Math Explorer

MATHEMATICS AND CIRCLES

Self-Similar Snowflakes

GOING IN CIRCLES

Hardy "discovers" Ramanujan

Vol. 6.2 Winter 2003

Math Explorer

Contents

Math Explorer
Texas State University-San Marcos
San Marcos, TX 78666
Phone: (512) 245-3439, Fax: (512) 245-1469
e-mail: max@txstate.edu
Visit our website: www.txstate.edu/mathworks
Math Explorer is published by
Texas State University
Texas Mathworks.

Ramanujan

by Hiroko Warshauer

Srinivasa Ramanujan was born in 1887 in Erode, India. Five years later, his family moved to Kumbakonam, near the city of Madras, where he attended school. A good student, by the age of 13 he already showed a keen interest in mathematics, especially regarding numbers. Though young Ramanujan did not have access to books that contained the mathematical developments of the past, he did come across a book called Synopsis of Elementary Results in Pure Mathematics by G. S. Carr, at his high school. Unaware of many mathematical results, Ramanujan posed numerous mathematical questions and tried to answer them in his own way. By the time he was 15, he had found his own method for solving quartic equations, and was trying to solve quintic equations.

Due to a lack of funds, Ramanujan could not continue his university studies immediately after high school. Regardless, he continued to work on mathematics, and kept a record of his ideas in a notebook. He eventually shared his ideas by writing to a renowned English mathematician, G. H. Hardy at Trinity College, Cambridge in England to discuss his work. Hardy realized that though many of Ramanujan's results were already known in Western mathematics circles, Ramanujan had posed and solved the problems independently and with little formal education. Ramanujan wrote to Hardy, "I have found a friend in you who views my labours sympathetically…"

Hardy invited Ramanujan to Trinity College in 1914 to collaborate on mathematical projects. The month long voyage followed by the difficulties adjusting to a new culture, climate, and religious environment posed extreme challenges for the young mathematician, who continued the practices of an orthodox Brahmin throughout his stay. Ramanujan suffered poor health while in England. During one of his hospital stays, Hardy mentioned while visiting that he had taken a taxi numbered 1729 - Ramanujan responded that it was a remarkable number. 1729 was the smallest integer that can be represented in two ways as the sum of two cubes.

$$1729 = 1^3 + 12^3 = 9^3 + 10^3.$$

Soon their work together led to many important results.

Wrote Hardy about Ramanujan, "In his favourite topics, like infinite series and continued fractions, he had no equal this century. His insight into algebraic formulae,…was truly amazing."

Cont. on pg 7

7

1. If you write out all the numbers from 1 to 1000, how many times will you write the number 7?

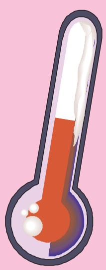
2. There are 100 students in the 5th grade. 1/4 of them play in the basketball league and 1/5 play in the soccer league. If 65 students are in neither league, how many are in both leagues?

3. The area of a rectangular patio is 180 square feet. If the lengths of the sides (in feet) are integers, what is the smallest possible perimeter for the patio?

4. How many ways can we choose 3 different integers from 1 to 9 so that the sum of the integers is 15?

5. What time will it be in 1000 hours if it is 1:00 pm now?

6. The year 2002 was a palindrome year, meaning if we reverse the digits 2-0-0-2, we get the same number. What is the next palindrome year?



7. To convert temperature from Fahrenheit, the scale used in the US, to Celsius, the scale used most commonly throughout the world, we subtract 32, divide by 9 and multiply by 5 in that order. So 95 degrees Fahrenheit = [(95 - 32) X 5] / 9 = 35 degrees Celsius. At what temperature is the number of degrees the same in both Fahrenheit and Celsius?

8. Ian started cycling on Monday and cycled 12 miles each day. Lance started on Tuesday, but by the time they both finished cycling on Friday, they had covered the same number of miles. How many miles did Lance cycle each day?

9. A sequence is formed starting with 1, 2, 3 as the first three terms. To get a new term, we always take the last three terms, add the first two of these and subtract the third. So the 4th term is 1 + 2 - 3 = 0, giving us 1, 2, 3, 0. The 5th term is 2 + 3 - 0 = 5. If the sequence continues in this pattern, find the 2004th term.

Answer: **1)** 300 **2)** 10 **3)** 54 feet **4)** 8 **5)** 5 am **6)** 2112 **7)** - 40° **8)** 15 **9)** -2000

Circles Everywhere

by David Snyder

In this article, you will learn about circles, and how to tell the difference between a geometric circle and a topological circle. You can observe them in your own body, at home, in the classroom , and at the store. Perhaps you will be astonished by how many circles you can find in the world around you!

