Math Explorer

MATH CYCLES

Rectangles Are Golden!

Heron's Formula

Circular Time Travel

Vol. 4.2 Winter 2001

Math Explorer

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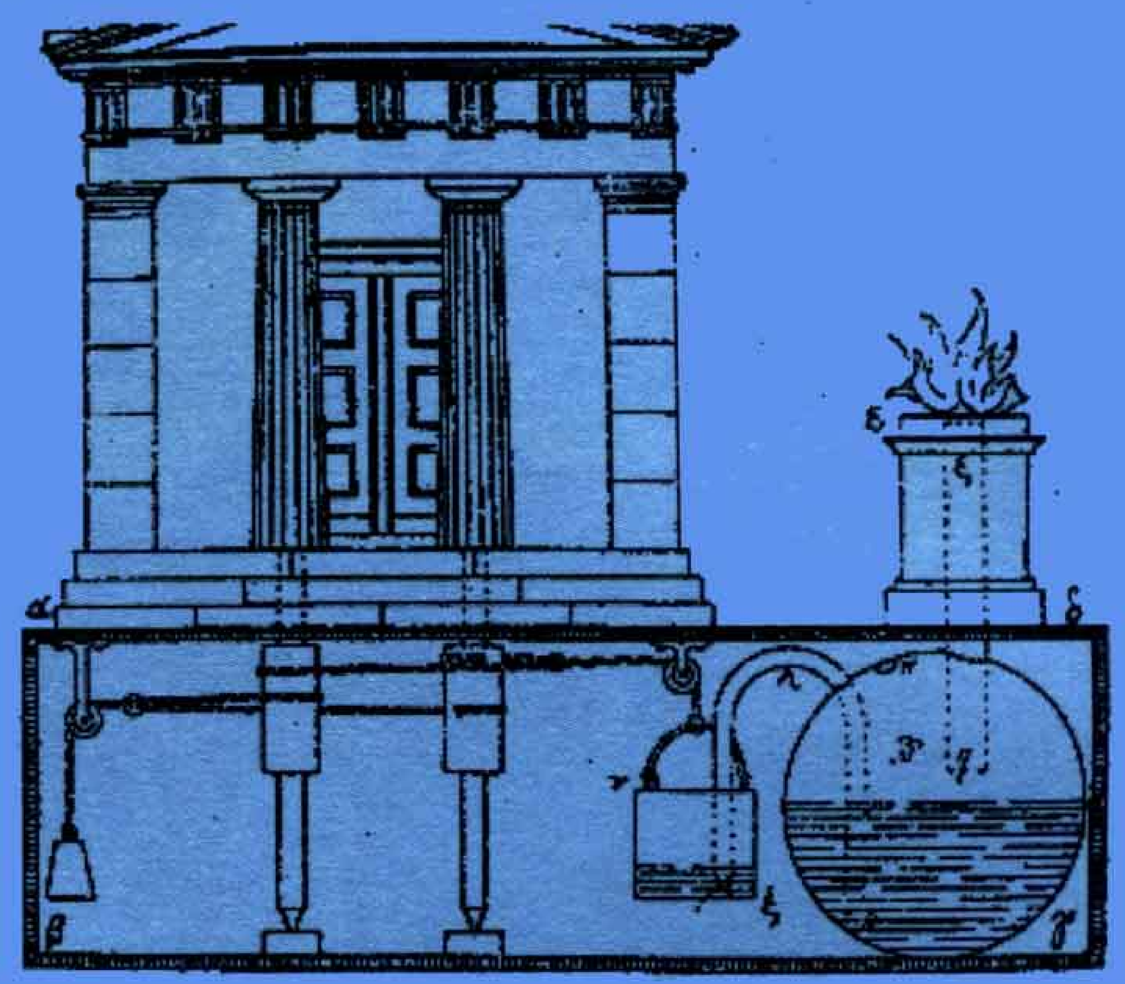
Heron of Alexandria

Jean Davis



The mathematician we know as Heron (or Hero) of Alexandria is a very difficult fellow to get our hands on, figuratively speaking, of course. Exactly when he lived is uncertain, with estimates ranging from 150 BC to 75 AD. Many scholars seem to prefer the later dates because Heron's apparent interest in the more practical side of mathematics is much more indicative of the Roman Empire than of ancient Greece. His name is also questionable. He has been referred to as Heron, Hero, Hiero, or sometimes Hron. These aliases could simply be translational errors, the results of various efforts to capture a phonetic spelling, or perhaps there was more than one person with this name. I suppose we will never know for sure.

Heron apparently lived and worked in the city of Alexandria on the northern coast of Egypt. For several centuries Alexandria had been the foremost seat of learning in the ancient world and the focus of mathematical activity. His emphasis was on applied problems in mathematics, engineering, and computing. Known for his study of mechanics, he is credited with many inventions operated by water, steam or compressed air. Some of them include a fountain, a fire engine, siphons, and a steam engine.

