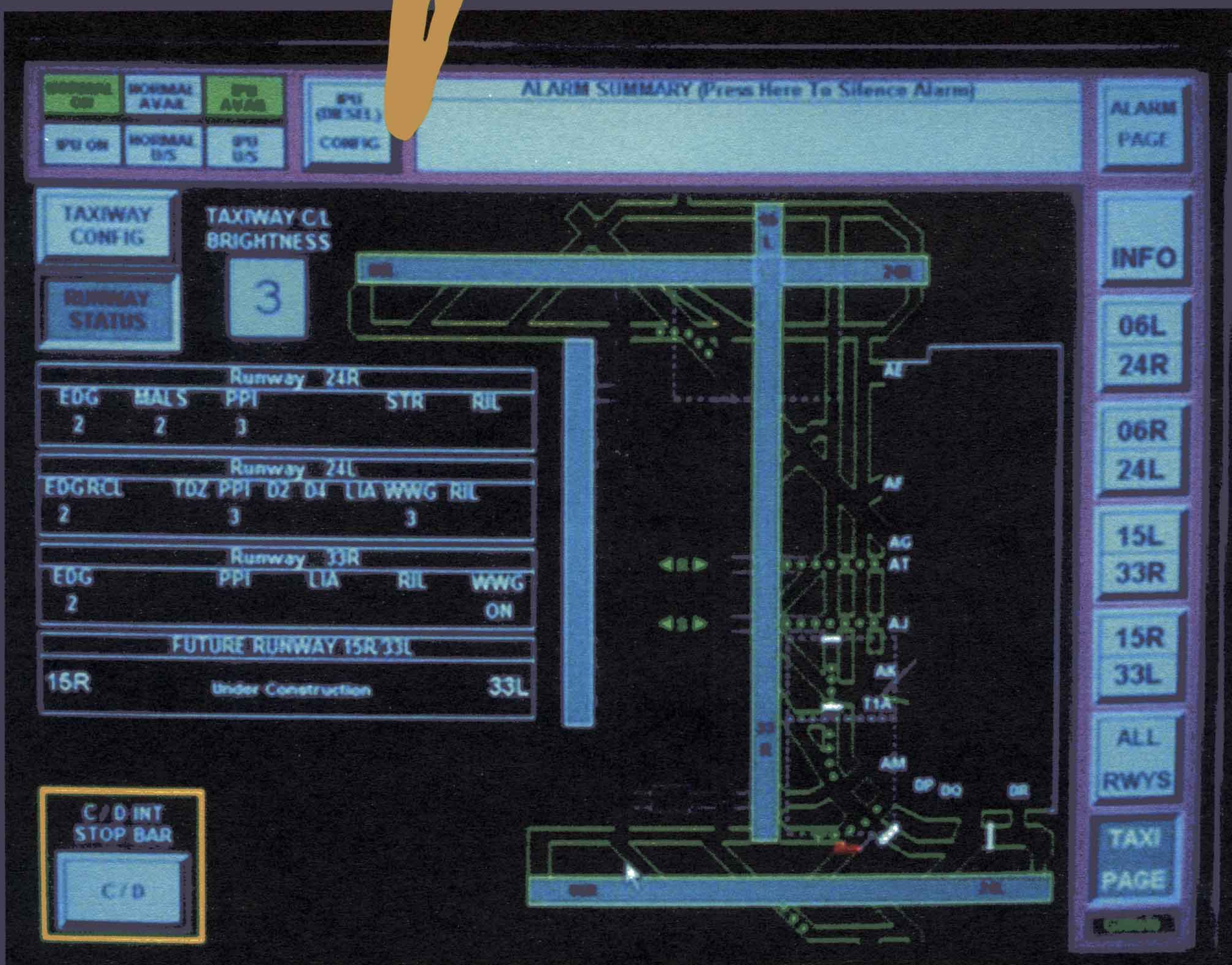
Math Explorer



MATH & COMPUTER SCIENCE

Turing Test measures **Intelligence!**

vending machine takes quarter!!