Geometric Circles

So, what is a circle? If your teacher asks your class to form a circle, you and your classmates form a shape that is round and has a center. Ideally, everyone is the same distance from the center of the circle. If so, the shape looks like Figure 1 below.

The center is shown as a dot in the middle. The circle is the round thing made of people. The distance from the center to anyone standing in the circle is called the *radius* of the circle.

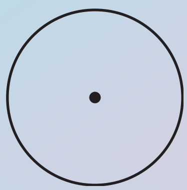
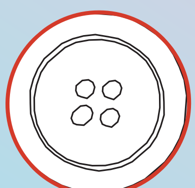
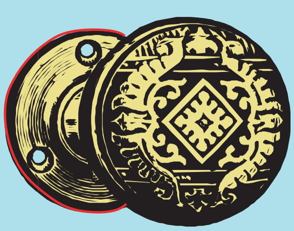


Figure 1

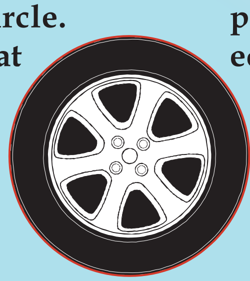
Look around you right now. Do you see any circles? Find as many as you can. Go to all the rooms in your house and make a list of what things you see that look like a circle.



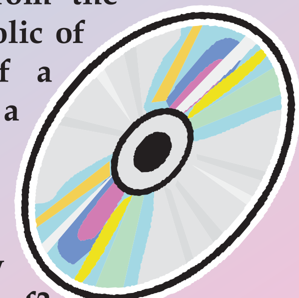


Right now I am looking around my office. I see a button that is round; the outer edge of the button is a circle. I see a metal ring in a school binder-it is a circle.

When I look in the mirror, I see that the rim of the pupil of my eye forms a circle. There is a circle where my doorknob touches the door. A car tire, too, forms a circle.



The circle pictured in figure 1 is called a geometric circle, since every point on the circle is *the same distance* from the center of the circle. If a train or horse travels around a track that is a geometric circle, then it is always the same distance from the center of the circle. When the Knights of The Round Table sat, they sat around a table shaped like a circle, because they considered themselves to be equal. Sitting an equal distance from the center of the table was symbolic of this equality. The edge of a compact disc or DVD forms a circle that spins around its center. These are examples of objects that have a geometric circle shape. How many examples can *you* think of?



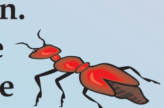
Topological Circles

In the picture below there are 5 objects. Look carefully at them. For each object, list 3 ways it is like a circle and 3 ways that it is different.

Figure 2

When you look at your lists, do you find a common way they are all alike? For each of the objects pictured, consider an ant crawling along the edge of the object in one direction.

Is there a central point that the ant stays the same distance from at all times? Will the ant return to the place she started from?



Lets consider a few other objects shown below.

Figure 3

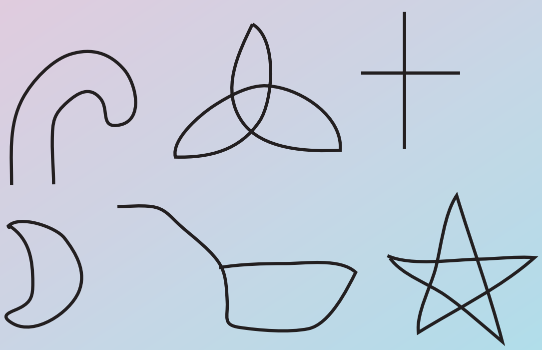
What makes the objects different from the ones in figure 2? How are they the same? What if the ant were crawling along the edge of one of these objects? Is there any way for the ant to get back to where she started without retracing her steps or without crossing over a place she had already been before?

So let's notice a few things about the circle:

1. It traces out a path in space
2. The path never crosses over itself
3. The path does return to where it started.

The 5 objects in figure 2 share this property of the circle. The 3 objects in figure 3 do not. A *topological circle* is any object that has the above three properties of the circle. Which of the following are topological circles? Explain Why or why not. Answers are given below.

Figure 4



Can you find anything in your home and classroom that is shaped like a topological circle? Here are some of the things that I found in my house: a rubber band, a necklace, the window frames, a star-shaped cookie cutter, an oval serving platter, my lips and a loop on my backpack. How many can *you* find?

