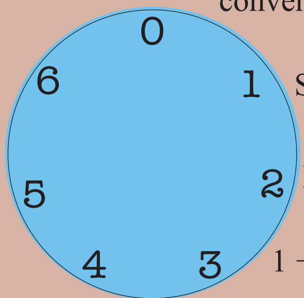
## MATH ODYSSEY

by Max Warshauer

Consider the numerals on most clocks. Usually the numerals go around a circular face and number from 1 to 12. Suppose we look at a similar circular face in which the numerals go from 0 to 6. Mathematicians call this set Z mod 7, thus $Z_7 = \{0,1,2,3,4,5,6\}$. The capital Z is often used to represent the set of integers, {0, 1, -1, 2, -2, 3, -3, …..}. Notice $Z_7$ is the set of possible remainders obtained if we divide any integer by 7. We are going to equip $Z_7$ with two basic operations, addition and multiplication. To describe these operations, it is convenient to arrange the elements of $Z_7$ in a circle as follows.




Suppose that we want to add two elements of $Z_7$, say add 5 to 6. Start at 6 on the circle and count clockwise a total of 5 steps. That brings us to 4. So we say $5 + 6 = 4$ in $Z_7$ . Here are some more examples of addition in $Z_7$.

$1 + 2 = 3 \qquad 6 + 6 = 5 \qquad 2 + 5 = 0 \qquad 3 + 4 + 5 + 6 = 4$

Subtraction is similar. For example to compute 2 - 6 start at 2 on the circle and count 6 units counterclockwise. This brings us to 3, and we say $2 - 6 = 3$. We could interpret this as saying that $-4 = 3$ in $Z_7$. (This does make sense because $3 + 4 = 0$ in $Z_7$.)

We multiply using the same idea. To multiply 6 by 2 just add 6 to itself twice: $6 \times 2 = 6 + 6 = 5$. To multiply 5 by 4, we add 5 to itself 4 times: $5 \times 4 = 5 + 5 + 5 + 5 = 6$ in $Z_7$. Alternatively we could first multiply them as integers to get $5 \times 4 = 20$, and then count 20 units clockwise from 0 to end up at 6 as before. Here are a few more examples of multiplication in $Z_7$.

$2 \times 3 = 6 \qquad 3 \times 4 = 5 \qquad 4 \times 6 = 3 \qquad 2 \times 3 \times 4 \times 5 = 1$

Now that we understand addition and multiplication, let's try to do more interesting calculations.

Can you find $^1/_2$ in $Z_7$? If we view $^1/_2$ as "1 divided by 2," can you find a number which when multiplied by 2 gives 1? Give it a try. (Answer below.) How about -5? (Answer also below). This is the fascinating world of modular arithmetic. You may like to continue exploring this idea on your clock at home in $Z_{12}$.

*Answers: 4 and 2*

Dear *Math Explorers*,

The new year is right around the corner. We close the 2005 year with an article that may help people make decisions by carefully analyzing the meaning of certain test results. Our biography takes us back to Baghdad, where our mathematician lived, and we also look at arithmetic on a set of numbers that resembles "working around the clock."

2006, our new year, is clearly not a prime number. Can you determine what whole numbers are factors of 2006? If you find some other interesting properties of 2006, please share them with us and we will post them on our website.

May you and your family have safe and happy holidays!