Abacus: You can **count** on us**!**

Vol. 3.1 Fall 2000

Math Explorer

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Math Explorer
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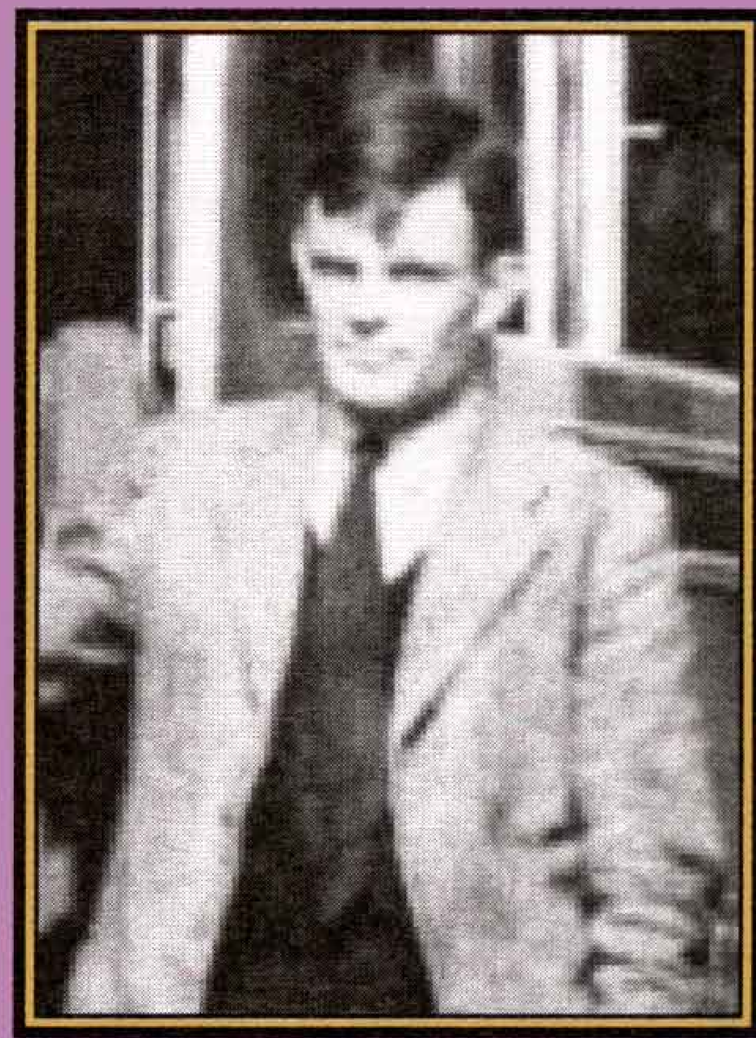
SWT

Alan Turing



by Carol Hazlewood

Alan Turing was born on June 23, 1912 in London, England. He developed an interest in science at a young age, and received a School Certificate in 1931,a degree from King's College at Cambridge in1934, and a Ph.D. from Princeton University in 1938.

In 1935, Turing started work on the Decidability problem: could there exist a definite method by which all mathematical questions could be decided, or solved? The answer needed a definition of "method" which would be not only precise but compelling. This is what Turing supplied—a theoretical machine able to perform certain precisely defined elementary operations on symbols. He presented convincing arguments that the scope of such a machine was sufficient to encompass everything that would count as a 'definite method.' The concept of the **Turing machine** has become the foundation of the modern theory of computation and computability.

During World War II, Turing worked full-time at the cryptanalytic headquarters in Bletchley Park, England. He made major contributions, using logic and statistics towards breaking the codes produced by the German Enigma machine.

After the war, Turing designed ACE, the Automatic Computing Engine, a single machine that could handle any programmed task, including symbolic computing. Turing did pioneering work on neural nets and envisioned remote terminals and programming languages at a time when there were none. He was one of the first people to use a computer to support his scientific research.

In 1950 he published a fundamental paper on Artificial Intelligence that describes the **Turing Test**. The Turing Test is a way to decide if a computer is intelligent: a human being and a computer are questioned by way of textual messages. If the interrogator cannot distinguish the person from the computer by questioning, then the computer is said to be intelligent. Turing used computers to study biological structures until his death in 1954.

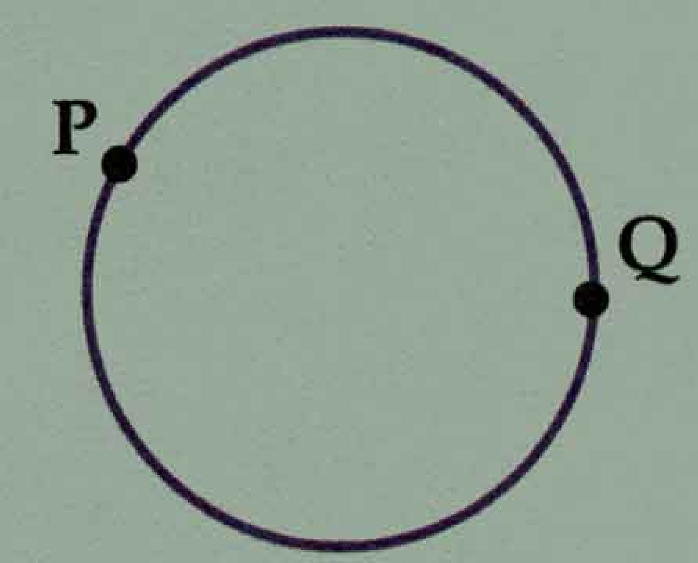
Reference: http://www.turing.org.uk/turing/bio/

Carol Hazlewood teaches computer science at Southwest Texas State University.

PROBLEMS OF THE MONTH

1. Consider the following sequence of numbers: 3, 4, 2, 3, 1, 3, ...
The first two terms of the sequence are 3 and 4. Each term after that is determined by taking the product of the two previous terms, dividing that product by 5 and writing down the remainder. For example, the fourth term is 3 because the product of 4 and 2 is 8 and the remainder when 8 is divided by 5 is 3. What is the 50th term of this sequence?