Answer to figure 4: Only the Crescent Moon

Activities

Drawing a Geometric Circle

For this activity, you will draw an almost perfect circle! You need a pencil, a paperclip (the bigger the better), a pad of paper and a pushpin. Push the pin in the center of the pad of paper - don't push too hard, you need to be able to see the pin between the paper and the handle of the pushpin. Now lay the paperclip on the pad of paper so that the pushpin goes though it. Scoot the paperclip so that one of the rounded ends is next to the pin. Place the point of the pencil in the other rounded end. Push with your pencil so that the one end is tight against the pin and the other end is tight against the pencil point. Keep the tension and rotate the paperclip around the pin with the point of our pencil drawing on the paper.

Drawing a topological circle

To draw a topological circle on paper, begin drawing a path at any point on the paper. The only rules are: 1) end your drawing where you started at and 2) never cross your path.

Creating a twisted circle

Cut a sheet of notebook paper lengthwise into 4 equal strips. With scotch tape, tape the 4 strips end-to-end into one long strip. Draw a straight line down the middle, then again on the other side. Now grab each end of the strip and bring them together, twisting one of the ends so the strip will have one twist in it. After making sure that the lines you drew match up, tape the ends together. Now the lines you drew will form one circle . How many times must your finger trace around the strip to get back to where it started? Now get some scissors ad cut the twisted strip along the middle, using the lines you drew as a guide. What happens?

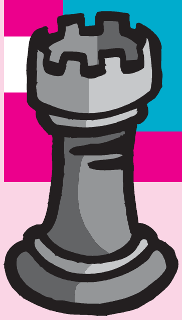
David Snyder is an Associate Professor of mathematics at Texas State University-San Marcos. He is a topologist by training and likes to travel in musical circles.

Puzzle Page

Math Explorers: We want to print your work! Send your original math games, puzzles, problems, and activities to
Texas Mathworks, 601University Dr., San Marcos, TX 78666

A chess board is 8 squares across by 8 squares down. Two of the opposite corners are missing.

Is it possible to cover this board using 31 rectangles that measure 1 square by 2 squares?



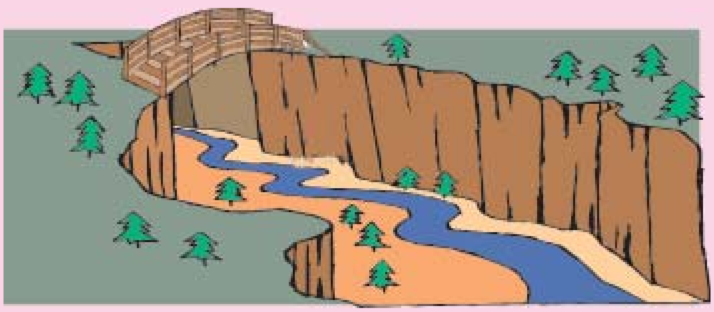
Word Search

Forwards or backwards, up, slanted, or down.
Where can the words in this puzzle be found?

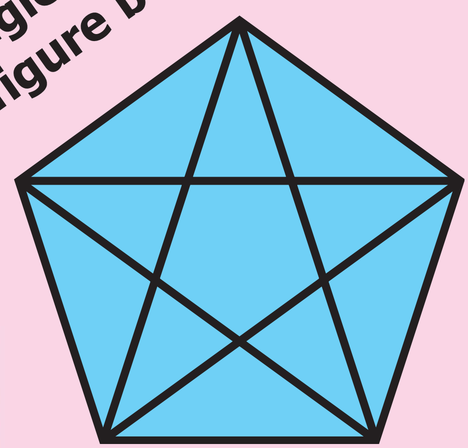
Topology
Geometric
Radius
Distance
Number
Circle
Deform
Fractal
Koch

M	R	R	A	D	I	U	S	T	O	R	E	L	S
T	O	P	O	L	O	G	Y	F	R	R	R	C	R
G	R	C	E	D	A	O	R	R	E	M	A	M	C
P	H	I	T	A	P	T	N	U	M	B	E	R	R
K	A	R	I	H	N	O	C	E	I	C	C	T	I
A	O	T	S	P	O	O	M	A	N	E	R	S	E
D	O	E	C	F	I	L	R	O	R	E	F	T	E
K	N	M	R	L	S	C	O	B	C	F	O	C	E
T	O	O	R	R	C	I	F	R	E	N	N	B	R
E	R	E	I	O	M	R	E	D	T	A	N	L	R
I	C	G	T	R	F	C	D	R	T	P	R	S	E
A	U	R	D	A	B	L	U	S	W	Y	L	E	E
K	O	C	H	O	L	E	I	S	C	I	N	L	O
N	A	A	C	E	L	D	R	I	E	C	E	A	I

Four people must cross a canyon at night on a rickety bridge. Only two people can be on the bridge at the same time. To cross, they must have a flashlight, and they only have one flashlight to share. Alone, the four people cross in 10, 5, 2, and 1 minutes. When two people cross together, they can only go as fast as the slower person. In 18 minutes a flash flood coming down the canyon will wash the bridge away. Can the four people get across in time? How can it be done?



