Math Explorer

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MATHEMATICS AND JUGGLING

Sudoku Puzzles?

JUGGLING THE NUMBERS

Saunders Mac Lane, 20th Century Mathematician

Vol. 8.1 Fall 2005

Math Explorer

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Saunders MacLane:

20th Century Mathematician

by Hiroko Warshauer

Many famous mathematicians, such as Pythagoras, Euclid, and even Fermat lived many hundreds of years ago. There are, however, mathematicians who lived in our lifetime and will be remembered well into the future. Saunders MacLane, was an important figure in the 20th century who laid the foundation for branches of mathematics that include algebraic topology, homology algebra, and category theory.

Along with his colleague Samuel Eilenberg, MacLane initially developed Category Theory as a language that explained transformations from one area of mathematics to another. He wrote the book, "Category Theory for the Working Mathematician," and Category Theory has since become a field of study in its own right. He and another colleague, Garrett Birkoff, published "Algebra" and "Survey of Modern Algebra," a leading textbook used by many university students today. The Eilenberg-MacLane spaces used in topology are named for the two famous mathematicians.

Born in Connecticut in 1909, MacLane attended Yale and the University of Chicago. He received his PhD from the Mathematisches Institut of Gottingen in Germany, an eminent mathematics center of that time. There he studied with two prominent mathematicians, Hermann Weyl and Paul Bernays. The political conditions in Germany in the early 1930's began to deteriorate as the Nazis came to power and many Jewish mathematicians or those with Jewish connections were removed from universities. MacLane quickly returned to the United States after defending his dissertation, "Abbreviated Proofs in the Logical Calculus."

After teaching at Harvard, Cornell and Columbia Universities, in 1947 he took a faculty position at the University of Chicago. There he went on to supervise 39 PhD students, many who themselves became prominent mathematicians, including Irving Kaplansky; John Thompson, a Fields Medalist; and David Eisenbud current president of the American Mathematical Society, a position which MacLane also held in 1973-1974.

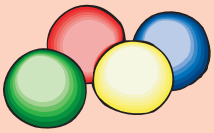
Eisenbud remembers his former teacher: "He was a figure of great honesty and integrity, who worked hard to advance research and to serve the mathematical community. His belief in the good, the right, and the rational, his care for the essence of mathematical ideas, his powerful enthusiasm, and his essential optimism were and are deeply attractive to me." Saunders MacLane died April 15, 2005 at the age of 95 in San Francisco.

References:

http://www-news.uchicago.edu/releases/05/050421.maclane.shtml
http://www-grops.dcs.st-and.ac.uk/~history/Mathematicians/MacLane.html
Focus: Newsletter of the Mathematical Association of America. August/September 2005. V. 25, no.6

PROBLEMS PAGE

1. Suppose you place 4 juggling balls, each a different color, in a hat. The juggling balls are red, blue, green and yellow. If you pick out two of these balls, what is the probability that neither ball is yellow?



2. A country has 1-cent, 2-cent, 5-cent and 10-cent coins. In how many ways can you make 15 cents using a combination of these coins?

3. Using only multiplication signs, parentheses, addition signs, and copies of the number 1, we can try to write each positive integer with as few 1s as possible. For example: $6 = (1 + 1) \times (1 + 1 + 1)$ so the integer 6 needs 5 ones. What is the smallest positive integer that needs 11 ones?

4. Hugo and August have a painting job to do. Working alone, Hugo could do the job in 6 hours. Working alone, August could do the job in 9 hours. If they work together, how soon could the job be completed?

***5**. Call an integer "happy" if the sum of its digits is 10. How many "happy" integers are there between 100 and 1000?

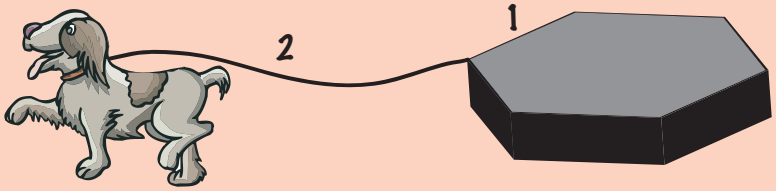
***6**. Compute the sum of a, b, and c given that:

$$\frac{a}{2} = \frac{b}{3} = \frac{c}{5}$$ and the product of a, b, and c is 1920.

