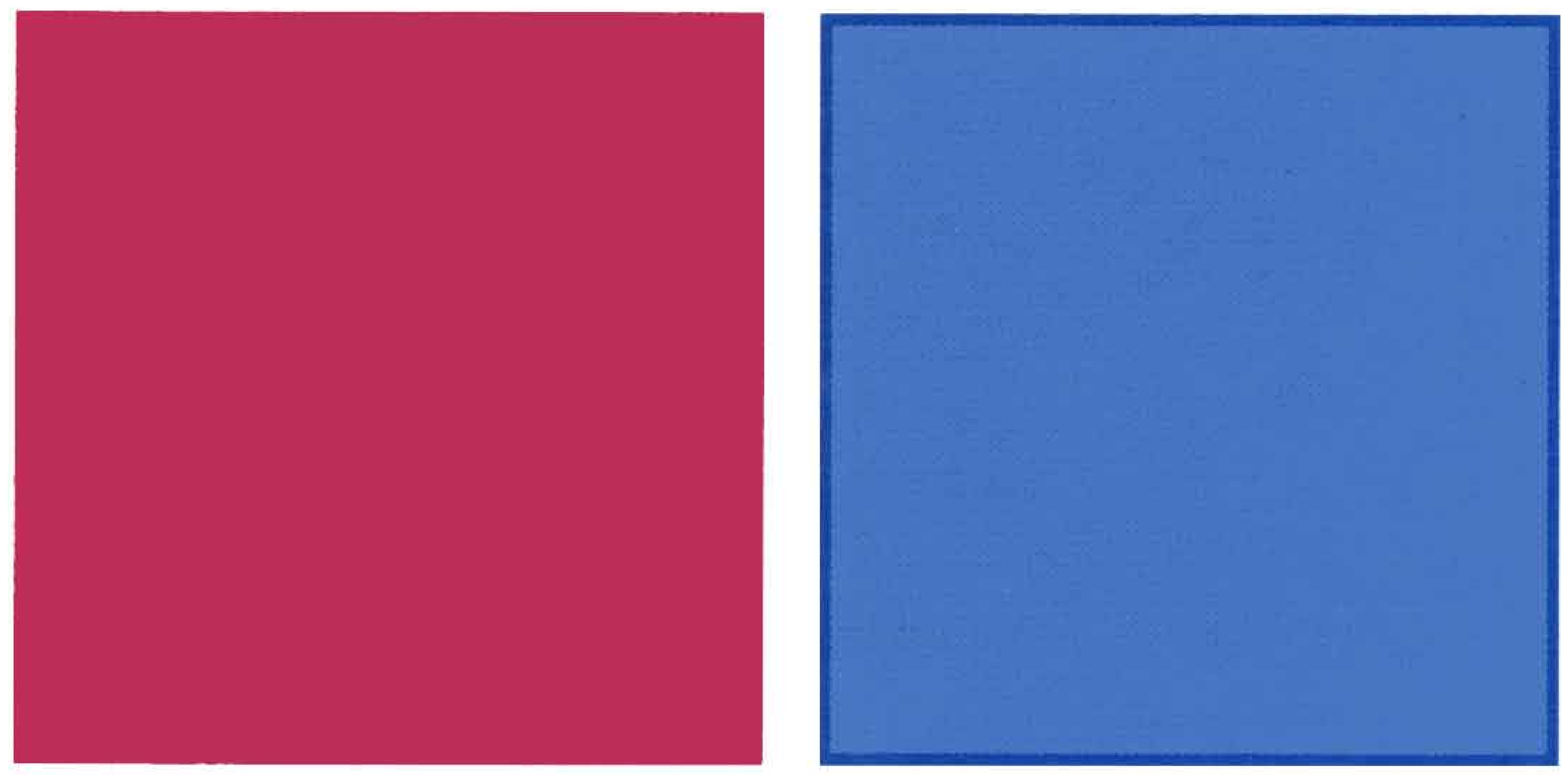
Now let's try to show that this is always true. Below you are given two sets of building blocks. The first set of building blocks has 4 right triangles and two squares, Square A and Square B, as pictured below.