Sincerely,

Hiroko K. Warshauer

Hiroko K. Warshauer, Executive Editor

Vol. 8.2 Winter 2005

# Math Explorer


MATHEMATICS AND DECISION MAKING

## MOD 7

## GET A SECOND OPINION!!

## Al-Khorezmi and Algorithms

# Math Explorer

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# Al-Khorezmi

by Hiroko K Warshauer

Have you heard the mathematical terms *algebra* and *algorithm*? Do you know where these words originated? Centuries after the decline of the Greek and Roman Empires, in the area we now call the Middle East, numerous mathematicians of medieval Islam made significant contributions. Mukhammad ibn Musa Al-Khorezmi (or al-Khorezmi) lived from about 780-850 and was the chief mathematician in the "House of Wisdom", an academy of sciences established in Baghdad (now the capital of Iraq) by the Caliph Al Ma'mun. Al-Khorezmi's family came from the oasis of Khorazem, at the southern end of the Aral Sea, in what is now Uzbekistan. He is credited with helping establish among the Arabs the Indian numbering system, which we now know as our Hindu-Arabic numerals, using decimal notation and the zero. Previous systems of writing numbers used letters, such as the Roman, Greek and Hebrew numerals.

Al-Khorezmi and other Arab mathematicians, including Thabit ibn Qurra and Abu Kamil, extended the mathematical area known as algebra beyond the Greeks and Babylonians. It is thought that the term "*algebra*" (in Arabic, al-Jabr) goes back to a text by al-Khorezmi entitled, "Kitab al muhtasar fi hisab al gabr w'al muqubalah." And when al-Khorezmi's book on the new Hindu-Arabic numeration system reached Europe, the Europeans called its use "algorism" or "*algorithm*," from the author's name. Today "algorithm" means a systematic method of calculation, and with the rise of computers, a great deal of interest lies in developing efficient computer algorithms.

Much of al-Khorezmi's writings are practical, with examples and solutions in a manual form. The problems often pertained to solving equations though as was the case with the mathematicians of this era, the solutions as well as the coefficients in the equations were positive. The Islamic mathematicians did not deal with negative numbers. Al-Khorezmi did not use symbols and wrote everything in words. For example, "What must be the square which, when increased by ten of its own roots, amounts to thirty-nine?" would be written with today's modern algebraic symbols as "$x^2 + 10x = 39$." In fact, al-Khorezmi writes the solution in words and also gives two geometric solutions that utilizes a method called "completing the square." This process is often used even today when working with quadratic equations. Even after 1200 years, we find the contributions of mathematics from the Islamic world are signficant and useful.



# Bulletin Board

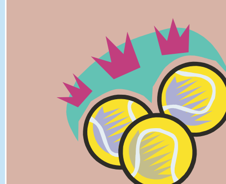
## Stamps Honor Four Mathematicians and Scientists


Richard P. Feynman, winner of the 1965 Nobel Prize in physics; Josiah Willard Gibbs a mathematical physicist; Barbara McClintock the 1983 Nobel Prize winner in medicine; and renowned mathematician John von Neumann are all featured in new 37-cent stamps issued by the US Postal Service.

## Fields Medal; the Mathematician's Nobel Prize

The Nobel Prize is awarded annually in six areas: physiology and medicine, chemistry, physics, economics, literature and peace. Mathematics is noticeably absent from this list. To recognize outstanding mathematical achievement, two gold Field Medals are awarded every four years at the International Congress of Mathematicians. Stay tuned for the announcement of the next Field Medalists in 2006.

## Math and Sports



What do billiards, water skiing and tennis have to do with mathematics? Visit this website to find out:
*http://sportsfigures.espn.com/sportsfigures/filmroom.htm*

## Check it out!

Visit these sites for fun puzzles and problems: *www.mathtv.org* and *www.archimedes-lab.org*
Mega-math includes lesson plans collected at Los Alamos laboratory.
Visit the website: *www.c3.lanl.gov/mega-math/*



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# How Worried Should You Be?

*by Max Warshauer*

People are often given medical tests to diagnose illnesses. When they get the results of these tests, they must then make decisions about how to proceed with their lives and what type of treatment they need. However, these medical tests are not perfect, and sometimes they give the wrong results. The reliability of a test is a measure of how likely it is that the test will give the correct result.