We know very little about his life, but quite a bit about his mathematics. Several manuscripts are attributed to Heron: *Dioptra*, a work on indirect measurement, including how to dig tunnels through mountains and how to measure the amount of water flowing from a spring, *Catoptrica*, *Mechanics*, and *Metrica*, a handbook of practical measurement discovered in 1896 in Constantinople as part of a manuscript dating from the 11th or 12th century.

It is a theorem from Metrica for which Heron is best known today. The formula, known as **Heron's formula**, gives the area of a triangle in terms of the lengths of the sides.

Let a, b, and c represent the sides of a triangle and s equal to half the perimeter of the triangle. Then the area is given by the formula

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

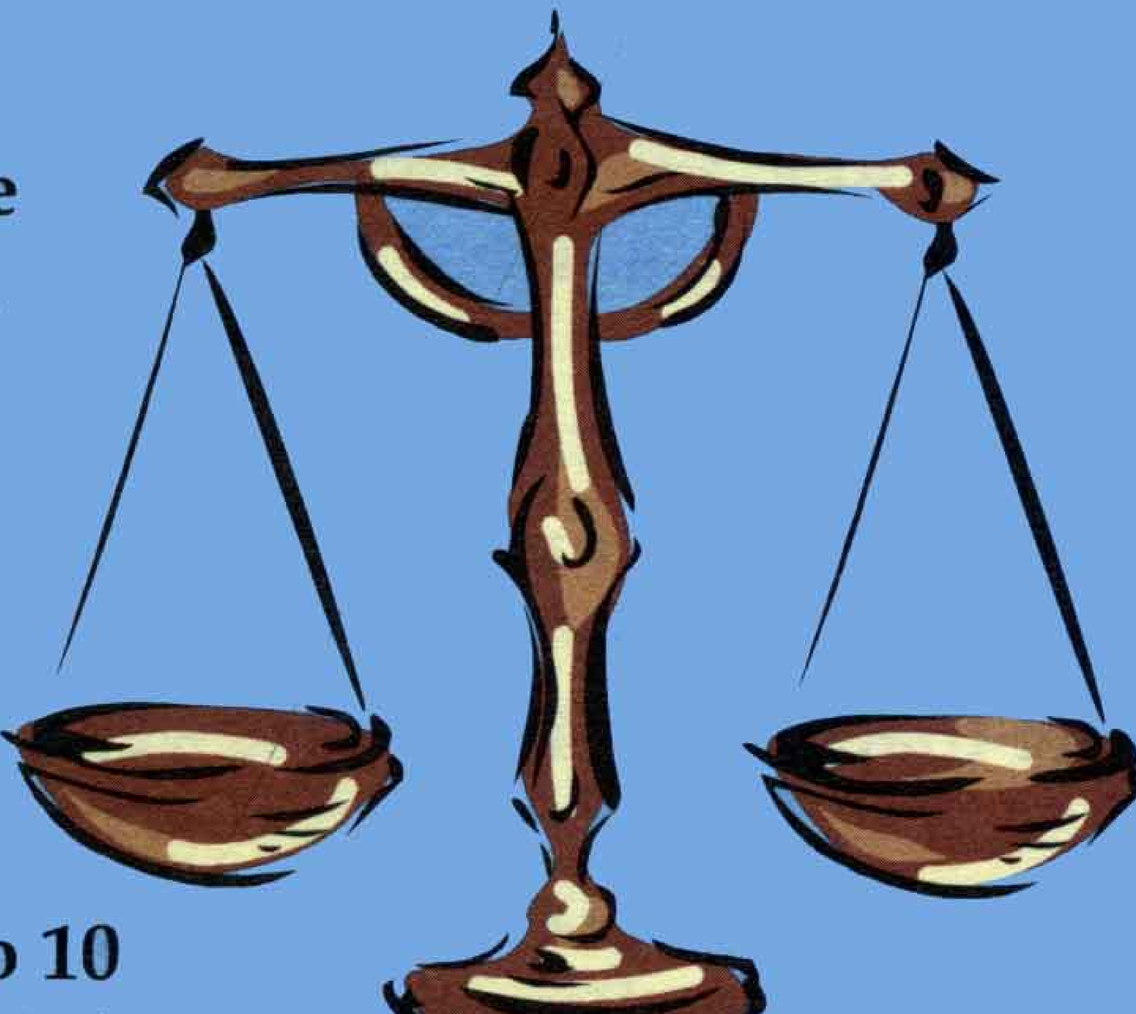
(continued on p.7)

Jean Davis teaches mathematics at Southwest Texas State University.

1. Sam and Joe were playing a game. At the end of each game, the loser gave the winner a penny. After a while, Joe had won 3 games and Sam had 3 more pennies than he did when he began. How many games did they play?

2. Jenny has 11 coins in her purse worth a total of $1.47. Each coin is a penny, nickel, dime or quarter. She has more dimes than nickels. What coins are in her purse?

3. Heather is 32 years old. The sum of her children's ages is 6. In 4 years, her age will be twice the total of the ages of these children. How many children does Heather have?



4. Four boys had a swimming race. The winner finished the race in one minute. Jeremy finished 3 seconds before Isaac. Aaron finished 4 seconds after Jacob, but 1 second before Isaac. What were the boys' times for the race?

