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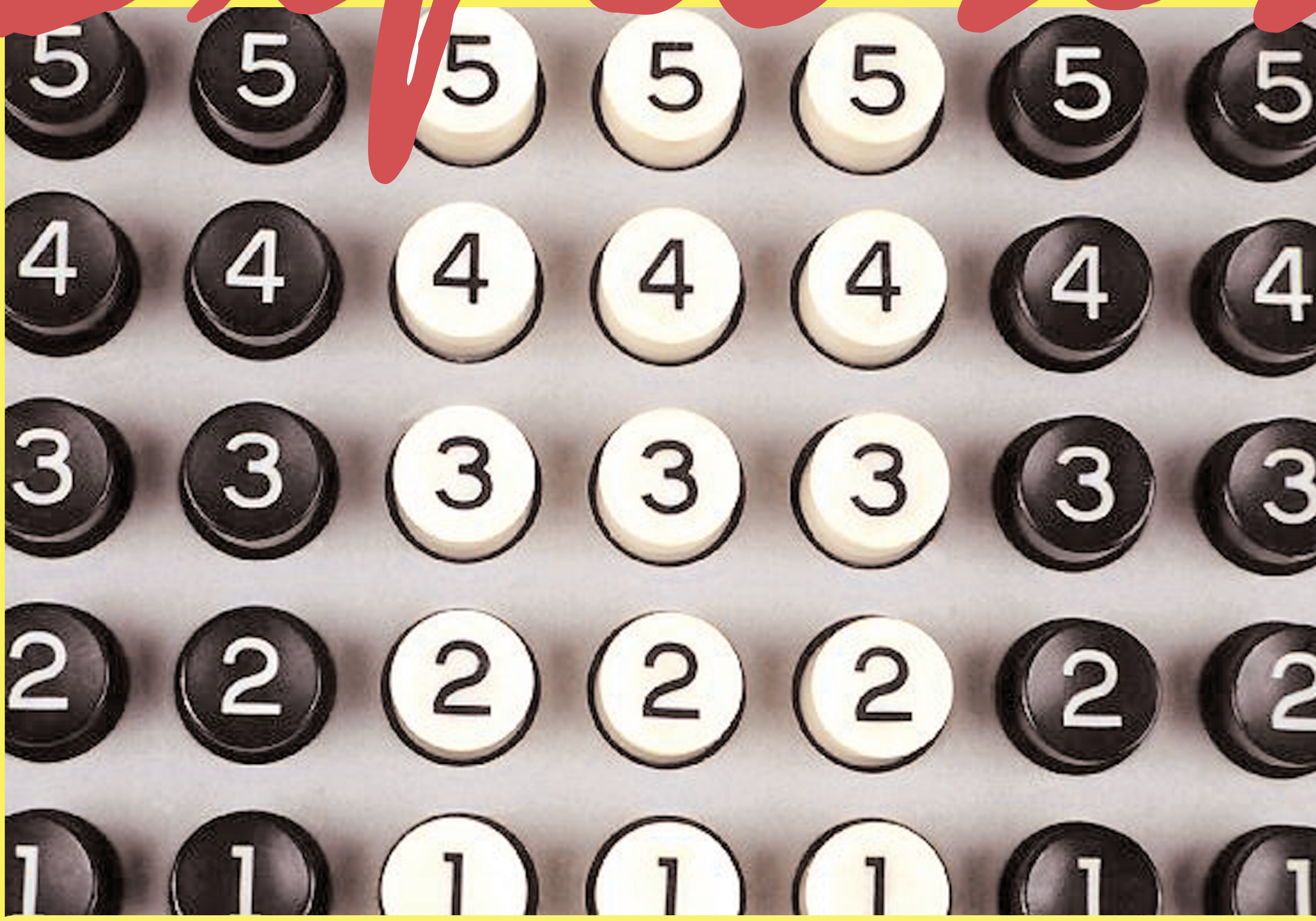

MATHEMATICS AND NONSTANDARD DIGITS

Wait a Minute...Think 60

Mathematics-Nonstandardized

Playfair, Scottish Mathematician

Vol. 8.4 Summer 2006

Math Explorer

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John Playfair

by Jean Davis

Many great works of mathematics and science have been criticized or even discarded completely because they were presented so poorly that no one could understand them. It takes a special person to take such a work, understand it, and rewrite it using clear language. John Playfair was such a man.

John Playfair was a Scottish mathematician born in 1748. Home-schooled until he was 14, he then attended the University of St. Andrews with the intention of becoming a minister. He was an exceptional student, particularly in mathematics and physics, earning his Master's degree by the time he was 17! He applied for a position at a university when he was only 18 years old. When he was unable to get a position, he became a parish minister, as his father had been. He still continued to study math and science, and kept company with distinguished scientists of that time.