Let's suppose that we have a medical test for a rare disease that is 99% reliable. What does this mean? The answer is that if we give the test to 100,000 people, then the test will give the correct answer 99% of the time. So if we have 100,000 people, the test will give the correct answer for (100,000) x .99 = 99,000 people, and the incorrect result for (100,000) x .01 = 1000 people. If the test tells you that you have the disease, then we say the result of the test is positive. If the test says you don't have the disease, then we say the result of the test is negative. Of course, you would hope that the test result is negative.

The question is, does this test tell you that you actually have this disease? In other words, if the test result is positive, how worried should you be?

The answer to this question depends on two factors. The first factor is the reliability of the test, which we discussed above. The second factor is how rare the disease in question is.



One can estimate how rare a disease is by studying medical data. Let's further suppose that this disease is indeed very rare, and that only .1% of the population has this disease. So again, if we have 100,000 people, then the total number of people who have this disease is (100,000) x (.001) = 100 people.

Let's return to our question—how worried should you be? When you are told that you have this disease, the question is if this test was administered to all 100,000 people, how many people would be told this same thing? And of the people who tested positive, how many actually do have the disease?

First, we split our population into two parts—the people who have the disease, and the people who do not have the disease. The number of people with the disease is 100, and the number of people who do not have the disease is 99,900. Second, we ask how many people from each group test positive and are told they have the disease.

From the 100 people who have the disease, a total of 99 people will be told they actually have the disease, since the test is 99% reliable. One person in this group will be told he does not have the disease, even though he actually does have the disease!

From the 99,900 people who do not have disease, (99,900) x .99 = 98,901 people will be told they do not have the disease, and (99,900) x (.01)=999 people will be told they do have this disease, even though they actually do not have the disease.

We can record these results in a table below:

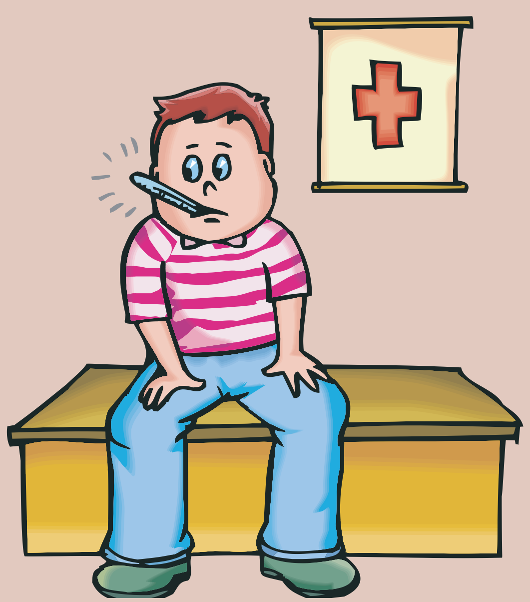
| Test Results | People who have the disease | People who do not have the disease | TOTAL |
|---|---|---|---|
| Positive<br>Person has the disease | 99 | 999 | 10,908 |
| Negative<br>Person does not have the disease | 1 | 98,901 | 98,902 |
| TOTAL | 100 | 99,900 | 100,000 |

So how many people from our total population of 100,000 people will be told they have this rare disease? The answer is that 99 + 999 = 1098 will be told they have the disease, and 98,902 will be told they do not have the disease. So again, how worried should you be if you are told you have this disease? Since only 99 people actually have the disease out of the total 1098 who are told they have this disease, **the probability that you actually have this disease is 99/1098= .0901... which is less than 10%**.

However, what about the other group? Suppose you are told that you do not have the disease. How certain can you be that you actually do not have the disease?

Well, in this group, there are 98,902 people, and only 1 of these people actually has this rare disease. So **the likelihood that you have this disease if the test tells you that you do not have the disease is 1/98,902=.00001011 which is less that 1/1000%**.

In short, if the test tells you that you do not have the disease, then you can be almost certain that you don't have the disease. If the test tells you that you do have the disease, then sound medical advice would be to get some further tests and not worry too much. The reason you should not worry so much is that you probably don't have the disease (90% probability) and the test has given what we call a false positive. The next time you get a medical test, be sure to ask how reliable the test is, and how rare the disease is. Then, when you get your medical results, you will know how worried you should be!