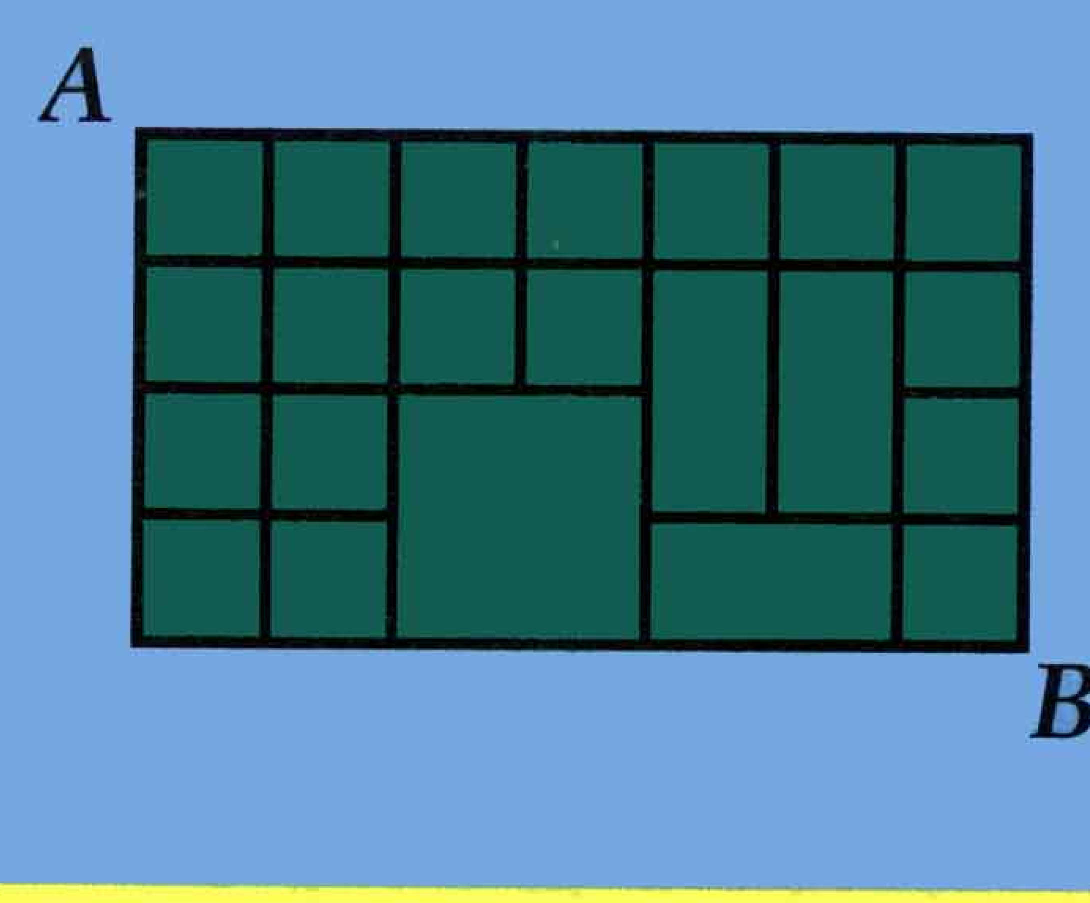
5. You have a balance scale and a 1-ounce and 3-ounce weight. You can weigh any object that is 1, 2, 3, or 4 ounces. Do you see how? You have money to buy one more weight, which can be any number of ounces from 1 ounce to 10 ounces. Which weight should you buy so as to be able to weigh as heavy an object as possible in ounces, with no gaps? (10 is not so good because you can weigh 14 = 10 + 1 + 3 but you cannot weigh 5.)

6. Find 6 consecutive numbers which add to 99.

7. How many equilateral triangle tiles with perimeter 1 foot are needed to tile an area that is an equilateral triangle with perimeter 12 feet?

8. Dad baked a cake. Josephine ate 1/4 of the cake. Then Omar ate 1/4 of the remainder. How much of the cake is left?

9. A palindrome is a whole number that reads the same forwards and backwards. If one ignores the colon, certain times on a digital clock are palindromes, (for example, 1:01 or 4:44). How many times during a 12-hour period will be palindromes?



Ingenuity: A bug crawls from A to B along the grid shown to the right. The bug may only move to the right and downward. How many different paths can the bug follow in going from A to B?