The second set of building blocks contains the same 4 right triangles and one square, Square C.



Each set of shapes will fit together without overlap to exactly cover the big squares below.



Cut out the shapes in the two sets, and see if you can figure out how. If you've figured out how to build the big squares above out of each set of building blocks, then you've discovered the key idea needed to prove the Pythagorean Theorem! Hint: To check your answer, look on the Bulletin Board.

How does this help prove the Pythagorean Theorem? What are we doing when we use our building blocks to make the identical squares? The fact that these two sets of building blocks combine to form identical squares with the same area shows that the areas of the building blocks must be equal.

Since the only difference between the sets of building blocks is the squares, Square *C* must have the same area as Square *A* and Square *B* combined. We can write this as an equation:

(Area Square A) + (Area Square B) = (Area Square C)

or

$$a^2 + b^2 = c^2$$

We have now proved the Pythagorean Theorem, which says that the sums of the squares of the lengths of the legs of a right triangle is equal to the square of the length of the hypotenuse.

Exploration: If a right triangle's sides all have integral length, then the set of three integers giving the sides' lengths is called a *Pythagorean triple*. According to the Pythagorean Theorem, if (***a*****,** ***b*****,** ***c***) is a Pythagorean triple, then $a^2 + b^2 = c^2$. For example, (3, 4, 5) is a Pythagorean triple, since $3^2 + 4^2 = 5^2$, i.e. 9 + 16 = 25. Can you find other *Pythagorean triples?*

Puzzle Page

Math Explorers,

We want to print your work! Send us your own math games, puzzles, problems, and activities. If we print them, we'll send you and your math teacher free *Math Explorer* pens.

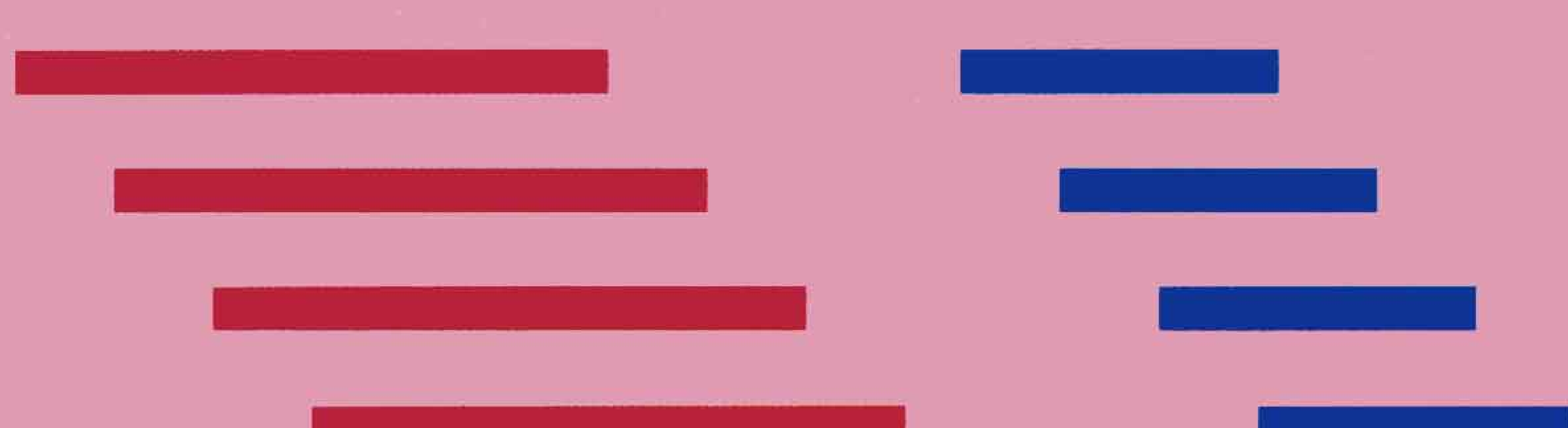
BREAKING THE CODE

Each letter below stands for a different digit. What numbers will make the problem work?

```
   S E N D
 + M O R E
 ---------
 M O N E Y
```

THREE SQUARES

Arrange the 8 sticks below to make three equal squares. The red sticks are exactly twice as long as the blue sticks.



