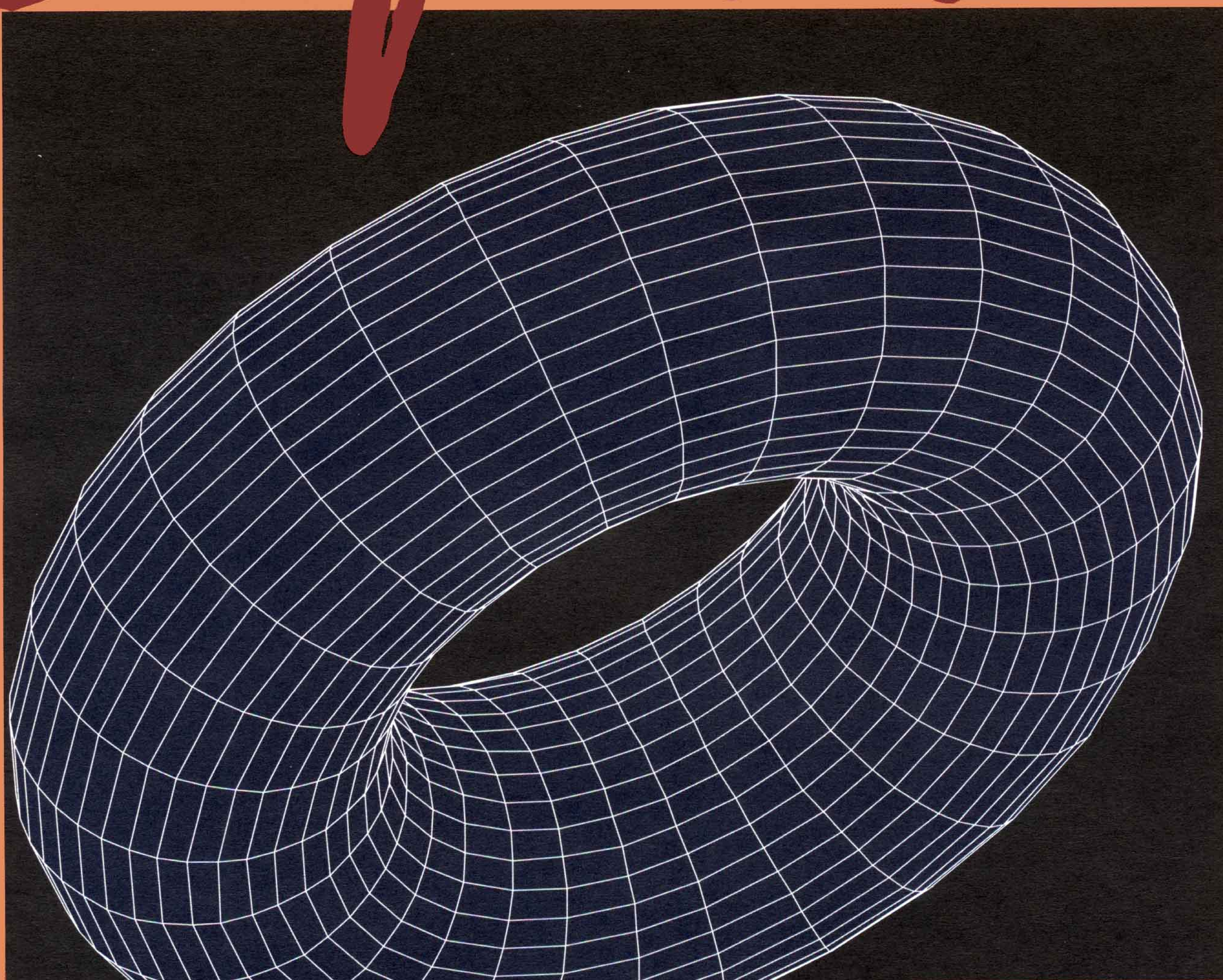
Math Explorer



MATH & TOPOLOGY

Trekking on a Torus!

Un wrap the Doughnut?

bernhard Riemann Surfaces!

