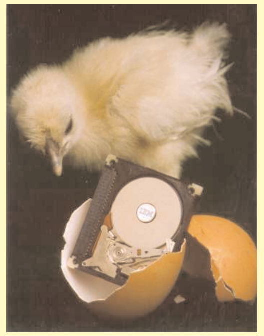
How Small is Small?

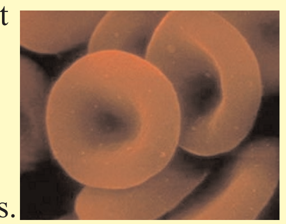


Nanotechnology is an area of interest to many scientist and engineers today-but do you know where the term nano comes from? Nano refers to the **nanometer**, a unit used to measure really small objects such as atoms.

Let's start with a **meter,** which makes a good choice for measuring your height or your room. A **kilometer** may be more suitable for a longer length, such as the distance from your home to school.

To measure smaller objects, such as a pinhead, a more appropriate unit would be a **millimeter**, which is 1/1000 of a meter. While we write $1000 = 10^3$, we write 1/1000 as 10^{-3}. Cells which are even smaller than pinheads are often measured in **micrometers**, which is 1/1000 of a millimeter or 1/1,000,000 of a meter. Notice that since 1 millimeter = 10^{-3} meter, then 1 micrometer = $(10^{-3})(10^{-3})$ meter = 10^{-6} meter.

To measure the size of molecules and individual atoms, the **nanometer** serves as a measurement unit. One nanometer is 1/1000 of a micrometer. Can you determine what fraction 1 nanometer is of a meter?



An individual atom such as hydrogen is less than one nanometer. DNA molecules are about 2.5 nanometers wide and red blood cells are about 10,000 nanometers. A man who is two meters tall is 2 billion nanometers tall!

Working at this almost invisible and certainly miniaturized environment, this technology holds the promise of smaller computers and stronger, lighter and more conductive materials. However, there are some health concerns from nanoparticles that may prove to be toxic or damaging to the environment and to animals and humans.

Already in use are some products of nanotechnology that include magnetic recording tapes, computer hard drives, bumpers on cars, sunscreens and cosmetics. Ever wonder what unit could be smaller than a nanometer?

References:
"*It's a small, small world*" by Rick Weiss, Austin American-Statesman, February 8, 2004
"*Virtual Nanotech*" by Alexandra Goho, Science News, February 7, 2004, pg. 87
"*Nanotech Under the Microscope*" by Anne Geske, Utne, July-August 2004, pg 15

Dear Math Explorers,

A new year is right around the corner and 2005 promises to be an interesting year. Though not prime, 2005 is certainly an odd number. Can you find other interesting features about this number? Share your observations with us and we'll post them on our website.

We enjoy bringing you articles and puzzles that intrigue you and bring hours of fun, too! We at Math Explorer wish you very happy holidays!

Sincerely,

Hiroko K. Warshauer

Hiroko K. Warshauer, Executive Editor

Vol. 7.2 Winter 2005

Math Explorer

MEASUREMENT AND MATHEMATICS

POLYGONS are all Ears?!?

HYPATIA, WOMAN MATHEMATICIAN

It's a small, small world - nanotechnology

Math Explorer

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Hypatia

by Jean Davis

The first woman mathematician was Hypatia, who lived from 370 to 415 A.D. Her father, Theon, was a professor of mathematics at Alexandria in Egypt. At the time, Alexandria was the greatest seat of learning in the world and Hypatia was immersed in an atmosphere of learning from her earliest years. She was an excellent student and was later asked to teach mathematics and philosophy at Alexandria.

Hypatia was truly her father's "golden girl." He trained her from childhood to be the ultimate example of the ideal woman. She was beautiful in appearance, physically fit, intellectually brilliant, kind, and virtuous. People spoke of her in glowing terms, calling her "mother, sister, reverend teacher."

Hypatia wrote mostly what we call commentaries or explanations of the works of others. These were very important, as they made it possible for other people to understand very difficult mathematics. Many of her writings were prepared as textbooks for her students.

Hypatia was particularly interested in a type of equation we call "diophantine equations." These are equations whose solutions are restricted to integers.