2. Suppose two points, P and Q, are drawn on a circle. Where would you place a third point (call it X) on the circle so that the area of the triangle formed by the three points would as large as possible? (Hint: Experiment with string, tacks or tape, etc.)



Send us your solutions! Every month, we will publish the best solutions on our website: www.mathexplorer.com. If we print your solutions, we will send you and your teacher free *Math Explorer* pens!

3. A list of numbers is wonderful if it has the following properties:
   a. The first number is 2 and the last number is 200.
   b. Each number in the list is either twice the number before it or five times the number before it.

How many different lists of numbers are wonderful?

4. Juan has three cube-shaped blocks. The first block has edges of length 1 inch, the second has edges of length 2 in. and the third block has edges of length 3 in. If Juan glues the three together to form a larger structure, what is the smallest possible surface area of this structure?

5. Diann draws a rectangle on a piece of paper. She measures the sides of the rectangle and draws another rectangle. The new rectangle is 50% wider and 25% longer than the old rectangle. What is the ratio of the area of the new rectangle to that of the area of the old rectangle?

6. Draw a model for a vending machine that dispenses orange and grape soda as well as cola. (See article on vending machines.)

7. Draw a model for a vending machine that accepts nickels and dimes as well as quarters.

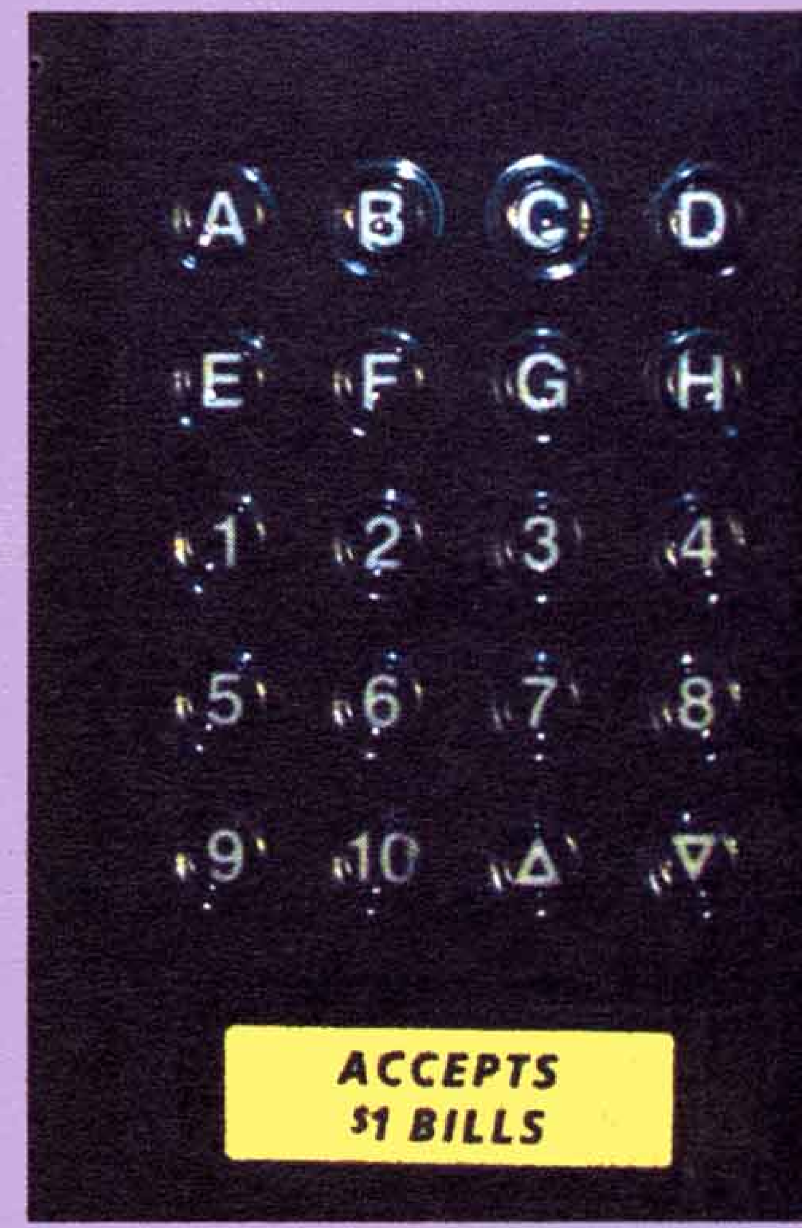
8. An artist has made 100 silk screen prints of a picture. His agent says that each silk screen is worth $20 now. But for each silk screen that is destroyed, the value of each remaining silk screen increases by $1. If he destroys a silk screen and sells the rest, will he make more money? How many silk screens would he need to destroy to make the most money?