Vol. 3.2 Winter 2000

Math Explorer

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Math Explorer

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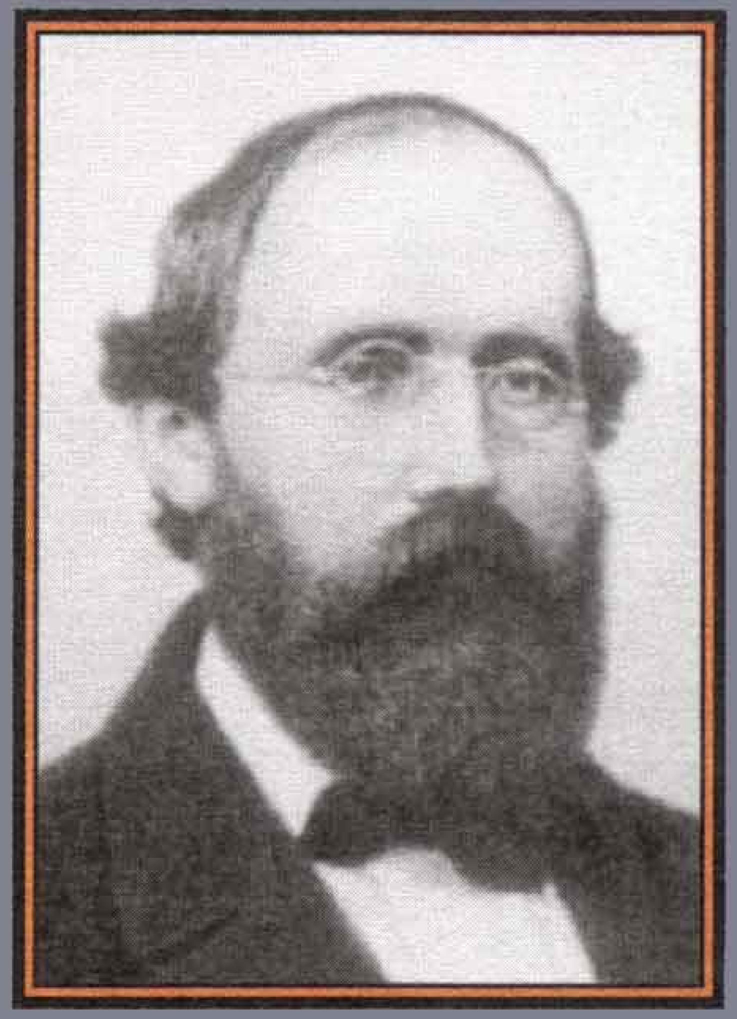
SWT

Georg Friedrich Bernhard Riemann



by Hiroko K. Warshauer

Georg Friedrich Bernhard Riemann was born in Germany on September 17, 1826. His father, a Lutheran minister, was Bernhard's teacher until he was 10 years old. As he continued his education in various schools, Bernhard showed a strong interest in mathematics. The director of his high school, called a gymnasium in Germany, observed the keen interest Bernhard showed and allowed him to study mathematics texts from his own library.

In 1846, Riemann enrolled at the University of Gottingen, known for its strong mathematics program. There he studied mathematics with Gauss, one of the greatest mathematicians of the century. Later, at the University of Berlin, Dirichlet, another notable mathematician, influenced Riemann. They both shared a preference for intuitive reasoning over lengthy computations in their mathematical discussions. Later Riemann would use an idea he learned from Dirichlet and call it the **Dirichlet Principle**.

Returning to the University of Gottingen, Riemann wrote his Ph.D. thesis under Gauss' supervision. The thesis studied the theory of complex variables and geometric properties on the connectivity of surfaces which we now call **Riemann surfaces**. It was considered a brilliant and original piece of work. In the years to follow, Riemann would make many contributions in the area of geometry and topology (originating from the study of geometry where distance is not relevant). His name is attached to numerous concepts studied in mathematics, including **Riemann integrals** and **Riemannian spaces**. Einstein used Riemann's ideas about the dimensions of real space and the geometry that describes it as a framework for his theory of relativity.