How many triangles are in the figure below?



Bulletin Board

The Educational Advancement Foundation provides grant to Texas Mathworks

The Educational Advancement Foundation (EAF) gave Texas Mathworks a grant of $43,364 to develop and implement a unique Discovery Learning Project for young students.

Mathworks team represents US in Hong Kong Math competition

Eileen Martin, Pranay Kothari, Stephanie Chan, and Jeffrey Chen will represent the Mathworks team in the 7th annual Primary Mathematics World Competition in Hong Kong this December.

New Intel Preservice Program

Mathworks received $50,000 from the Intel Foundation to link preservice education majors at Texas State University with inservice teachers, building a continuum of teachers dedicated to exciting, inquiry-based instruction.

Check the Web

Looking for challenging math problems? Go to http://mathforum.org/students for problems of the week at various levels.

Ramanujan...continued from pg. 2

Ramanujan earned the equivalent of a Ph. D from Cambridge in 1920 and was honored as a fellow of the prestigious Royal Society of London. Ramanujan died at the young age of 32.

References:
http://scienceworld.wolfram.com/biography/Ramanujan.html
http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Ramanujan.html
http://www.uz.ac.zw/science/maths/zimaths/raamanhdy.htm

MATH ODYSSEY

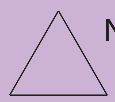
by Hiroko Warshauer

Koch Snowflakes

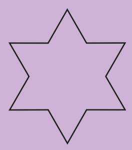
Have you ever observed the shape of a snowflake? The tiny ice crystals are usually symmetrical and hexagonal, yet it is said that no two of them are alike. Since many readers in Texas usually do not get a chance to see a snowflake, much less catch one on their tongue, we will create our own snowflakes and study an interesting phenomenon.

The snowflake that we study is called the Koch Snowflake, named after Niels Fabian Helge von Koch, a Swedish mathematician, who in 1904 studied some of the interesting properties associated with this shape. The Koch Snowflake is an example of a fractal. A fractal is an object which is self-similar. Nature provides many examples of fractals, such as ferns, cauliflower, even coast lines. Though often in photographs coastlines appear to be jagged edges, upon closer inspection small sections of the coastline may actually be smaller replicas of the larger coastline. Such is the case with the Koch snowflake. Let's see how.

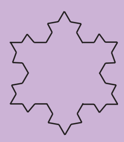
1. Begin with an equilateral triangle with sides of dimension 3 units.
Let's call this snowflake N=1.


N=1

2. Trisect each side of the equilateral triangle and remove the middle third. Construct another equilateral triangle on the outside of each side using the length of the removed middle third. It should look like a "Star of David".
This is snowflake N=2.


N=2

3. Repeat step 2 with each of the twelve sides of the "snowflake". We get snowflake N=3.


N=3

If we continue step 3 repeatedly, notice we get snowflake, N=4, N=5 and so on.

Let's investigate one aspect of the snowflakes at each stage. What is the perimeter of the snowflake N=1? Yes, it's 9 units. What is the perimeter of snowflake N=2? With a little calculation you probably come up with 12 units. Find the perimeter of N=3. You may wish to make a chart to keep a record and possibly find a pattern for the new perimeter at each iteration (this just means when you do the repeated step). As you continue to create more and more protrusions on the original equilateral triangle, can you see the perimeter getting larger and larger? In fact, there is no limit to how large the perimeter can get, and we say it has infinite perimeter!

N	Perimeter
1	9
2	12
3	
4	
5	

References:
http://scidiv.bcc.ctc.edu/Math/Snowflake.html
http://math.rice.edu/~lanius/frac/koch.html
http://teacherlink.org/content/math/activities/mw-snowflake/guide.html

Dear Math Explorers,

Snowflakes are a rare sight in Central Texas but read about the Koch Snowflakes in the Math Odyssey to learn about this interesting shape. Our main article explores the difference between geometric and topological circles and the puzzles and problems offer challenges for our readers over the holidays.

The year 2004 is a leap year. Do you know how many leap years we'll have in the millennium? May each day of the new year hold fun math problems to challenge you!

Sincerely,

Hiroko K. Warshauer

Hiroko K. Warshauer, executive editor