***7**. Buses from town A to town B leave every hour on the hour. (for example: 6:00, 7:00, ...) Buses from town B to town A leave every hour on the half hour. (for example: 6:30, 7:30,...) The trip from town A to town B takes 5 hours. Assume the buses travel on the same road. If you get on a bus from town A, how many buses from town B do you pass on the road?

***8**. Oliver has a 7-digit phone number ABCDEFG. The sum of the number formed by the first 4 digits ABCD and the number formed by the last 3 digits EFG is 9063. The sum of the number formed by the first 3 digits ABC and the number formed by the last 4 digits DEFG is 2529. What is Oliver's phone number?

***9**. Porky's doghouse has a regular hexagonal base that measures one meter on each side. Porky is tethered to a 2 meter rope which is fixed to a vertex. What is the area of the region outside the doghouse that Porky can reach? Calculate an approximate answer by using $\pi = 3.14$ or $22/7$.



**These problems are from the 9th Primary World Mathematics Contest.*

Answer: 1) 1/2 2) 22 3) 22 4) 3 hours 36 minutes 5) 54 6) 40 7) 10 8) 8371692 9) 9.42 or 66/7

Juggling Mathematically

by Bill Finkenkeller

Have you ever watched a juggler perform silly tricks? Jugglers have an array of objects that they toss in the air, from balls, clubs, hats, and even flaming torches. Did you know that jugglers can use mathematics to help them come up with new patterns that they can use for tricks? Let's examine how this is done.

A juggling pattern made through a simple pattern of numbers is called a **site swap**. To make a site swap, some assumptions about the type of juggling will be used:

1) **A juggler has 2 hands** (sorry octopus jugglers)
2) **A juggler alternates moving his/her hands:** So a juggler throws first from his right hand then from his left, alternating constantly.
3) **A juggler can only have one ball in his hand at a time.**

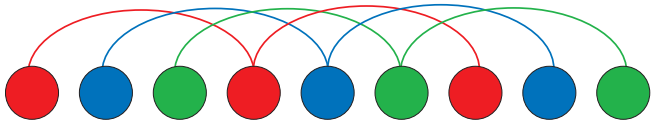
The simplest juggling pattern, the 3-ball **cascade**, happens to be a site swap. When a juggler throws a ball in a 3-ball cascade, that ball will be thrown again exactly 3 throws later. So we denote this throw by a 3. Then the next ball is also thrown again 3 beats after it is thrown the first time, so we denote this throw by a 3. In fact, all throws are the same 3 beats long. So this site swap can be denoted as 33333… , or simply as 3.

We can write out the beats in a diagram showing each throw and each ball in the pattern. The pattern of throws is a simple left, right, left, right… and we can indicate each ball by a different color. The 3-ball cascade with red, blue, and green balls would look like this:

L R L R L R L R L…
3 3 3 3 3 3 3 3 3

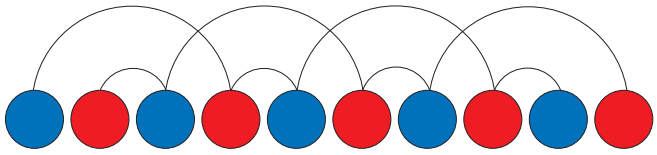
The first L indicates the first throw of the red ball from the left hand. Then the red R indicates that on the 3rd throw the red ball is thrown again from the right hand.

Consider the corresponding diagram below.



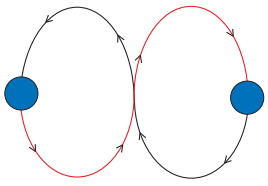
We can make more complicated patterns by using multiple numbers. A throw that lasts for more beats also goes higher than a throw that lasts for only a small number of beats. A pattern that you could try right now is 31. Note that a 1 means you simply hand the ball to the other hand to be thrown next. It looks like this:

R L R L R L R L R L…

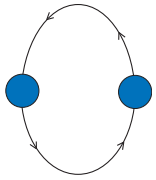


3 1 3 1 3 1 3 1 3 1

This pattern uses only 2 balls and is called a **shower**. A shower pattern is a pattern with balls going in a circle rather than in the weaving cascade pattern.