9. How many base 2 numbers are there with exactly two 1's and no more than 8 digits?

10. Draw a model for a vending machine that accepts dollars as well as quarters and gives change by adding some states that remember how much change to give. *(See article on Vending Machines.)*

HOW DOES A VENDING MACHINE WORK?

by Carol Hazlewood

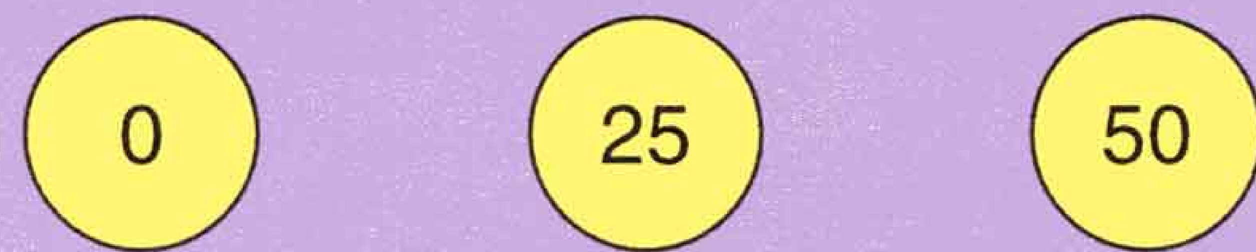


Let's take the customer's point of view: you walk up to the machine, deposit two quarters, push the button for your favorite soft drink, and a can rolls out.

Vending machines are large, heavy, and expensive. Let's build a **model** *-- a design on paper that shows how our vending machine works. We will first describe the important features of the vending machine, such as how much money has been deposited since the last soda was dispensed. Then we will describe how the features can change. For example, dropping a quarter into the machine will alter the amount of money that has been deposited. We will start with a simple design. We can add more features later.*

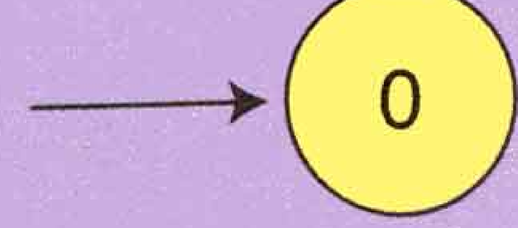
The owner of the machine wants to make sure that soft drinks are dispensed only after 50¢ is collected. We want the machine to be able to tell the differences if 0¢, 25¢, or 50¢ has been deposited. In fact, being able to know how much money has been deposited is one of the most important features of the machine, and we have to find a way to put this in our model. We will say that the machine is in a different **state** *for each amount of money that can be deposited. The states will be part of our model. In other words, we are going to remember how much money has been deposited by creating a state for each possible amount of money. Let's say, for now, that only quarters can be inserted into the machine. Then we will have a state to remember 0, another state to remember 25 and a third state to remember 50. We will show a state as a circle with a label inside:*

Three States



How much money has been deposited toward the purchase of your soda when you walk up to the machine? The customer before you probably didn't deposit coins and then walk away, so we'll suppose no money has been deposited -- that is, the machine is in state 0. Our **start state** *will be 0. We will add an arrow from nowhere into the state to show it is the start state:*

Start State



We'll make one rule that we must have a start state and another rule that we can't have more than one start state.

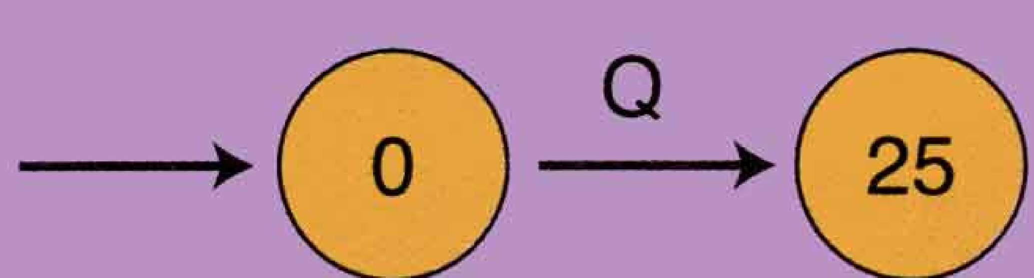
Next, we will make a "dispense soda" action of the vending machine into a state. Let's say our machine only serves cola. Getting the cold soda is our goal. The "dispense cola" state (dc) is the last state, or **final state**, *that the machine reaches during the sale of a soda. We may want to be able to tell the difference between the ending, "dispense cola," state and the other,* *intermediate, states that the machine may be in, so we'll use two circles for a final state:*

Final State

We will allow a model to have more than one final state.