Example: In what ways can a person make change for a dollar using only nickels, dimes and quarters?

If n = the number of nickels
d = the number of dimes
q = the number of quarters

The equation becomes
$$5n + 10d + 25q = 100$$

One solution is $n = 6, d = 2, q = 2$

Hypatia was a beautiful , highly intelligent woman and a very popular teacher. She achieved things that, as far as we know, no woman before her even dreamed of doing. Her death in 415 signaled the end of the Golden Age of Greek mathematics.

Resources: Burton, David M., The History of Mathematics, 5th edition. McGraw-Hill Companies, Inc. New York
http://www-history.mcs.st-andrews.ac.uk/Mathematicians/Hypatia.html

Jean Davis has been a member of the Mathematics faculty at Texas State since 1979. She loves telling people about mathematics and is particularly interested in the history of mathematics.

Bulletin Board

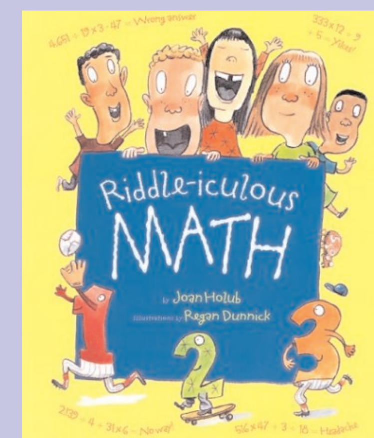
Grandparents are Great!

Can you determine how many great-great-great-grandparents you have? Do you know their names? When they lived? Where? To learn more about genealogy, check out the Library of Congress site at www.loc.gov/rr/genealogy

Virtual Manipulatives

Visit the National Library of Virtual manipulatives for interactive math at http://matti.usu.edu/hlvm/grade_g_z.html

Math Chuckles While You Read



Riddle-iculous MATH by Joan Holub and Regan Dunnick is a book that offers fun riddles and mathematical jokes.

Math Bytes-Check it out!

Math Olympiads offers monthly math contests for students in grades 4-8. For more information, visit www.moems.org

The Mathematical Association of America sponsors the **American Mathematics Competition** for students in grades 6-12 to enrich the problem solving and mathematical experiences for students. Visit its website at www.unl.edu/amc/whatswhat.html

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ANGLES, TRIANGLES AND EARS

by Max Warshauer

A **simple polygon** is a closed figure with *n* sides, where none of the sides overlap. For example, Figure A below is a simple polygon, while Figure B is not.

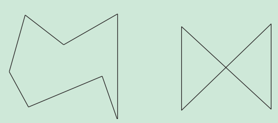


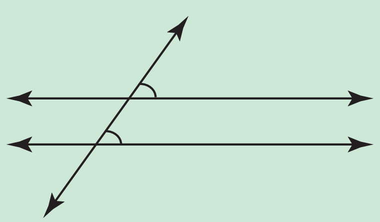
Fig. A   Fig. B

Have you ever thought about adding up the angles in a polygon? Do you always get the same answer if two polygons have the same number of sides? In order to study problems in mathematics, we often begin by looking at simple examples first to see if there is a pattern. So we will begin by looking at the simple polygon with the least number of sides—the triangle. Triangles are like the building blocks of polygons.

To study this problem, we will need a fundamental property of parallel lines:

Corresponding Angle Property

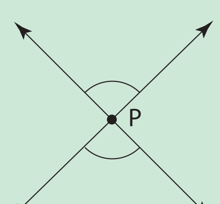
If 2 parallel lines are cut by a straight line (called a transversal), then the corresponding angles are equal.



We also will need the:

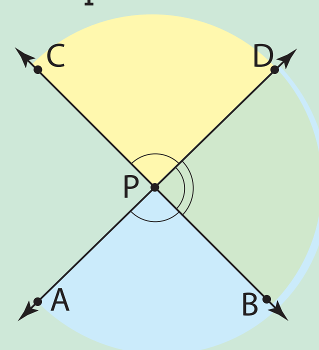
Vertical Angle Property

If two lines intersect at point P, then the vertical angles are equal.



Do you see why the Vertical Angle Property is true?