In 1785 he was finally able to secure a position at the University of Edinburgh as chairman of the Department of Mathematics. In 1805, he became chairman of the Department of Natural Philosophy, what we now call physics.

In the 18th century, the study of geometry was primarily a study of ***The Elements*** of Euclid, a Greek mathematician who lived around 300 BC. Playfair published an edition of ***The Elements*** for his students, in which he improved the notations and gave explanations to make things easier to understand. In doing this, he restated the famous "**parallel postulate**" to read:

> *Given a line and a point not on the line, it is possible to draw exactly one line through the given point parallel to the line.*

Now this form of the axiom was not new, and Playfair gave credit to the mathematician Proclus. Nevertheless, it has been known as **"Playfair's Axiom"** ever since. Playfair's book was very popular and made the understanding of geometry possible for many students.

The second major accomplishment of Playfair was in the field of geology, which is the science of the earth. When his good friend, James Hutton, died, Playfair wrote a tribute to James, defending James' book on geology. Playfair was able to restate clearly parts of Hutton's book that were hard to understand. He helped create the modern science of geology and also gained great reputation for his friend James.

John Playfair earned his reputation primarily as an "explainer", rather than a "discoverer." He consolidated the learning of the past, and combined it with the new knowledge of his own time. This made it possible for others to continue with future research in mathematics and physics.

www-history.mcs.st-andrews.ac.uk/ history/Mathematicians/Playfair.html
www.1911encyclopedia.org/P/PL/PLAYFAIR_JOHN.htm

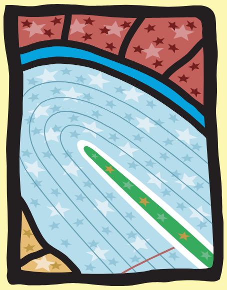
Jean Davis is on the mathematics faculty at Texas State University. She is particularly interested in the history of mathematics.

1. An urn contains 4 red balls, 5 green balls, and 3 white balls. Isabelle takes the balls out of the urn one by one, each time choosing a ball at random. What is the probability that she will remove all the white balls before removing any of the red balls?

2. Becky tosses a coin 5 times. What is the probability that she will get *heads* twice in a row at some point?

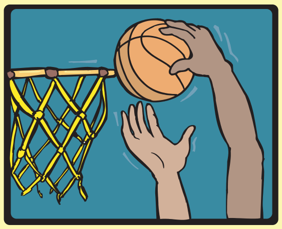
3. How many positive integers divide evenly into 2006?

4. The letters **a, b, c, d,** and **e** all represent different digits in the following equation. Find their values: 25450/**abac** = **de**

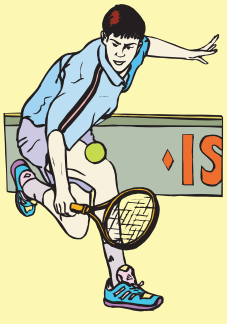


5. Notice that $3^2 + 3^3 = 9 + 27 = 36 = 6^2$. Find the next integer n such that $n^2 + n^3$ is a perfect square.

6. Sofia and Isaac run laps on a 1/4 mile track. They start at the same time but not at the same place on the track. They both run for 30 minutes. Sofia runs at 8.2 miles per hour and Isaac runs at 6.7 miles per hour. How many times will Sofia pass Isaac during the run?



7. The Bobcat basketball team scored 15 points in the first quarter of a game. In basketball, each time you score you get 1 point, 2 points or 3 points. How many different scoring sequences could have led them to get 15 points?



8. At the end of the basketball game, the team had scored 55 points. Heather wanted to know how many times they had scored 1 point, how many times they scored 2 points and how many times they scored 3 points. How many possible answers are there to Heather's question?

9. Aaron won a set in tennis against Jacob by 6 games to 4. Of the 10 games Aaron played, how many possibilities are there for which 6 games he won? How many possibilities are there if Aaron was never behind in the set?