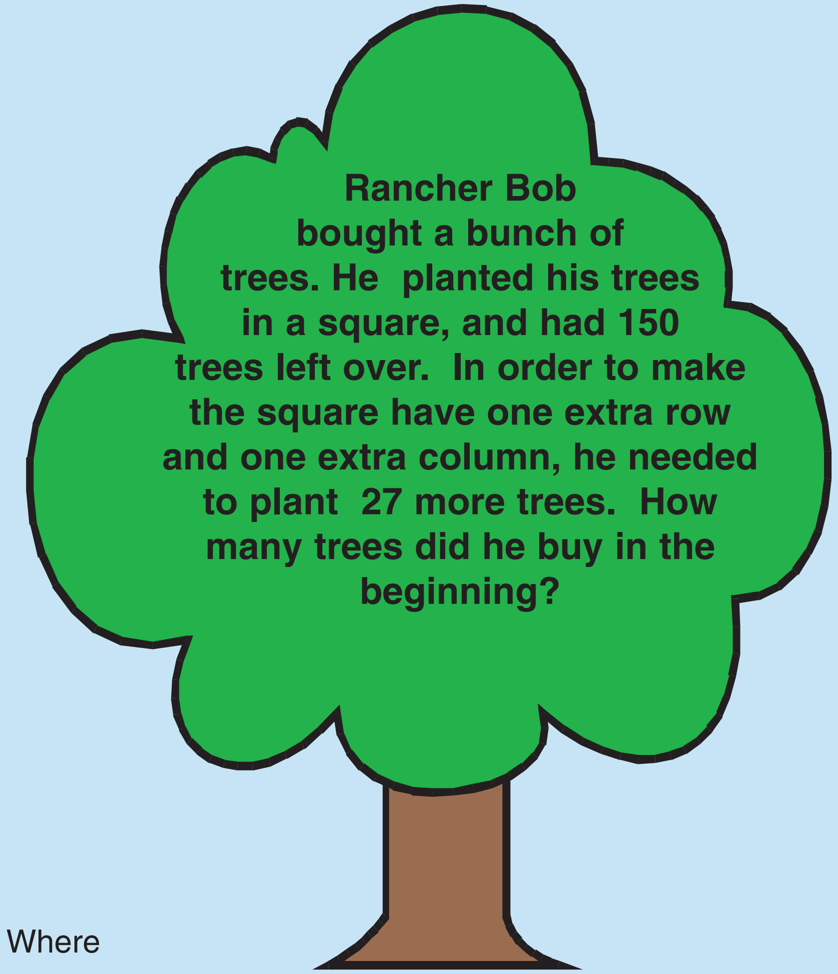


*Max Warshauer is a professor of mathematics at Texas State University-San Marcos and director of Texas Mathworks. He loves working on math problems!*

# Puzzle Page

*Math Explorers:* We want to print your work! Send your original math games, puzzles, problems, and activities to:
Texas Mathworks, 601 University Dr., San Marcos, TX 78666

Farmer Joe has several cows and chickens. He observed that they had a total of 64 eyes and 104 legs. How many cows did he have, and how many chickens?



**Rancher Bob bought a bunch of trees. He planted his trees in a square, and had 150 trees left over. In order to make the square have one extra row and one extra column, he needed to plant 27 more trees. How many trees did he buy in the beginning?**

## *Word Search*

Forwards or backwards, up, slanted, or down. Where can the words in this puzzle be found?