Consider the picture below:



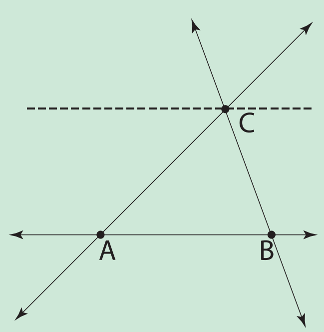
Notice that ∠APB and ∠CPD are vertical angles. But ∠APD is a straight angle so the measure of angle APD, written, m∠APD, equals 180°. Also ∠BPC is a straight angle, so m∠BPC = 180° also.

Hence, 180° =m∠APB + m∠BPD = m∠CPD + m∠BPD

And so we have, ∠APB ≅ ∠CPD

Now let's return to our basic triangle problem of why the angles in a triangle always add up to 180°.

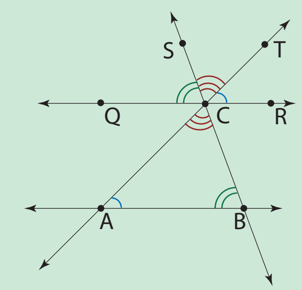
If we begin with our triangle, we can then extend each of the sides. Next, we construct a line through C parallel to the base AB.



Once we have done this, we can use our Corresponding Angle Property. We have the transversal CA that cuts our two parallel lines. Hence, we can say that ∠TCR ≅ ∠CAB.

Similarly, we have the transversal CB that cuts our two parallel lines. Hence we can also say that ∠SCQ ≅ ∠CBA.

Further, the Vertical Angle Property tells us



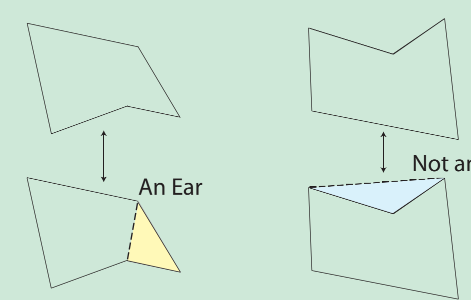
that ∠ACB ≅ ∠SCT.

Putting these pieces together, the angles of our triangle equal

m∠QCS + m∠SCT + m∠TCR = 180°.

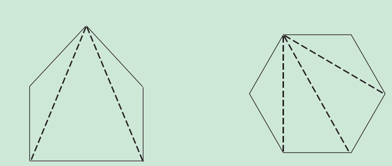
Since the sum of the angles in a triangle always adds up to 180°, we can next consider a quadrilateral, which is a simple polygon with 4 sides. This time, we can divide the quadrilateral into two triangles. The sum of the angles in each triangle is 180°, so the sum of the angles for the quadrilateral is 360°. Next we could consider polygons with even more sides. Can you guess what the sum of the degrees of all the angles in a polygon will be?

To think about these more complicated polygons, we introduce the idea of the "ears" of a polygon. An **ear** of a polygon is formed by two successive sides and a connecting



diagonal which is entirely inside of the polygon. This is illustrated in the pictures below:

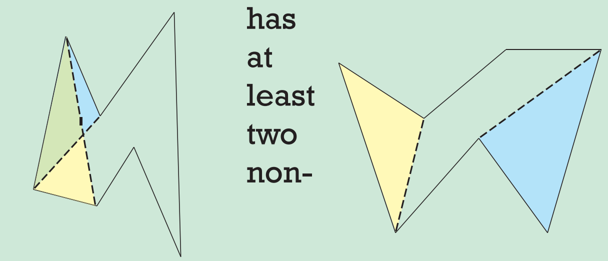
When we form an ear of a polygon, we are really constructing a triangle inside of the polygon. If we continue the process of forming ears for these interior polygons, then



eventually we will partition our polygon into triangles. For example, the pentagon below is partitioned into 3 triangles as shown, while the hexagon is partitioned into 4 triangles.

To add up the measures of all of the angles in these polygons, we can add up the angles in all of the triangles. And as we have already seen, each triangle will contribute 180° to the total.