Cascade

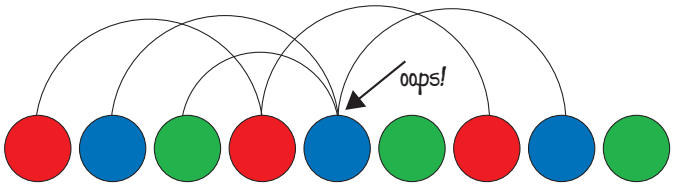


Shower

Now we will examine what keeps a random sequence of numbers from being a site swap. Remember that a juggler cannot have 2 balls in his hand at the same time. We will illustrate with the pattern 332. We attempt to color this pattern with 3 balls:

L	R	L	R	L? ...
3	3	2	3	3 ...

What color should the black L be? We know that the blue ball thrown from the right hand should be thrown 3 beats later and this is exactly the black L. But we also know that the green ball should be thrown 2 beats after the first green L. This is exactly the black L. Thus we have a conflict in that the black L cannot have both the green and blue ball at the same time. Look at where we have a problem.



Actually, this conflict happens whenever the sum of the sequence of numbers is not divisible by the number of numbers. For instance, 332 is not a valid pattern because 3+3+2 = 8 and 8 is not divisible by 3 (there are 3 numbers). 4522 is not valid because 4+5+2+2 = 13 and 13 is not divisible by 4.

However, the conflict of colors above happens even though we have a sequence that adds correctly. 432 is an example where the sum of the numbers (9) is divisible by the number of numbers (3), but the pattern is not a site swap because more than one ball lands in the same hand at the same time.

The number of balls involved in a trick is a good indication of the trick's difficulty. A site swap using 6 balls is going to be almost impossible, while one with 4 balls can be done by a good juggler.

A cool fact about site swaps is that we can know how many balls are involved simply by doing the same math we did to see if the pattern exists. We add up the numbers and divide by the number of numbers-that is how many balls are used. So in the pattern 31, (3+1)/2 = 2 would have given us the correct number of balls. And in a pattern such as 453, we can do the same process (4+5+3)/3 = 4 to know there are 4 balls. So a pattern such as 71 (a shower of 4 balls) can be considered easier than the pattern 5 (a cascade of 5 balls) just by doing the math.

Are the following site swaps valid? How many balls do they use?

441	501
52	91
7333	344
435	531

Next time you see a juggler doing amazing tricks, see if you can determine the site swap pattern he/she is using. And have fun learning to juggle!

Bill Finkenkeller has been juggling since he was 13 and now juggles dangerous things like fire against the advice of his mother. He recently graduated from Rice University and is now a graduate student in Mathematics at Texas A&M University.

Puzzle Page

Math Explorers: We want to print your work! Send your original math games, puzzles, problems, and activities to Texas Mathworks, 601University Dr., San Marcos, TX 78666

If Amy bicycles up a mountain at an average speed of 10 mph and then goes back down at 20 mph, what is her average speed?

In the following problem, each letter represents a different numerical digit. No letter represents a zero. What number does each letter represent?

```
  WRONG
+ WRONG
-------
  RIGHT
```

Word Search

Forwards or backwards, up, slanted, or down. Where can the words in this puzzle be found?

Juggle
Cascade
Site
Swap
Shower
Multiple
Category
Algebra
Sudoku

C J E T I S S I J L K S B A
F S U F O V E Z U P P H V D
Q P Y G N J E A V X H O M G
D R R N G J Q L S H E W R B
O L O N Y L X O P D Q E K S
U E G O R U E T A I Z R U G
P H E D F Y R C Y F T D W G
Z E T Z L T S O B X O L R C
R P A A Y A N I N K H Q U B
F I C S C V P G U K C N B M
G P P Q B Y U J D C O T B J
Z R A O I K T U U V G F T K
Y A Q W A D C Q M K F Q R N
T T D D S A L G E B R A D E