1) 9 2) 2 pennies, 1 nickel, 4 dimes, 4 quarters 3) 3 4) Jacob 1 min, Jeremy 1 min 2 sec, Aaron 1 min, 4 sec, Isaac 1 min 5 sec 5) 9 oz 6) 14, 15, 16, 17, 18, 19 7) 144 8) 9/16 9) 57 Ingenuity) 101

Time Will Tell: Clock Arithmetic

by Hiroko Warshauer

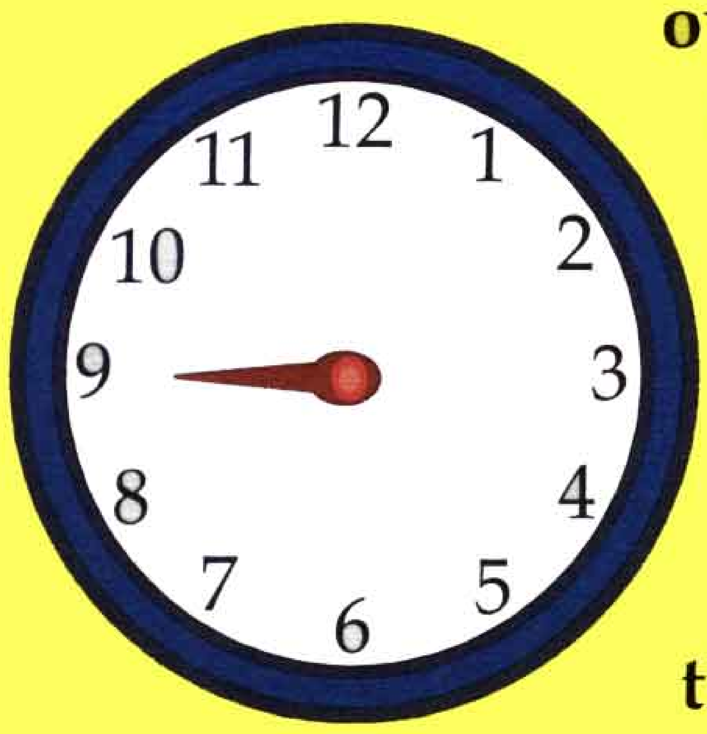
"To see the world in a grain of sand
And a heaven in a wild flower,
Hold infinity in the palm of your hand
And eternity in an hour."
-- William Blake

Question 1: Can 9 + 8 = 5?

Question 2: If you must take your medication every 8 hours and you just took your medication at 9 am, at what time must you take your next dose?

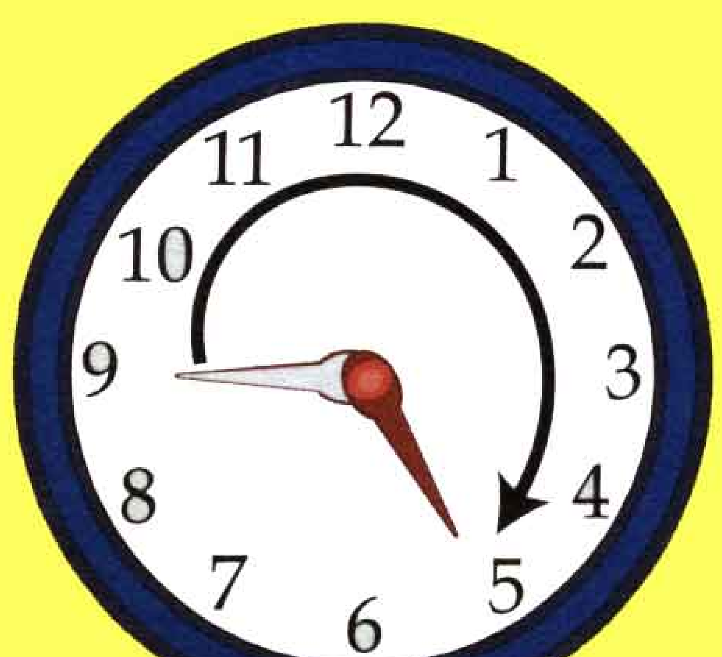
Question 3: If December 25, 2001 is a Tuesday, on what day will December 25, 2002 fall? Note that 2002 is not a leap year.

These are some of the questions that we can answer using an area of mathematics called *clock arithmetic* or *modular arithmetic*. These questions involve a pattern that we observe in natural processes. The processes form cycles over periods of time, such as the seasons, moon phases, weeks, months and years.