Thirteen years after beginning his studies at Gottingen, Riemann became the chair of mathematics there following Dirichlet's death. An important unsolved

(continued p.7)

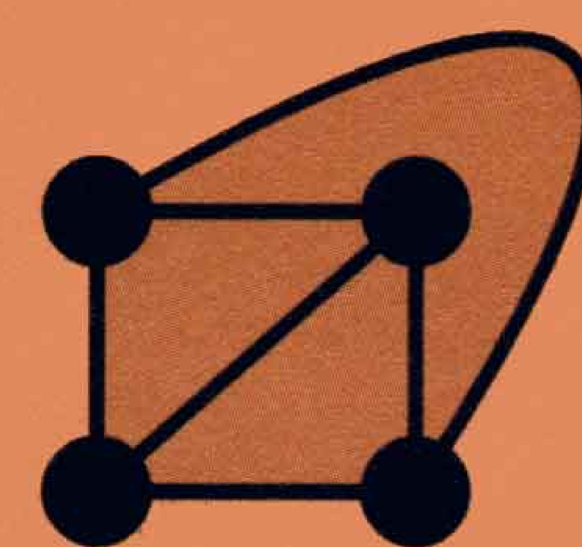
PROBLEMS OF THE MONTH

1. In the year 2000, Jody's birthday falls on a Wednesday in the month of May. In the years 2001 through 2050, how many of Jody's birthdays will fall during the weekend (on Friday, Saturday or Sunday)?

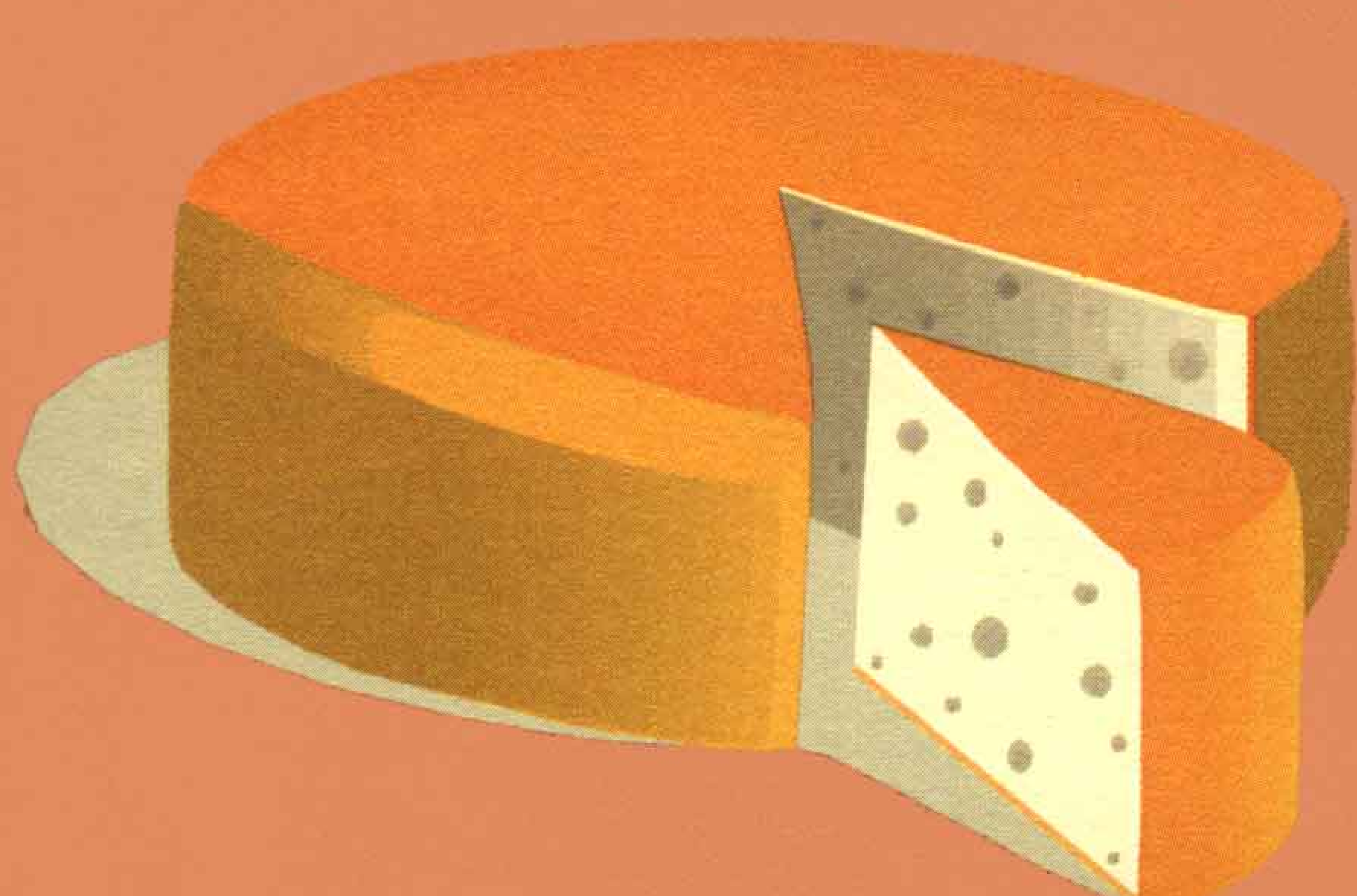
2. Suppose n is an integer so that $1/n + 1/(n+1) + 1/(n+2) = 253/1092$. What is n?