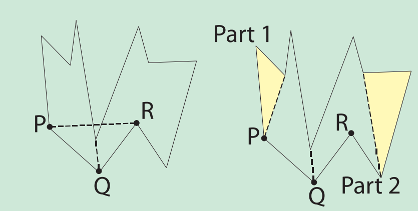
There is one part of this explanation that needs a little more discussion—namely how do you know that you can always find an "ear" in a simple polygon? Here is the clever idea that was first discovered by Gary Meisters in 1975. He proved the **Two Ears Theorem,** which says that any polygon with 4 or more sides always has at least two non-



overlapping ears. By non-overlapping, we mean that the ears do not intersect as shown in the pictures below:

Overlapping ears   Non-overlapping ears

To see why this is true, begin with three successive points P,Q,R in a polygon where the diagonal PR passes through the interior of the



polygon. If this forms an ear, then we have begun to reduce the problem. If not, we pick a vertex of the polygon inside PQR that is closest to point Q and connect it to Q, This will divide the polygon into two smaller polygons, each with fewer edges than before.

We can thus reduce the problem of finding ears on an *n*-sided polygon to finding ears on a polygon with fewer edges, showing

Max Warshauer is a professor of mathematics at Texas State University-San Marcos and director of Texas Mathworks. He loves working on math problems!

Puzzle Page

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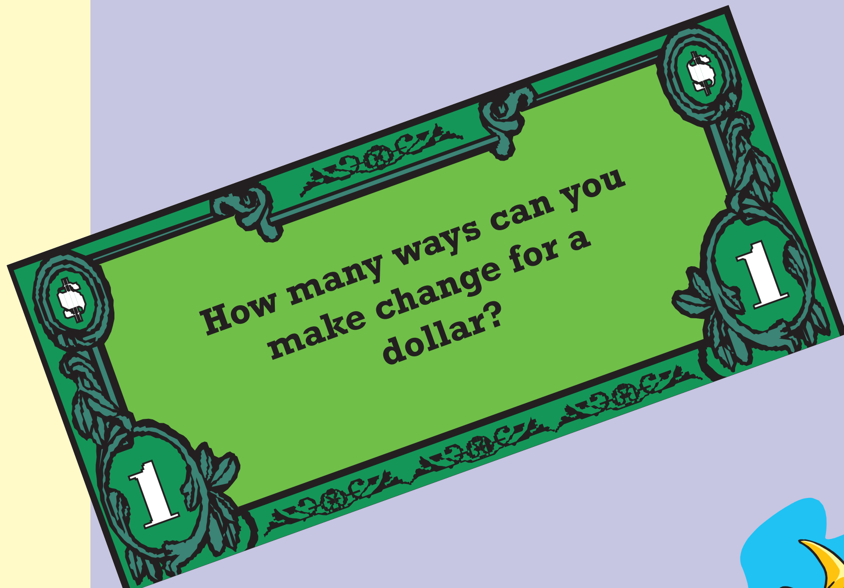


Alisa, Carla, Joost, Kent, and Lizzy are professional deep-sea divers. Alisa is at a depth of 210 meters. Joost is 47 meters above Alisa. Carla is 87 meters below Joost. Kent is 30 meters below Lizzy. Lizzy is 110 meters above Alisa.

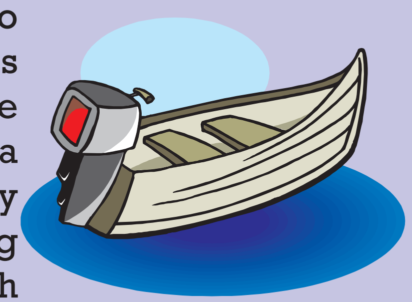
At what depth (in meters below sea level) is each of the divers?

Word Search

Forwards or backwards, up, slanted, or down.
Where can the words in this puzzle be found?