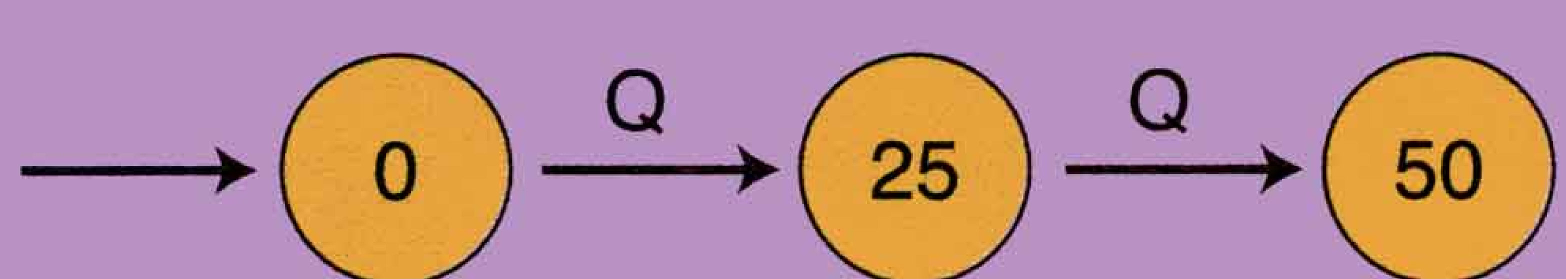
*Let's see how events can change the state of the vending machine. We will call these events **inputs**. The vending machine will change state if it receives an input -- a coin is deposited, or a "dispense" button is pushed. For example, if the machine is in state 0 and a quarter (Q) is deposited, the machine should change state, or make a **transition**, to state 25. In our model we will show this by connecting the states with an arrow to show there is a transition. In addition, we will label the arrow with the input that causes the transition.*

Depositing the First Quarter



A second quarter takes the machine to state 50:

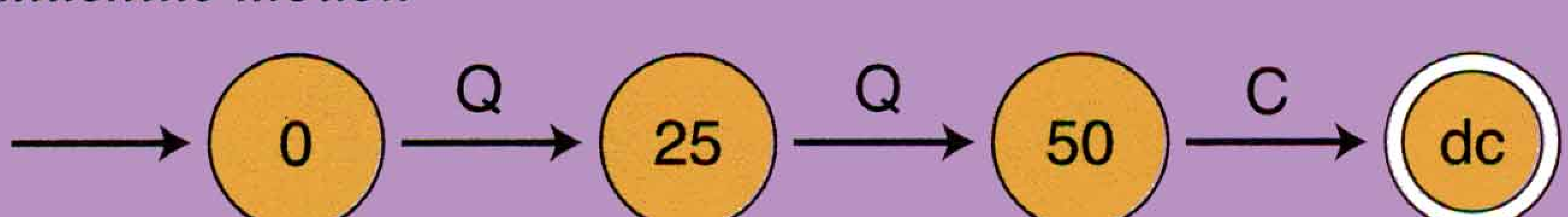
Depositing the Second Quarter



Now pushing the cola button (C) will send the machine to the "dispense cola" state:

Pushing the Cola Button

The last diagram describes our basic vending machine model.



Our model can be improved. How can we add more flavors? Can you draw a diagram for this? What happens if the machine is out of cola and we push the cola button? Can you see the final state? What happens if we deposit three quarters before we push the button? Should the machine accept nickels and dimes as well as quarters? Should it give change? The key to adding these features is to think about the states you will need.

*Our vending machine model has all the parts needed to be a **finite state automaton** (pronounced aw-TOM-a-ton and its plural is automata (aw-TOM-a-ta). We'll call it an FSA for short. An FSA has five parts: states, a start state, final states, input, and transitions. Once you have described the five parts, your FSA is complete.*

FSA's were invented about 45 years ago by electrical engineers. They are used by computer scientists to model computer circuits and to build the compilers that translate programs written in languages like C or C++ into a form that the computer hardware can understand.

*Computer scientists often use concepts from mathematics to express their own ideas. **Sets** and **functions** are important objects in mathematics. The states of an FSA form a set. A transition associating a state with another state; is a function. The diagrams are **graphs**. Mathematicians have been studying graphs for a long time. The **Four-Color Problem**, a very famous graph theory problem, was solved about a decade ago and a computer was used to get part of the solution.*

*In 1900 mathematician David Hilbert published a list of interesting unsolved problems. One of these was the **Decidability Problem**. The **Turing machine** provides the foundation for the solution of this problem. These are just some of the many connections found between mathematics and computer science.*

Puzzle Page

Math Explorers: We want to print your work! Send us original math games, puzzles, problems, and activities. If we print them, we'll send you and your math teacher free *Math Explorer* pens.

FIVE 2's

Use the operations +, - , x, /, and parenthesis to combine the five 2's and make each equation below true. For example, we can use five 2's to make 0 like this:

(2 +	2 -	2 -	2) /	2	= 0
2	2	2	2	2	= 1
2	2	2	2	2	= 2
2	2	2	2	2	= 3
2	2	2	2	2	= 4
2	2	2	2	2	= 5
2	2	2	2	2	= 6
2	2	2	2	2	= 7
2	2	2	2	2	= 8
2	2	2	2	2	= 9
2	2	2	2	2	= 10

Two strange men live in Strange Country. Peter lies every Monday, Tuesday, and Wednesday, but tells the truth on all the other days. Paul lies on every Thursday, Friday, and Saturday, but tells the truth on all the other days. One day they said:

Peter: "Yesterday I had one of my lying days."

Paul: "Me, too."

What day of the week was that?