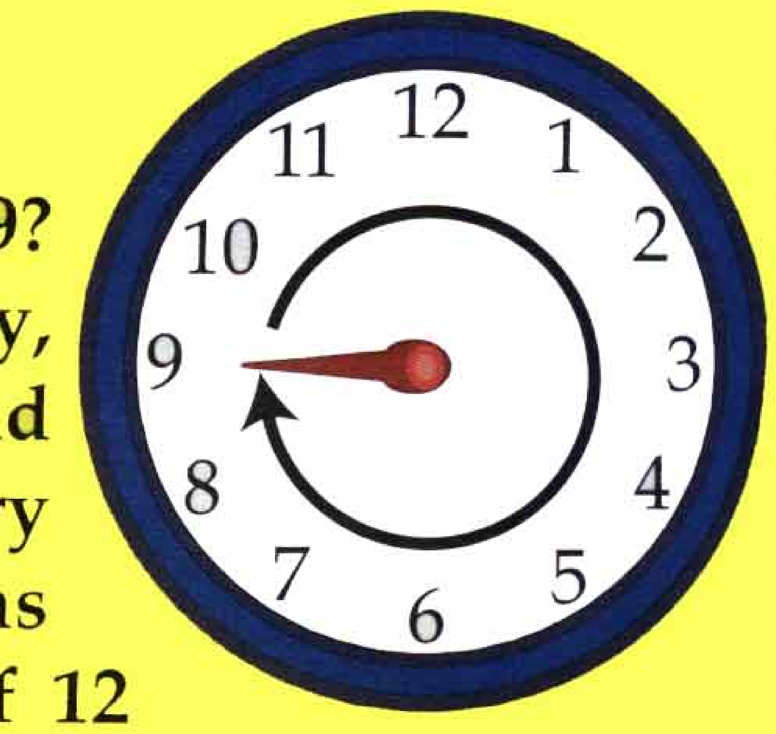


Let's look at Question 2 using a clock. If we take one dose at 9 am, then after 8 hours time the clock will read 5 pm, our answer to Question 2.

In our 12-hour clock 9 + 8 = 5, so the answer to the Question 1 is yes!

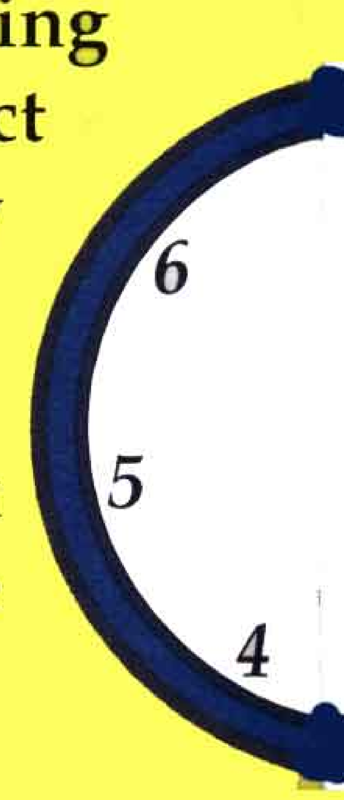


Notice that if you should take your medication every 12 hours, then you would take your dosage at 9:00 am, then 9:00 pm and so on. 9 + 12 = 9. In our 12-hour clock, adding 12 is like adding 0. This is an example of *clock arithmetic.*



What happens if we add 24 to 9? Did you get 9 + 24 = 9? Precisely, since 9 + 24 = 9 + 12 + 12 and adding 12 is like adding 0 every time. In fact, what happens when we add any multiple of 12 such as the numbers 36, 48, 60, 72,...to 9? It is like adding 0 to 9.

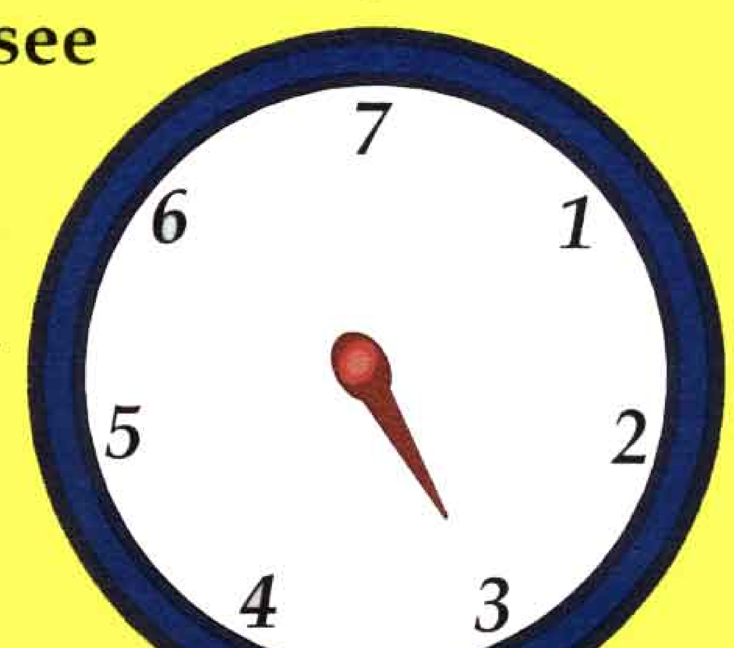
Now what happens if we add a number to 9 that is not a multiple of 12, such as 13? Since 9 + 13 = 9 + 12 + 1, adding 13 to 9 is like adding 1 to 9. All the numbers 13, 25, 37, 49, 61, ... act like a 1 when added to 9. Check to see why that is.



Let's see what happens to our clock arithmetic if our clock happens to look like this.

Notice every 7 hours we come back to where we started. In other words, adding 7 is the same as adding 0. If it is 3 o'clock now, 9 hours from now it will be 5 o'clock as you can see from this clock. Let's see why.

3 + 9 = 3 + 7 + 2 = 3 + 2 = 5 using 7-hour clock arithmetic.



What does 2 + 15 equal?

Could there be any use for such a strange clock? Notice on a calendar that if today is Sunday then 7 days later we'll be back to Sunday again. Clock arithmetic is at work here!