FOUR 2'S

Use the operations

+ − × ÷

and parentheses to combine the four twos and make each equation true. For example, we could use four twos to make 0 by:

(2 − 2) × (2 + 2) = 0

2 2 2 2 = 1

2 2 2 2 = 2

2 2 2 2 = 3

2 2 2 2 = 4

2 2 2 2 = 5

2 2 2 2 = 6

2 2 2 2 = 8

2 2 2 2 = 10

BULLETIN BOARD

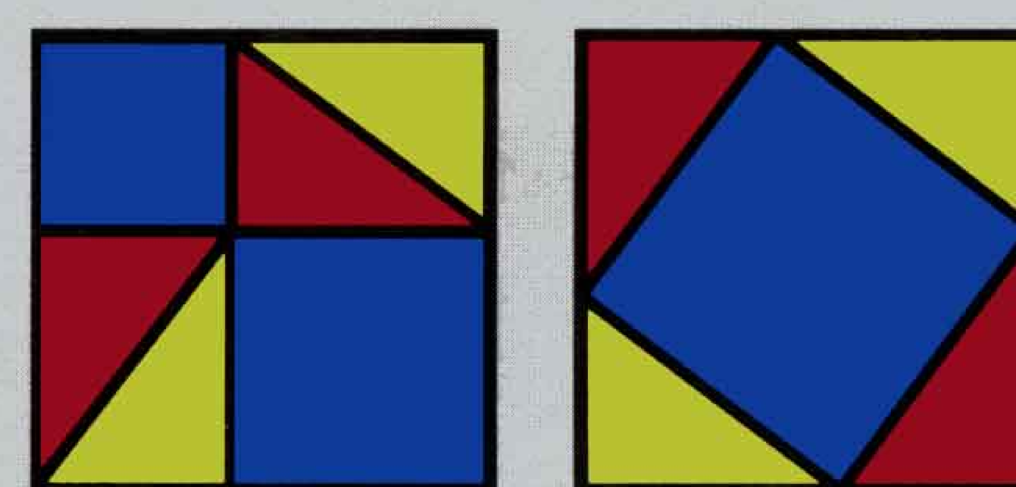
MATH COMPETITIONS

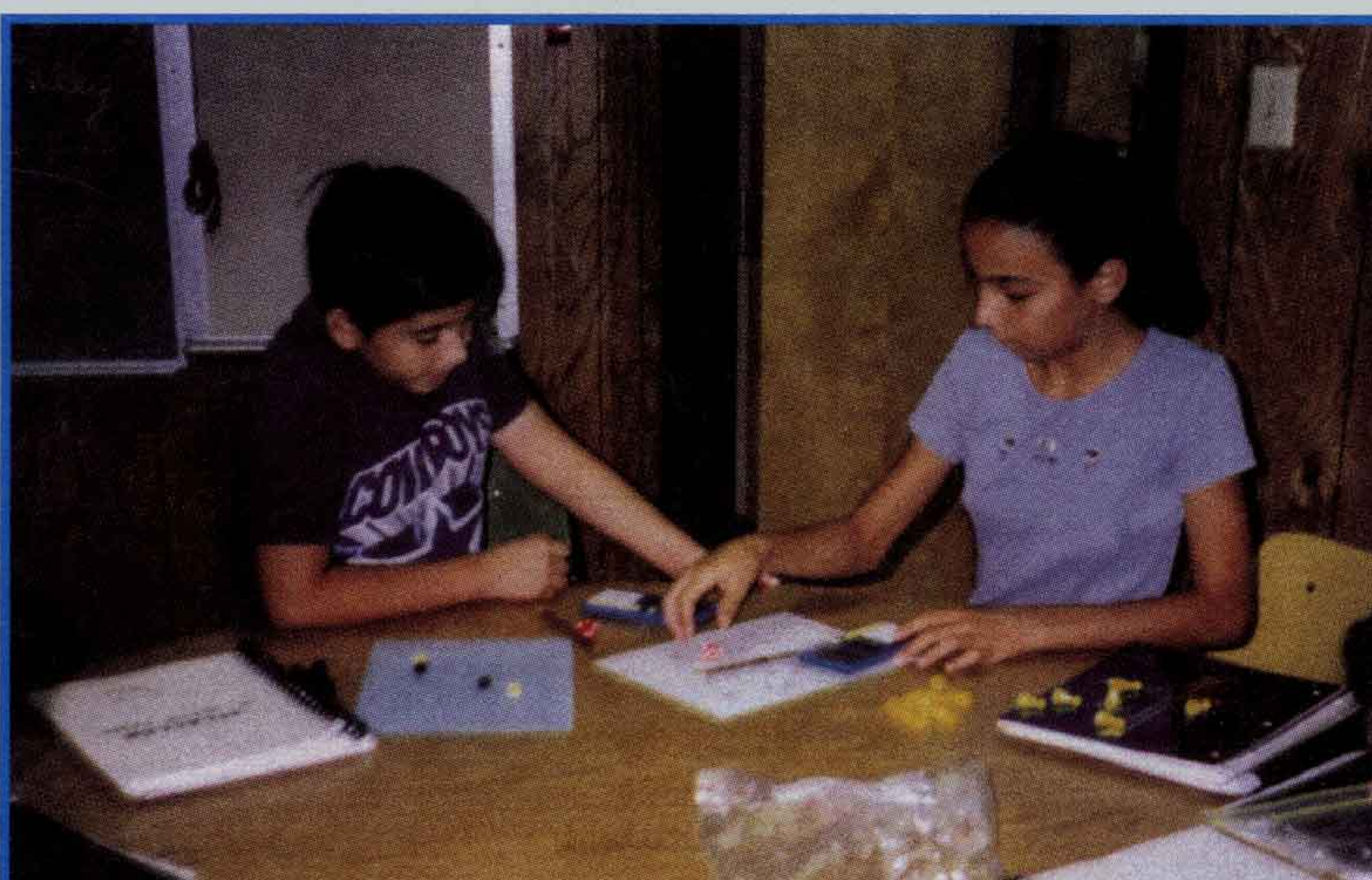
The American Junior High School Exam is great preparation for problem solving and high school. For information and registration materials, write:

Prof. Walter E. Mientka
AMC Executive Director
Dept. of Mathematics and Statistics
University of Nebraska
Lincoln, NE 68588-0658

or look on the web for information about the American High School Math Exam, AHSME, probably the most famous exam of all, at http://www.unl.edu/amc/ahsme.html