10. Find the next number in the sequence: 1, 2, 2, 4, 2, 4, 2, 4, 6, 2, ...
(Hint: it has something to do with prime numbers)

Answer: **1)** 1/35 **2)** 19/35 **3)** 8 **4)** a=1 b=0 c=8 d=2 e=5 **5)** 8 **6)** 3 **7)** 7622 **8)** 280 **9)** 126;42 **10)** 6

Nonstandard Digits

by Daniel Shapiro

The ten digits from 0 to 9 are used in the standard **base ten** system to represent all whole numbers. For instance, 8572 represents the number

$$8 \cdot 1000 + 5 \cdot 100 + 7 \cdot 10 + 2.$$

This "place value" method has each digit count a different value depending on its place in the number. So with 8572, 2 is the units digit, 7 is the tens digit, 5 is the hundreds digit and 8 is the thousands digit. As we move left in the number, each digit counts quantities ten times as large as in the previous step. Mathematicians usually use exponents to represent these quantities, with 10^2 standing for one-hundred (10·10) and 10^3 standing for one-thousand (10·10·10). Then our example number is $8572 = 8 \cdot 10^3 + 5 \cdot 10^2 + 7 \cdot 10 + 2$.

The usual ten digits represent the first ten numbers, counting whole numbers in order from zero up to nine. With those ten digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) and the place-value system, we can write down any whole number. Students learn this in pre-school.

But wait a minute! Do we have to pick that particular set of digits to make this work? What if we introduce a new digit **X** which stands for ten. This extra digit provides two representations for some numbers (after all, X = 10). To balance this, let's throw out the digit 0. Then the only way to represent ten is by the single digit X. This **X-system** has these ten digits:

1, 2, 3, 4, 5, 6, 7, 8, 9, X.

Remember, we're still using the usual base ten system, but counting becomes a bit different since no digit "zero" is allowed. The number after 19 cannot be 20 since that uses a zero: instead we write it as **1X** (after *nine-teen* comes *ten-teen*?). Of course, after 1X come 21, 22, 23, and so on. Similarly we count ... , 27, 28, 29, 2X, 31, 32, ... , avoiding the digit zero.

What about one-hundred, which cannot be 100 any more since zero is forbidden? That becomes "ninety-ten" or **9X**. The next numbers after that are **X1, X2, . . .** The largest 2-digit number will be **XX** (one-hundred-ten) followed by the first 3-digit number, **111**.

The usual methods of addition and multiplication (with carrying) work as usual in this **X-system**. For instance:

$$\begin{array}{r} 1\ 1 \\ 2\ X\ 5 \\ +\quad 9\ 8 \\ \hline 3\ X\ 3 \end{array} \qquad\qquad \begin{array}{r} 1\ 6 \\ 5\ 2\ X \\ \times\qquad 7 \\ \hline 3\ 6\ X\ X \end{array}$$

Here's an explanation of the above multiplication problem on the right. *(We will leave the addition problem for you to figure out.)* Multiply 7 by the unit digit X to get 6X (sixty-ten, or seventy). Write down the X and carry the 6. Multiply 7 by the tens digit 2 and add the carried 6 to get 1X (ten-teen, or twenty). Write down the X and carry the 1. Multiply 7 by the hundreds digit 5 and add the carried 1 to get 36. Then the answer is 36XX. With standard notation this multiplication problem becomes

$$530 \cdot 7 = 3710.$$

That worked pretty well! Let's begin again, trying a different set of digits. Starting from the standard digits let's introduce the new digit **J** to represent the number -1 (negative one). Then for example 2J6 represents 2·102 + J·10 + 6, which equals 200 - 10 + 6, which is 196 in the standard system. Similarly 1J = 1·10 + J = 10 - 1 = 9. To avoid two representations of the same number, let's drop the digit 9 from the system. Then in this **"J-system"** we use the ten digits:

J, 0, 1, 2, 3, 4, 5, 6, 7, 8.

In this J-system we can still write down every whole number. For instance 19 is now represented as 2J. How can we write eighty-nine or ninety, knowing that the digit 9 is forbidden? Well, since nine is expressed as 1J, ninety should be 1J0. (To multiply a number by 10 just adjoin a zero on the right).*** To get eighty-nine, the number just before this one, we can just subtract 1 (which is the same as adding J):

89 = 1J0 + J = 1JJ.

To check, calculate 1JJ = $1 \cdot 10^2$ + J·10 + J = 100 - 10 - 1 = 89. To verify that addition methods still work, let's add 1 to ninety-eight. Since 98 equals 1J8, we obtain:

$$\begin{array}{r} 1\,J\,8 \\ +\quad\;\; 1 \\ \hline 1\,0\,J \end{array}$$

Here we first add units digits: 8 + 1 = 1J, write down J and carry 1. In the tens place, since 1 + J = 0, there are no further carries and we get the answer written above. To check this is the correct value, ninety-nine, calculate:

10J = 1·100 + 0·10 + J = 100 + 0 - 1 = 99.

Can you write down the numbers, in order, from 85 to 105 in this J-system?