Using all the digits 1, 2, 3, 4, 5, 6, 7, 8, and 9, complete the mutiplication problem below.

```
   2 _ _
 X   _ 8
---------
 5 _ _ _
```

Bulletin Board

Junior Summer Math Camp a Success

Over 200 students attended the Tenth Annual Mathworks' Junior Summer Math Camp held at Hernandez Intermediate School in San Marcos, Texas. Here is what some campers had to say:

I like math camp because it is fun with math, making new friends and learning with good teachers.
-Timothy, 4th Grade

I love this camp, it's fun and makes me think. I love to make my brain hurt.
-Alexandra, 6th Grade

I love math! This camp is awesome! I never want to leave!
-Tucker, 4th Grade

Congratulations Jeffrey and Mark!

Congratulations to former Junior Math Campers, Mark Zhang and Jeffrey Chan for their outstanding achievement in this year's National MathCounts competition. The mathletes were honored by the president at the White House this past May. For more information on the MathCounts program for your school, visit *www.mathcounts.org*

White House photo by Paul Morse

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Sudoku

Have you ever done a crossword puzzle? You fill in the empty boxes with letters that eventually form words, across or down, that satisfy the clue. Now, did you know that there's is a numerical game that resembles a crossword puzzle. No clues are written, but you fill the empty boxes with numbers that satisfy certain rules. The game, called Sudoku (a Japanese word), is popular in Japan and the United Kingdom and is gaining popularity in the United States.

A puzzle consists of a 9 by 9 square grid within which there are nine 3 by 3 squares (called regions) indicated with dark lines. The objective of the game is to fill in each box with digits from 1 through 9 so that every row and every column contains the digits from 1 to 9, no digit repeated and not necessarily in order. Furthermore, each region must contain all 9 digits. Each Sudoku puzzle comes with some of the squares already containing numbers. These are the clues. Let's get started with the following puzzle.

	1				2	3	4	
			4	5	6			
				3		6	7	8
		6	2		9			
8		7				1		9
			8		5	4		
4	3	5		9				
			6	2	7			
	6	2	5				9	

You may have already noticed that the top right corner box must be a 5. Why? We certainly need one in the upper right region. It can't be in the second row, since the second row already has a 5 along the big square. The last row of the top right region is all filled so it must be at the top right corner.

Let's continue working together in this rightmost region, though there is no set place where you should or must begin. Notice that the rightmost column has a 9 as does the second from the right column. This forces the third column of the top region's middle term to be 9. Do you see why? You can't repeat a digit in any column but within each region you must use all the nine digits. So we've filled in another box.

With a similar strategy, or some you come up with through your reasoning, try finishing the remainder of the puzzle. Once complete, you can check your answer below. The following websites have additional Sudoku puzzles you can download or play online.

Have fun, but be careful-you might get hooked!

References:
http://www.sudoku.com
http://www.monterosa.co.uk/sudoku
http://www.suntimes.com/sudoku

6	1	9	7	8	2	3	4	5
3	7	8	4	5	6	9	1	2
2	5	4	9	3	1	6	7	8
5	4	6	2	1	9	7	8	3
8	2	7	3	6	4	1	5	9
1	9	3	8	7	5	4	2	6
4	3	5	1	9	8	2	6	7
9	8	1	6	2	7	5	3	4
7	6	2	5	4	3	8	9	1

Dear *Math Explorers*,

Welcome to the 2005-2006 season of Math Explorer magazine. We look forward to bringing you interesting articles such as in this issue where we look at mathematics and juggling! We continue to include challenging problems and puzzles for your exploration. Our biographies will include a range of mathematicians, even those who lived within our lifetime. Saunders MacLane, who passed away in April was an eminent mathematician and enthusiastic teacher. I had the good fortune to take two of his classes as an undergraduate at the University of Chicago. He was an inspiring teacher who generously shared his love of mathematics with students and to the larger community. As students, you too will surely encounter teachers from whom you will learn a great deal.

We always welcome hearing from you with any solutions, ideas or suggestions. We look forward to sharing our journey into mathematics with you in the months ahead.

Sincerely,

Hiroko K. Warshauer

Hiroko K. Warshauer, Executive Editor