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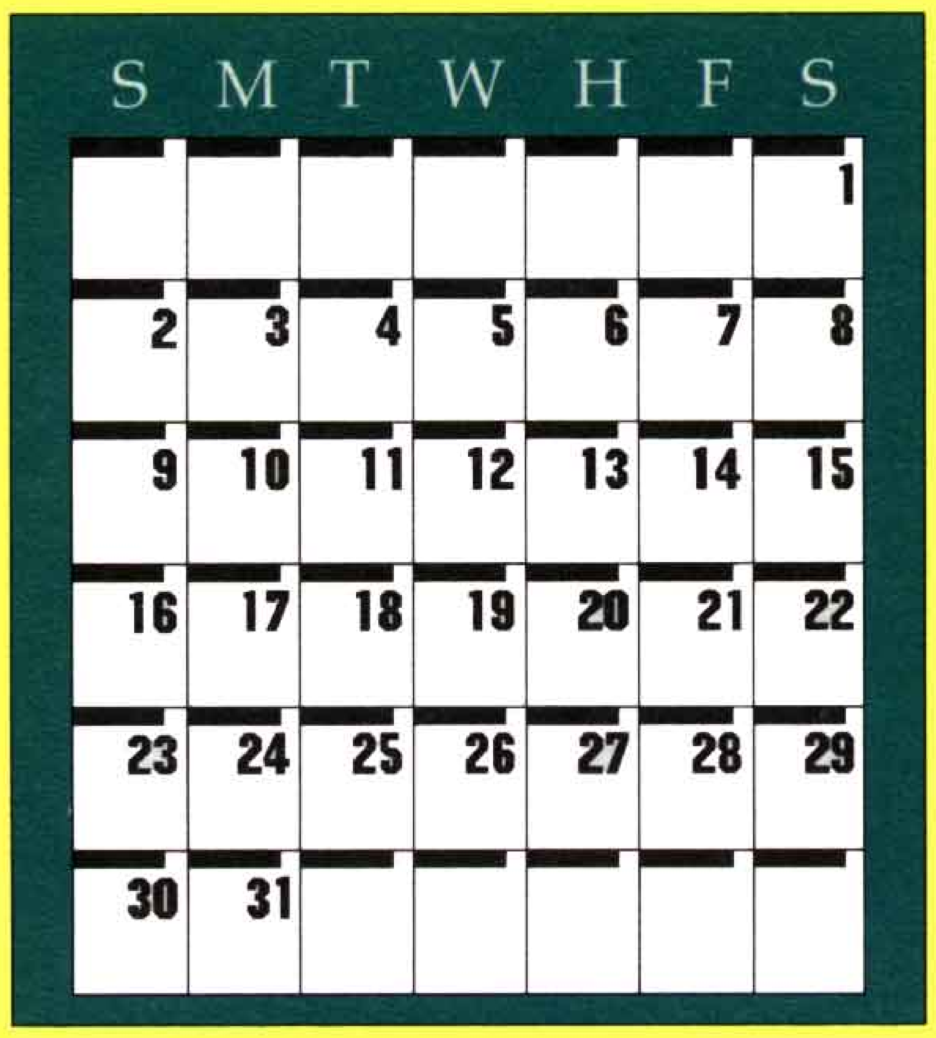
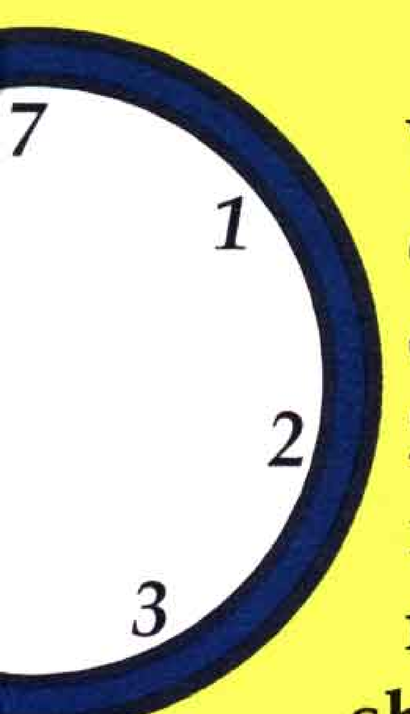
This leads us to an area of mathematics called number theory and in particular modular arithmetic. Remember that in our last clock arithmetic adding 7 is the same as adding 0. In mathematical symbols we write $7 \equiv 0 \bmod 7$, which we read as "7 is congruent to 0 mod 7". We see from our work above $12 \equiv 5 \bmod 7$ (remember $12 = 3 + 9 = 3 + 7 + 2 = 3 + 2 = 5$). Looking at the calendar of December, 2001 notice to which day December 5 and 12 correspond.



Can you see why $26 \equiv 5 \bmod 7$?

There is another pattern about congruent numbers we can observe.

$7 \equiv 0 \bmod 7$ and $7 - 0 = 7$
$12 \equiv 5 \bmod 7$ and $12 - 5 = 7$
$26 \equiv 5 \bmod 7$ and $26 - 5 = 21 = 7 \times 3$



What can you say about numbers that are congruent to 5 mod 7? Looking at the calendar you might notice where 5, 12 and 26 are located. You would be right if you note that the differences between the numbers and 5 are multiples of 7. We should not be restricted to just numbers on the calendar. Using larger numbers, $47 \equiv 5 \bmod 7$ since $47 - 5 = 42 = 7 \times 6$ but 42 is not since $42 - 5 = 37$ and 37 is not a multiple of 7.