Bulletin Board

Thanks to our sponsors

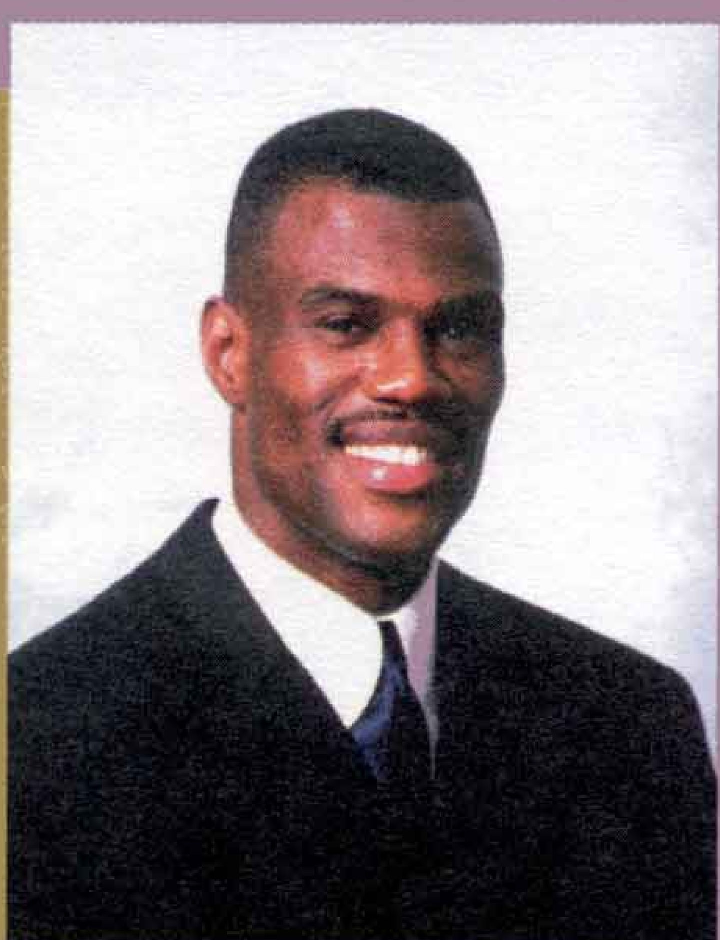
Southwestern Bell Communications has provided private support for the SWT Math Camps and has helped open new camps in the Rio Grande Valley.

Did you hear this?

Did you hear about Al Gore's new band? They call themselves the algorithms.

David Robinson Joins Math Camp Advisory Board

Welcome to our newest Math Camp Advisory Board member, San Antonio Spur's basketball star David Robinson. David graduated from the Naval Academy in 1987 with a bachelor's degree in math. He writes:



"If you want to succeed in life, education is the key...especially math education. There are many opportunities in the areas of math, science and technology. With discipline, initiative, integrity and belief in yourself, you will achieve your dreams."

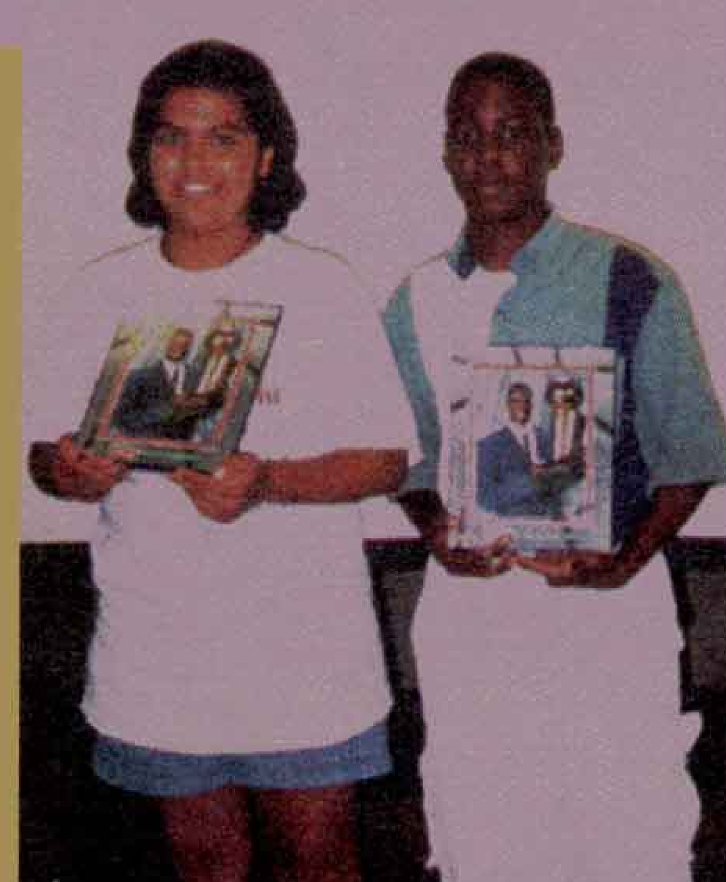
-- David Robinson
Bachelors Degree in Mathematics
Naval Academy 1987
And NBA Center for the San Antonio Spurs

Congratulations Sadie and Fredrick!!

Sadie Lee Castillo, age 10, and Fredrick Womack, age 12, are the *David Robinson Math Scholars* from the 2000 SWT Junior Summer Math Camp at Hernandez Elementary in San Marcos. Sadie is a 5th grader at Maria Hernandez in San Marcos and Fredrick is a 7th grader at Webb Elementary in Austin.