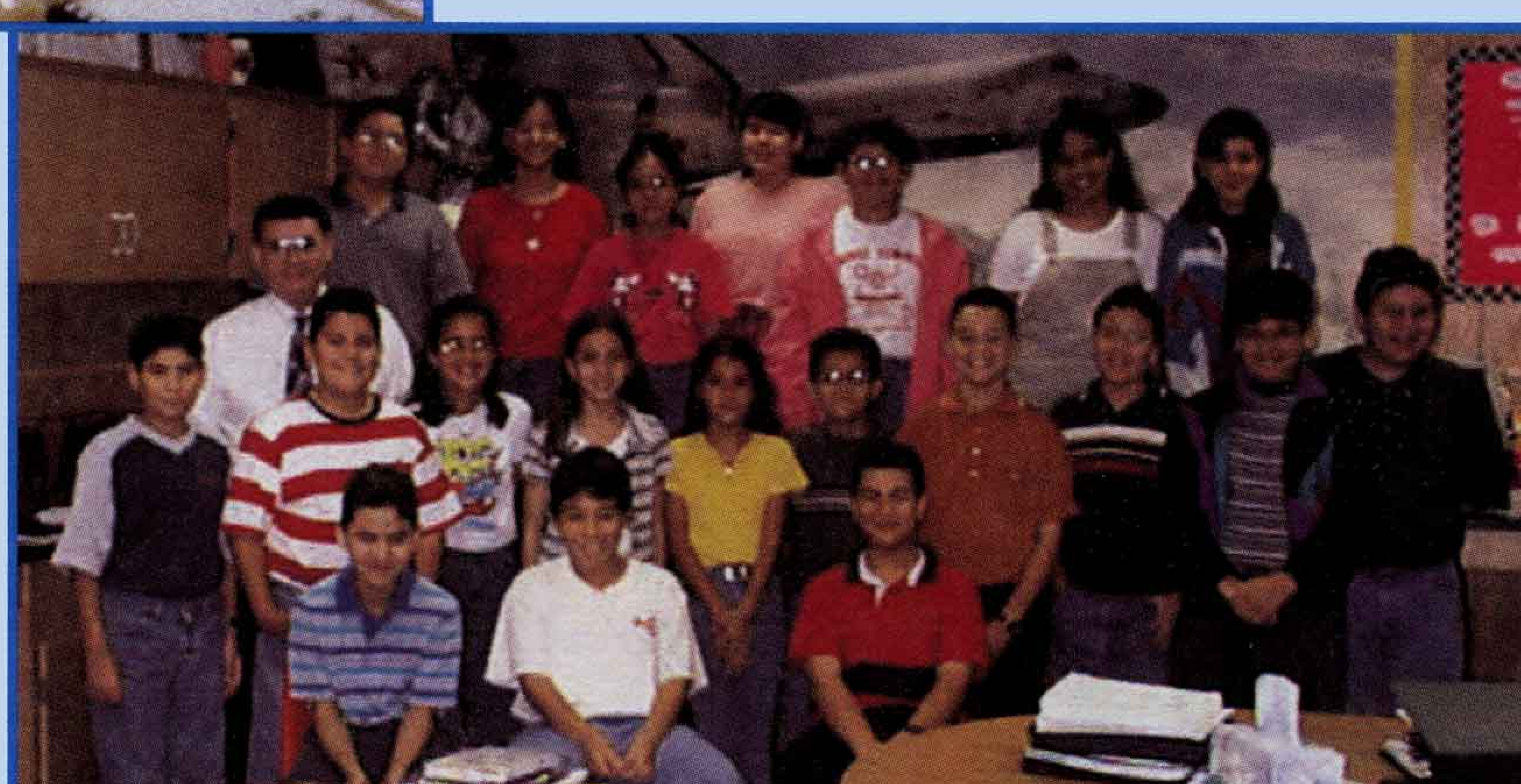
Math Reader is another kid's magazine for elementary students published by the SWT Math Institute for Talented Youth. It has fun problems you might also enjoy exploring. Check it out!





Students at Brown Middle School in McAllen, Texas are enjoying exploring math problems together. Pictured on the left are two of Eduardo Reyna's sixth grade students. Eduardo and Judy Carlin began a Junior Summer Math Camp at Brown Middle School in 1998, and plan to expand the program in 1999.

Pictured on the right are Eduardo Reyna and his sixth grade students from Brown Middle School. These students are using the Pythagorean Theorem (see page 4) to solve application problems.



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Math Notes

Dear Reader,

Welcome to our new magazine! *Math Explorer* is a magazine designed for intermediate students. I hope you'll have an exciting time exploring new math problems and sharing ideas with each other.

Math Notes is our Reader's Showcase. Write us with news from your school; about math events you've enjoyed; or with your own puzzles, activities, and problems. Please include:

• Your name • Your teacher's name • Any related pictures.

We'll publish as many letters as we can each month. I hope to hear from you soon.

Sincerely,

Max Warshauer

Max Warshauer