Algorithm
Reliability
Baghdad
Modular
Remainder
Probability
Positive
Likelihood
Negative

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H | X | S | D | L | F | K | H | Q | C | V | Z | T | M |
| P | O | S | I | T | I | V | E | S | O | T | I | Y | R |
| Z | D | M | U | R | B | K | T | U | R | Z | E | E | G |
| E | O | O | R | I | S | T | R | E | T | F | L | M | G |
| S | M | W | O | A | H | F | D | P | F | I | R | H | J |
| I | Z | Z | C | H | L | N | V | D | A | W | E | T | M |
| N | Y | T | I | L | I | B | A | B | O | R | P | I | O |
| L | E | Y | N | A | B | L | I | K | J | U | F | R | D |
| M | Y | G | M | Y | B | L | E | F | J | X | O | O | U |
| D | M | E | A | A | I | H | N | K | E | Q | Y | G | L |
| H | R | W | G | T | A | T | L | D | I | I | U | L | A |
| P | X | D | Y | H | I | F | N | X | K | L | Z | A | R |
| H | A | P | R | D | Y | V | N | E | Z | M | U | E | A |
| D | Q | U | Z | O | M | G | E | X | A | U | P | K | G |


If you have a digital watch, how many "palindromes" are there between 12:00 am and 12:00 pm?

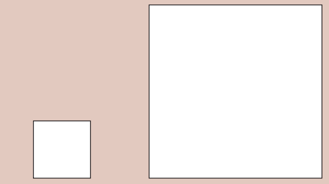
Example: 12:21 is a palindrome since it is the same forwards and backwards.





**1.** Jacob made a list of all the whole numbers between 1 and 100. How many times did he use the numberal 2?

**2.** A side of square T is three times the length of a side of square S. How many times greater is the area of square T than the area of square S?

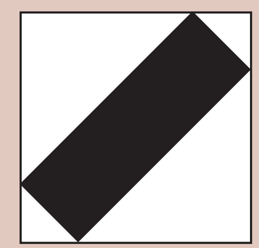



**3.** If $a$ and $b$ are positive integers and $65 = a^2 - b^2$ what is the value of $a$?

**4.** In how many different ways can 3 X's and 2 O's be placed in different squares of a 3-by-3 tic-tac-toe board, if the 3 X's must be in a line?

***5.** If the last number of the third row is 15, find the last number in the 80th row of this pattern:

$$1 + 2 = 3$$
$$4 + 5 + 6 = 7 + 8$$
$$9 + 10 + 11 + 12 = 13 + 14 + 15$$

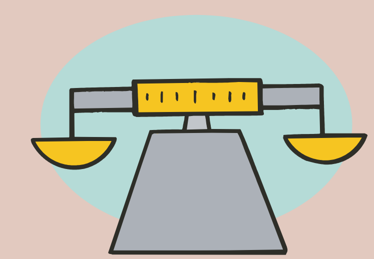


***6.** An isosceles right triangle is removed from each corner of a square piece of paper so that a rectangle of unequal sides remains. If the sum of the areas of the cut-off pieces is 200 $cm^2$ and the lengths of the legs of the triangles cut off are integers, find the area of the rectangle.


***7.** Some people in Hong Kong express 2/8 as Feb 8th and others express 2/8 as Aug 2nd. This can be confusing when we write 2/8, as we don't know whether it is Feb 8th or Aug 2nd. However, it is easy to understand 9/22 or 22/9 as Sept 22nd, because there are only 12 months in a year. How many dates in a year can cause this confusion?

***8.** There are four consecutive positive integers (natural numbers) less than 2005 such that the first (smallest) number is a multiple of 9 and the last number is a multiple of 11. What is the first of these four numbers?

***9.** On a balance scale, 3 green balls balance 6 blue balls, 2 yellow balls balance 5 blue balls and 6 blue balls balance 4 white balls. How many blue balls are needed to balance 4 green, 2 yellow and 2 white balls?



**These problems appeared in the 9th Primary Mathematics World Contest held in Hong Kong, July 2005*

**Answers:** 1) 20 2) 9 3) 9 4) 120 5) 6560 6) 56 7) 132 8) 257 9) 16