Sadie writes:
"I enjoy the feeling that I get when I meet a challenge that I'm faced with."

Fredrick said of camp:
"I enjoyed working in groups because it allowed me an opportunity to search out the solution to the problems with others, allowing free thinking on all parts."



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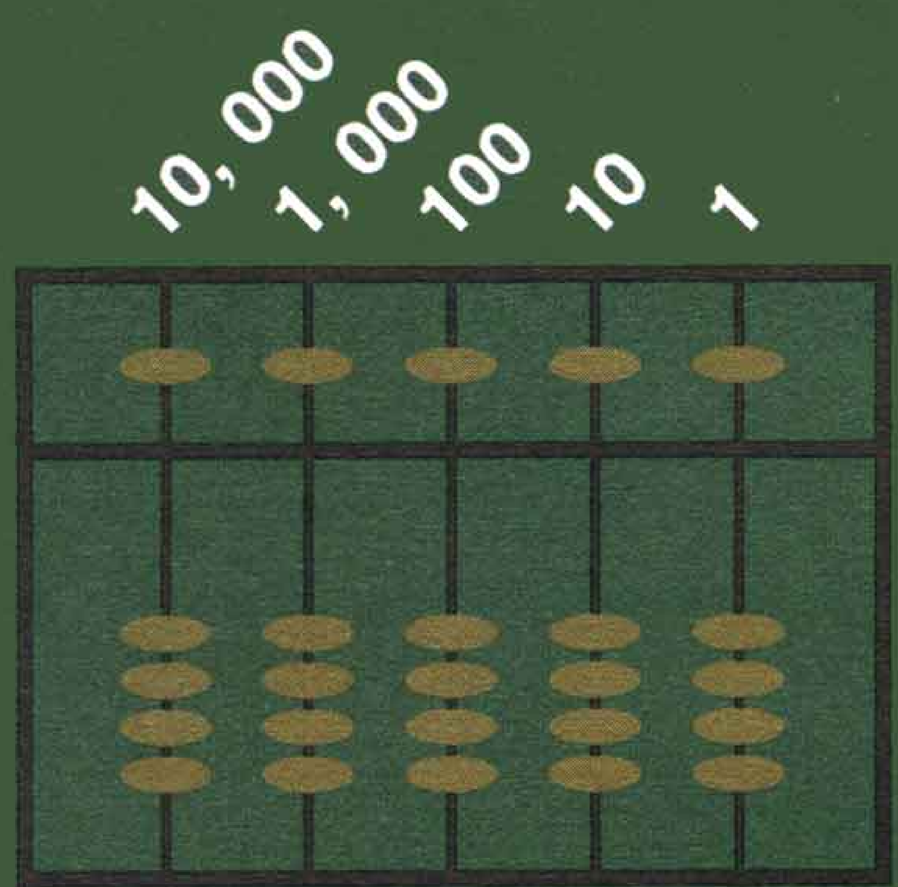
MATH ODYSSEY

by Laura Chavkin and Hiroko Warshauer

Abacus

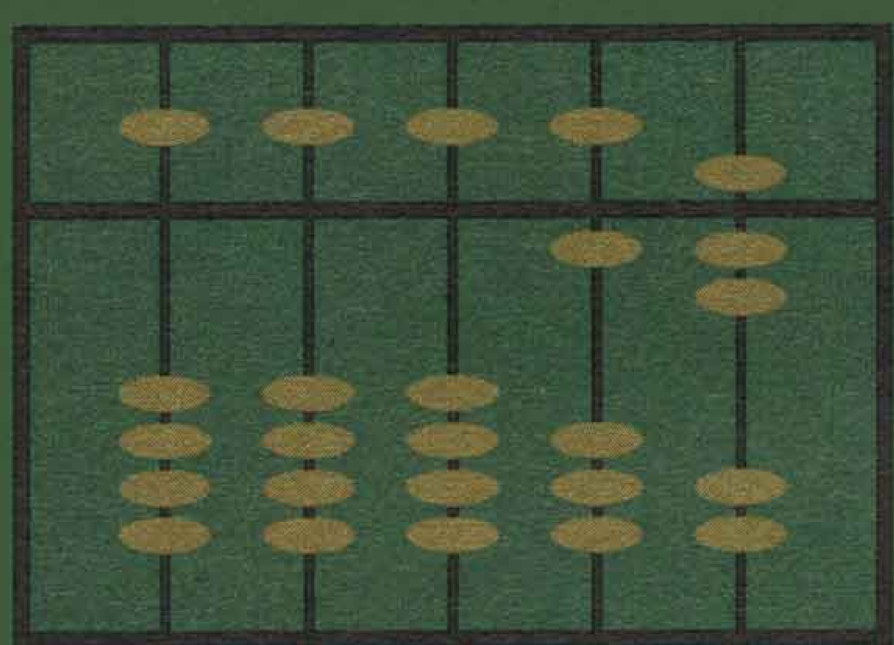
Did you ever try to imagine how people figured out complicated calculations in the days before calculators and computers were invented? People have been inventing new tools to make calculations easier and more efficient for hundreds of years. The abacus is one example of an early calculating machine. It was invented in 14th century China and used by many cultures including the Japanese and Romans since its invention.