Angle
Parallel
Vertical
Corresponding
Transversal
Polygon
Triangle
Hypatia

S	U	F	S	A	D	C	N	J	X	U	A	L	D
K	I	B	G	U	U	O	B	T	A	T	G	A	O
C	N	M	Y	U	G	R	M	R	R	H	V	C	I
I	U	F	M	Y	Z	R	G	A	N	R	Y	I	M
U	I	T	L	L	Y	E	N	H	Y	Z	M	T	A
L	L	O	D	Z	E	S	Y	W	L	Q	P	R	M
G	P	S	C	B	V	P	A	N	G	L	E	E	J
C	J	A	L	E	A	O	A	K	X	B	Z	V	L
E	R	F	R	T	I	N	B	R	P	D	R	U	N
I	P	S	I	R	M	D	M	O	A	J	M	L	V
M	A	A	H	G	O	I	N	Z	W	L	O	F	A
L	O	O	E	L	G	N	A	I	R	T	L	F	F
S	H	Z	J	Z	L	G	W	V	L	Q	S	E	J
Y	G	W	G	O	U	I	U	M	L	F	W	E	L



There are two motorboats on opposite sides of a river. They start moving towards each other, but at different speeds. When they pass each other the first time they are 700 yards from one shoreline. They continue to the opposite shore, turn around and start moving towards each other again. When they pass the second time, they are 300 yards from the other shoreline. How wide is the river?

PROBLEMS PAGE

*1. A structure is built with identical cubes. Figure 1 is the top view, Figure 2 is the front view and Figure 3 is the side view. What is the least number of identical cubes required to build this structure?

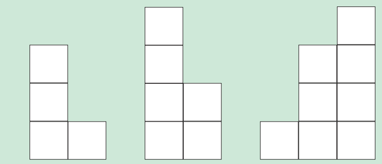
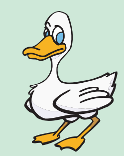


Figure 1 Figure 2 Figure 3

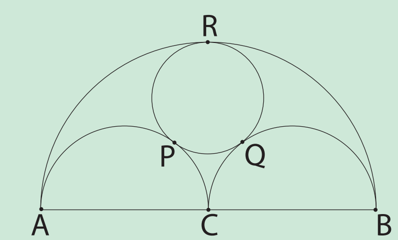
2. What is the difference between the sum of the first 30 even counting numbers and the sum of the first 30 odd counting numbers?

3. A storage tank is 1/4 full. When 5 gallons are removed from the tank, the tank will be 1/5 full. What is the capacity of the tank?



4.Three ducks and two ducklings weigh 32 kg. Four ducks and three ducklings weigh 44 kg. All ducks weigh the same and all ducklings weigh the same. What is the weight of two ducks and one duckling?

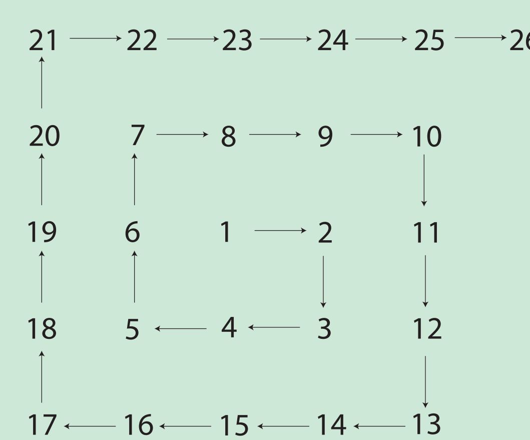
*5.In the following figure, AB is a diameter of a circle with center C. Two semi-circles APC and CQB are drawn on AB. The circle PQR touches all the three semi-circles. If AB = 28 cm, find the radius of the circle PQR.



*6. A positive number leaves a remainder of 1 when divided by 6, 7 or 8. It is divisible by 5. Find the smallest possible value for this number.

7. The edge of a cube is 8 cm. All the faces are painted orange. It is then cut into small cubes with an edge of 1 cm. How many small cubes have exactly two orange faces?

*8. If today were Sunday, July 18, 2004, which day of the week was January 1, 1999?



*9. Arrange the Natural Numbers 1, 2, 3, 4, ... in the order as shown in the figure. The numbers 2, 3, 5, 7, 10, ... are called the "turning number", as the arrow-in and arrow-out of these numbers changes direction at the corner. How many "turning numbers" are there between 529 and 1000?

These problems appeared in the 8th Primary Mathematics World Contest held in Hong Kong July 2004

Answers: 1) 10 2) 30 3) 100 gals 4) 20 kgs 5) 14/3 cm 6) 505 7) 72 8) Friday 9) 18