Numbers starting with J are negative. For instance: J7 = J·10 + 7 = -10 + 7 = -3. Similarly, J4 = -6 and JJ = ¬11. For a larger number let's try J43J. This equals

J·103 + 4·102 + 3·10 + J
= -1000 + 400 + 30 - 1
= -571.

Are you surprised that every negative number can be expressed in the J-system without using any minus signs? For instance, let's try -284. Since 284 goes to the hundreds place, the trick is to go one step higher (one-thousand) and compute the difference: 1000 - 284 = 716. Subtracting 1000 from both sides we find: -284 = -1000 + 716. Since J is -1, this becomes:

-284 = J000 + 716 = J716.

This expresses the negative number -284 without any minus sign.

Lets try -371. With the same method, we first find 1000 - 371 = 629. Then -371 = -1000 + 629. Converting those two terms to the J-system (where 9 isn't a digit), we get J000 and 63J, and we obtain the conversion: -371 = J000 + 63J = J63J.

This method will work for every negative number! That is, every integer (positive or negative) can be represented in exactly one way in the J-system, without using any minus signs. This is certainly a nicer property than the old system that uses ten digits along with an extra "minus" symbol.

Wouldn't life be simpler if everyone converted to the J-system?

Exercises:

1. How is -12 represented in the J-system? One method is to start with the expression -12 = -100 + 88. Another method is to subtract 1 from -11. That is the same as adding J to JJ. Do this addition, remembering that J + J = -12 = J8.

2. We have looked at the X-system and the J-system. Think of another set of ten digits that could be the basis of another system. How hard is it to count, add, and multiply in your new system?

3. All examples in this article have been in base ten. Do these ideas work for other bases? For instance, it's interesting to investigate the base three system with digits J, 0, 1. Starting from zero and counting upwards, we get:

0, 1, 1J, 10, 11, 1JJ, 1J0, 1J1, 10J, 100, ...

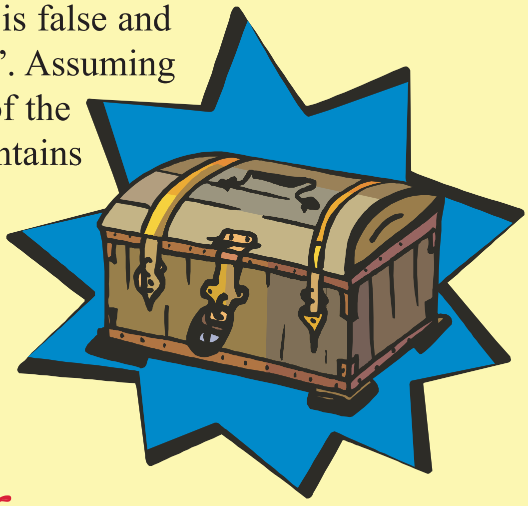
***Why does this work?

Daniel Shapiro is a professor of Mathematics at Ohio State University. He is also the director of the Ross Mathematics program.

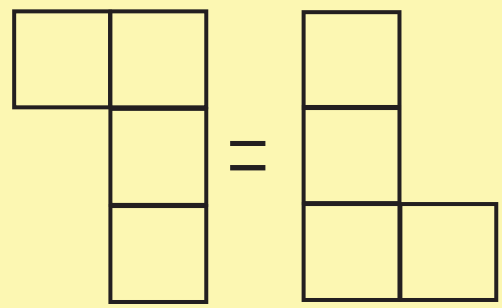
Puzzle Page

Math Explorers: We want to print your work! Send your original math games, puzzles, problems, and activities to:
Texas Mathworks, 601University Dr., San Marcos, TX 78666

Two chests are labeled **A** and **B**. A sign on box **A** says “ The sign on box **B** is true and the gold is in box **A**”. A sign on box **B** says “ The sign on box **A** is false and the gold is in Box **A**”. Assuming there is gold in one of the boxes, which box contains the gold?



How many different shapes can you make with 4 squares joined side to side? (*Rotating the picture doesn’t make it a new figure.)*



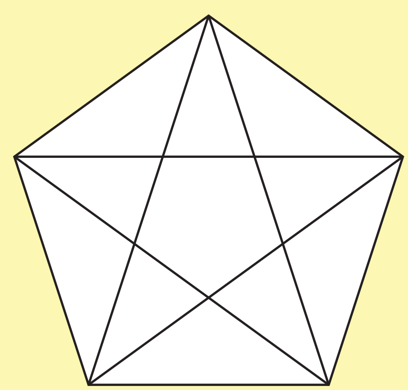
Word Search

Forwards or backwards, up, slanted, or down.
Where can the words in this puzzle be found?