The abacus can be used to add, subtract, multiply and divide as well as more complicated operations like finding square roots or expressing fractions. The abacus is important for storing numbers as well as for calculating.

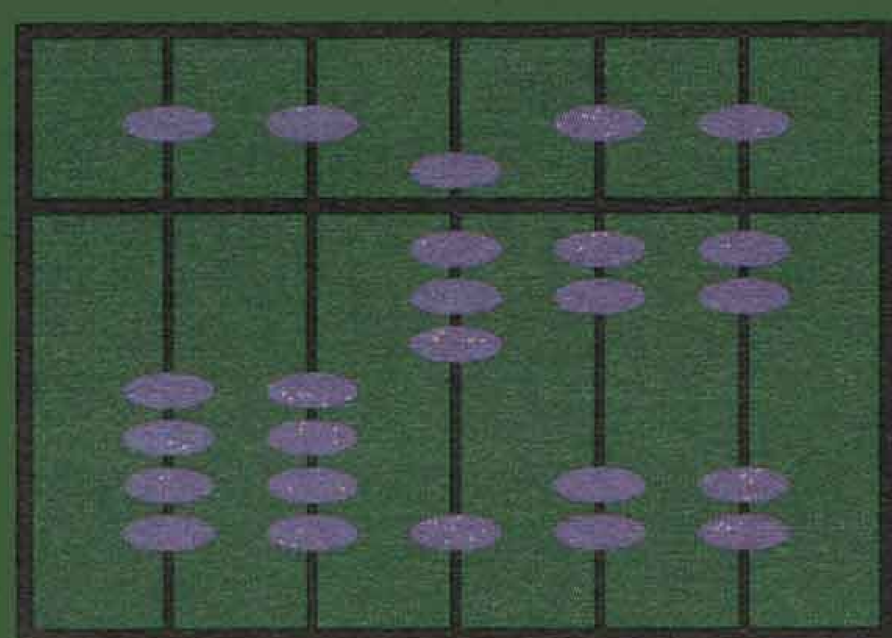


How to Read an Abacus

There is a horizontal center bar with rows of beads above and below this bar. Each vertical row of beads represents a multiple of 10 (10,000, 1,000, 100, 10 and 1). The beads below the center bar represents one unit of that row and the beads above represents five units of that row. Beads pushed toward the center bar add values, away from the center bar subtract values.

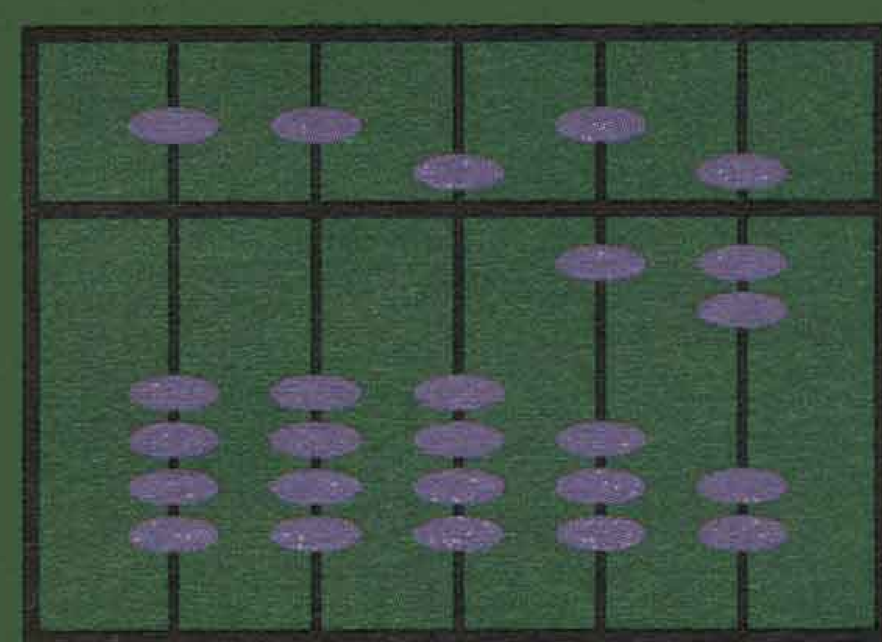


This setting represents 17.



What value (number) does the setting to the left represent?

We have subtracted 305 from the above example. Now what value does the setting to the right represent?



References: http://www.qu-journal.com/abacusTest.html
http://www.cut-the-knot.com/blue/Abacus.html

We at *Math Explorer* welcome you to an exciting new year in math exploration. Through the seasons, we want to bring you fun articles to read, challenging problems to work and intriguing puzzles to solve.

Please share any solutions, ideas or comments with us. You can visit our website to access any past issues and see the solutions to the problems and puzzles. We hope you'll visit often!

Sincerely,

Hiroko K. Warshauer, editor