In general, when are two numbers a and b congruent to a number n?

In Question 3 we ask on what day of the week Christmas will fall next year.

Let's see how modular arithmetic can help us get the answer. We know that there are 365 days in a non-leap year and that there are 52 weeks in a year. $365 = 52 \times 7 + 1$ so $365 \equiv 1 \bmod 7$. This tells us that December 25, 2002 is 52 weeks and 1 day after December 25, 2001. December 25, 2001 is on a Tuesday so we can conclude that December 25, 2002 must be one day after Tuesday, or on Wednesday. We have answered Question 3.

On what day is your birthday this year? Can you determine the day of the year you will turn 16? Try using modular arithmetic. Remember to be careful of any leap years along the way!

Among the many places where modular arithmetic is used is in the area of encryption (coding)and identification numbers. Notice the UPC bar codes on all the cereal boxes, cans and packages in your pantry. These are identification numbers that code information about what the item is and a "self-check" or error-detection component.

Modular arithmetic is used in checking the bank identification numbers that appear on personal checks. Each US bank has an 8 digit identification number which is printed on all personal checks along with a 9th check digit at the end. To check that the identification number is correct, each of the digits is given specified weights that have been set by the industry. The sum of these weighted numbers must be congruent to 0 mod 10.

For example, the Bank of America has the bank identification number 113000023. Using the specified weights, we look at the sum of the weight times each bank ID digit.

$7 \times 1 + 3 \times 1 + 9 \times 3 + 7 \times 0 + 3 \times 0 + 9 \times 0 + 7 \times 0 + 3 \times 2 + 9 \times 3 = 70.$

$70 \equiv 0 \bmod 10$ and hence the error-detection method tells us this should be the correct bank identification number. Had the second 1 changed places with the next to the last 2, we would not catch an error using this method. The self-check is not foolproof.

This is just one of many places where modular arithmetic is used.

Reference: Burger and Starbird, "The Heart of Mathematics", Key College Publishing, 2001.

Puzzle Page

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Word Search

Forwards or backwards, up, slanted, or down.

Where can the words in this puzzle be found?

Modular
Cycles
Multiple
Congruent
Codes
Heron
Triangle
Area
Golden
Ratio

N	O	I	S	N	E	M	I	D	D	R	O	O	E
O	M	H	T	E	S	E	Q	U	E	N	C	E	A
N	D	I	G	E	T	E	W	U	I	O	T	E	F
F	O	B	A	V	R	R	A	G	O	I	X	R	U
E	S	E	Q	U	E	A	C	E	A	S	A	Y	N
E	T	F	E	T	Y	E	T	E	E	C	R	T	D
D	I	D	B	C	A	R	B	I	T	V	B	I	M
B	N	E	A	N	S	T	A	A	O	I	E	R	B
A	F	E	X	P	O	N	L	N	T	N	G	A	R
C	I	T	O	R	B	L	E	D	N	A	M	L	R
K	N	G	C	O	Q	E	I	O	T	B	A	I	K
Q	I	P	A	T	T	E	R	N	S	I	A	M	C
U	T	V	I	T	U	C	S	O	A	H	C	I	S
J	E	E	L	O	O	B	I	E	R	O	C	S	P

Can You Make 20?

Using at most:

6 one's: ___________

6 two's: ___________

6 three's: ___________

6 four's: ___________

6 five's: ___________

6 six's: ___________

6 seven's: ___________

6 eight's: ___________

6 nine's: ___________

ex: $11 + 11 - 1 - 1 = 20$ or $(11 - 1) * (1 + 1) = 20$

Alyssa has a 16-ounce drink that she wants to divide evenly with her friend. To do this, she has two other containers to help -- one holding 11 ounces and the other 6 ounces. How can she do this without spilling any of her drink? How many operations (an operation is pouring liquid from one container to another) are needed?

In order to cross a desert, it takes 6 days. Each day, a person needs to drink 1 gallon of water to survive. However, each traveler can only carry 4 gallons of water. How many travelers would it take to get one person across the desert without having anyone die in the desert. (If a people are either at the start or end of the trip they can find water. Otherwise they must use the supplies they brought along.) How could they do it?

Bulletin Board

Math Counts on the Internet

http://mathcounts.org
gives you access to Go Figure! Math Challenge, Problem of the Week, Competition information and more.

Check It Out

The Maths File Game Show uses animated games to do topics in number, algebra, measurement and more.
www.bbc.co.uk/education/mathsfile/

She Does Math!

Have you read the book, "She Does Math" Real-Life Problems from Women on the Job by Marla Parker, editor published by The Mathematical Association of America? The book presents the career histories of 38 professional women and math problems written by them.

Math Camp 2002

For information on the SWT Summer Math Camps for 2002 contact SWT Mathworks at (512)245-3439.