Babylonian
Digits
Standard
Numerals
Sexigesimal
Decimal
Playfair
Exponents

L	S	E	E	R	T	A	D	P	R	N	E	N	L
A	A	T	S	D	Q	B	L	S	A	X	B	U	W
M	S	M	A	P	J	A	E	I	C	D	U	M	C
I	C	T	K	N	Y	U	N	C	L	P	U	E	R
S	F	X	I	F	D	O	H	D	D	J	F	R	N
E	R	B	A	G	L	A	N	W	C	K	W	A	X
G	A	I	W	Y	I	X	R	O	B	Y	L	L	E
A	R	W	B	A	C	D	Q	D	E	B	E	S	A
X	L	A	C	P	H	L	E	J	F	G	L	C	V
E	B	E	X	P	O	N	E	N	T	S	I	O	U
S	V	G	J	B	I	C	T	G	Z	R	U	F	C
M	D	A	G	J	R	C	B	B	L	N	D	H	Q
N	L	V	F	L	A	M	I	C	E	D	M	Q	B
I	Y	B	Q	J	W	K	Z	G	Z	T	O	A	K

How many triangles can be found in the diagram below?



Bulletin Board

Sudoku Craze

Sudoku, the popular puzzle with numbers has a version with shapes. Visit: *http://math.about.com/od/recreationalmath/ss/KidsSudoku.htm*
And here is a site with numbers: *http://www.websudoku.com/*

Prime Time

What is the Twin Prime Conjecture? It is a 2,300-year-mystery! Check out the following to learn more, song included: *http://www.pbs.org/wgbh/nova/sciencenow/3302/02.html*

Stamping through Mathematics

For some summer reading, consider a history of mathematics book that describes the many mathematicians included on stamps and their contribution in mathematics. Visit: *www.springer-ny.com* for information on the book, "*Stamping through Mathematics*," by Robin J. Wilson.

Check It Out!

Want to learn more about mathematics and its connection to other fields of study? Visit: *http://www.plus.maths.org.uk/*

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MATH ODYSSEY

by Hiroko Warshauer

Babylonian Math

Most of us know that there are 60 minutes in one hour and that there are 60 seconds in one minute. From that information, we can conclude that there are 3600 seconds in one hour. In terms of dividing up the hour unit into sixty parts and the minute unit into sixty parts, this tradition traces back in history to a civilization of the distant past, that of the Babylonians.

The Babylonian civilization flourished about 4000 years ago in the region along the lower Tigris and Euphrates Rivers, in present-day Iraq. While our numeration system is a **base 10** or **decimal system**, the Babylonian numeration system was based on groupings of 60, called **sexigesimal**. For example, they used 60 minutes for an hour.

Their writing was on clay tablets using a stylus, and the script is referred to as cuneiform, with wedge-shaped symbols. Tablets found in the 1800s date back to the period of Hammurabi (2000 BC) that contain the sexigesimal notation.

v represents one and **<** represents ten. The Babylonians also used a positional system where the far right represented numbers from 1 to 59. The position to its left is the 60's place so that 73 would look like the following in Babylonian numeration: **v <vvv** that is one 60 and 13 more. The system was also additive, so that 73 = 60 + 13. Can you write 782? Can you see how it might relate to the question: how many hours is 782 minutes?

While our decimal system takes fraction parts as tenths (or 1/10), hundredths (or 1/100), thousandths (or 1/1000) and so on, the Babylonian system represented fractional parts as 1/60, 1/3600, and so on. For example, we represent 3 hours 13 minutes and 57 seconds as 3;13,57 or 3 + 13/60 + 57/3600. Notice that the fraction 1/3 is equivalent to 20/60 so in Babylonian notation, we can write 0;20. Can you write 3/5 in the sexigesimal notation? What does 0;30 equal as a fraction in base 10?

Dear *Math Explorers*,

Summer is right around the corner, and for many of us, a sign of another school year coming to an end. We hope you can use your summer months to continue exploring the mathematics around you. Daniel Shapiro, director of the Ross Summer Mathematics Program and professor of mathematics at Ohio State University, shares his insights into mathematics with this issue’s main article on nonstandard digits. We trust it will have you thinking about just how a number system works!

We are looking forward to another exciting summer working with Math Campers here in San Marcos and in sites throughout Texas. Perhaps we will see you there. Certainly, we look forward to next fall for another year of math explorations. Send us your comments, suggestions, or ideas for next year's issues. Have a safe and fun summer.

Sincerely,

Hiroko K. Warshauer

Hiroko K. Warshauer, Executive Editor