Did you hear this?

What has 18 legs and catches flies?

A baseball team

Heron (cont'd)

This formula would be very unusual if it originated in ancient Greece because the number under the square root is given as a product of 4 things, a very ungeometrical concept. Euclid's work in Greek times, for example, contains products of only 2 or 3 things. Heron gives a proof of his theorem as well as a description of how to calculate the square root. While it is thought that Archimedes may be the original creator of the formula, it is Heron's proof that survives. A thorough and very dramatic presentation of Heron's proof is given in Chapter 5 of William Dunham's intriguing book, ***Journey Through Genius***.

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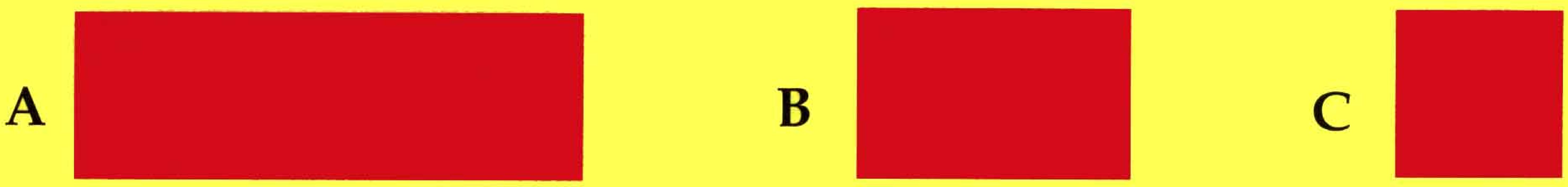
MATH ODYSSEY

by Hiroko K. Warshauer

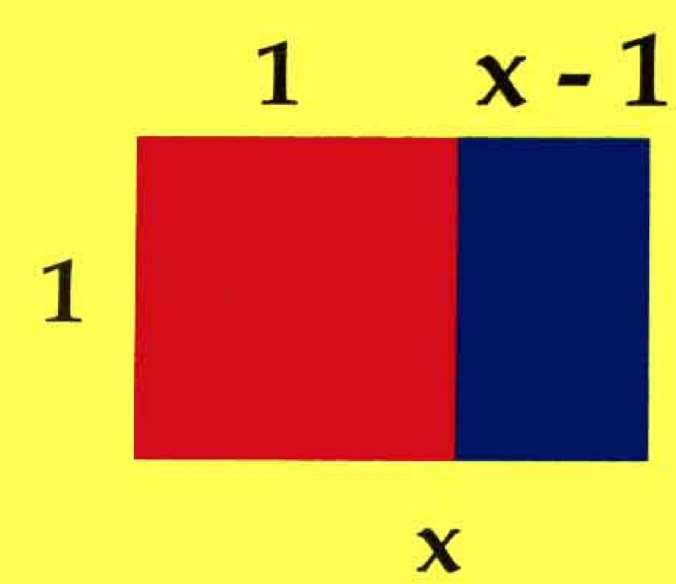
What can be special about a rectangle? What size rectangles are most beautiful and pleasing to the eye? A rectangle is, after all, just a quadrilateral whose opposite sides are parallel and whose adjacent sides form right angles.

There are very special kinds of rectangles that artists like Leonardo da Vinci, Seurat and Mondrian and architects from the designers of the Parthenon to Le Corbusier have incorporated in their works. Notice the three rectangles A, B, and C below.

Does one rectangle appeal to you more than the others?



Did you say B? Any particular reason?



Suppose rectangle B has dimensions 1 by x and we cut off a 1 by 1 square from it. The remaining rectangle (blue) has sides which are in the same proportion as the original rectangle B, that is

$$\frac{x}{1} = \frac{1}{x-1}$$

We call this a *golden rectangle.*

Solving the equation $x^2 - x - 1 = 0$, algebraically or using the calculator, we get two values for x. One value of x is equal to $(1 + \sqrt{5})/2$ or approximately 1.6.

Any rectangle, then, which has the dimensions: width = n and length = 1.6n will be a golden rectangle. The ratio 1.6 to 1 is called the *golden ratio* or *golden mean.*

Notice that the rectangle to the right of the cut out square is also a golden rectangle. If we cut a square off it, we should have an even smaller golden rectangle. This process can continue infinitely.

Does this page have the dimensions for a golden rectangle? Some examples approximating a golden rectangle include a 4 by 6 photo and a 3 by 5 index card. See if you can find other golden rectangles around you, both large and small.

References: *www.mcs.survey.ac.uk/personal/R.Knott/Fibonacci* *www.q-net.net.au/~lolita/symmetry.htm*

Winter in the Northern Hemisphere is the season when we see the ending of a calendar year and the beginning of a new. Just as seasons and years form cycles, in this issue of *Math Explorer* we look at number patterns that form cycles in our article on clock or modular arithmetic. We hope this will only be the beginning of a topic that you may want to explore.

We wish you Happy Holidays and the best in the (palindrome) year 2002!

Sincerely,

Hiroko K. Warshauer

Hiroko K. Warshauer, editor