3. Suppose we made days metric, so that each day is divided into 10 metric hours, each hour into 100 metric minutes and each metric minute into 100 metric seconds. How many metric seconds would be in a day? What is the ratio of the length of a metric second to that of an old fashioned second?

4. If you draw 4 points on a page, it is possible to draw curves to connect every point to every other point without any curves crossing each other. With 5 points you have to have crossings on the page. Draw a picture of a doughnut with 5 points and 10 curves drawn on it so that each point is connected to each other point by a curve, and no curves cross each other. Try it with 6 points and 15 curves on the doughnut also!



5. Suppose that three points are selected on the boundary of a square with side of length 1. If the three points are connected by line segments to form a triangle, what is the largest possible area of such a triangle? How many triangles will give you this area?



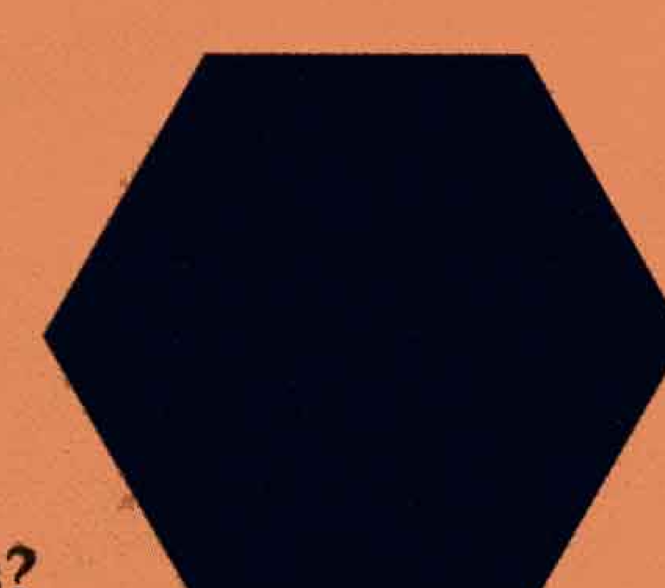
6. A farmer sells cheese in two sizes, 13 pound and 8 pound cylinders. If you are having a big party and want exactly 170 pounds of cheese, how many of each size cheese cylinders will give you the exact amount of cheese? Is there more than one way to buy the cheese?

7. How many ways can you give someone 85 cents only using nickels, dimes and quarters?

8. What are the next three members of the following sequence: 1, 5, 13, 29, 61, ___, ___, ___ ? Can you find a pattern in this sequence that would help you determine the 20th member of the sequence?

9. Find a pair of prime numbers that add up to 78. How many such pairs are there?

Ingenuity: A regular hexagon is a hexagon that has six sides of equal length and all the angles are the same (congruent). Suppose that eight regular hexagons of equal size are joined together so that each hexagon shares at least one side with another hexagon. After the hexagons are joined together, a new shape appears. What is the least number of sides this new figure could have? What is the greatest possible number of sides? Assume the new shape is in one piece, not two or more pieces.



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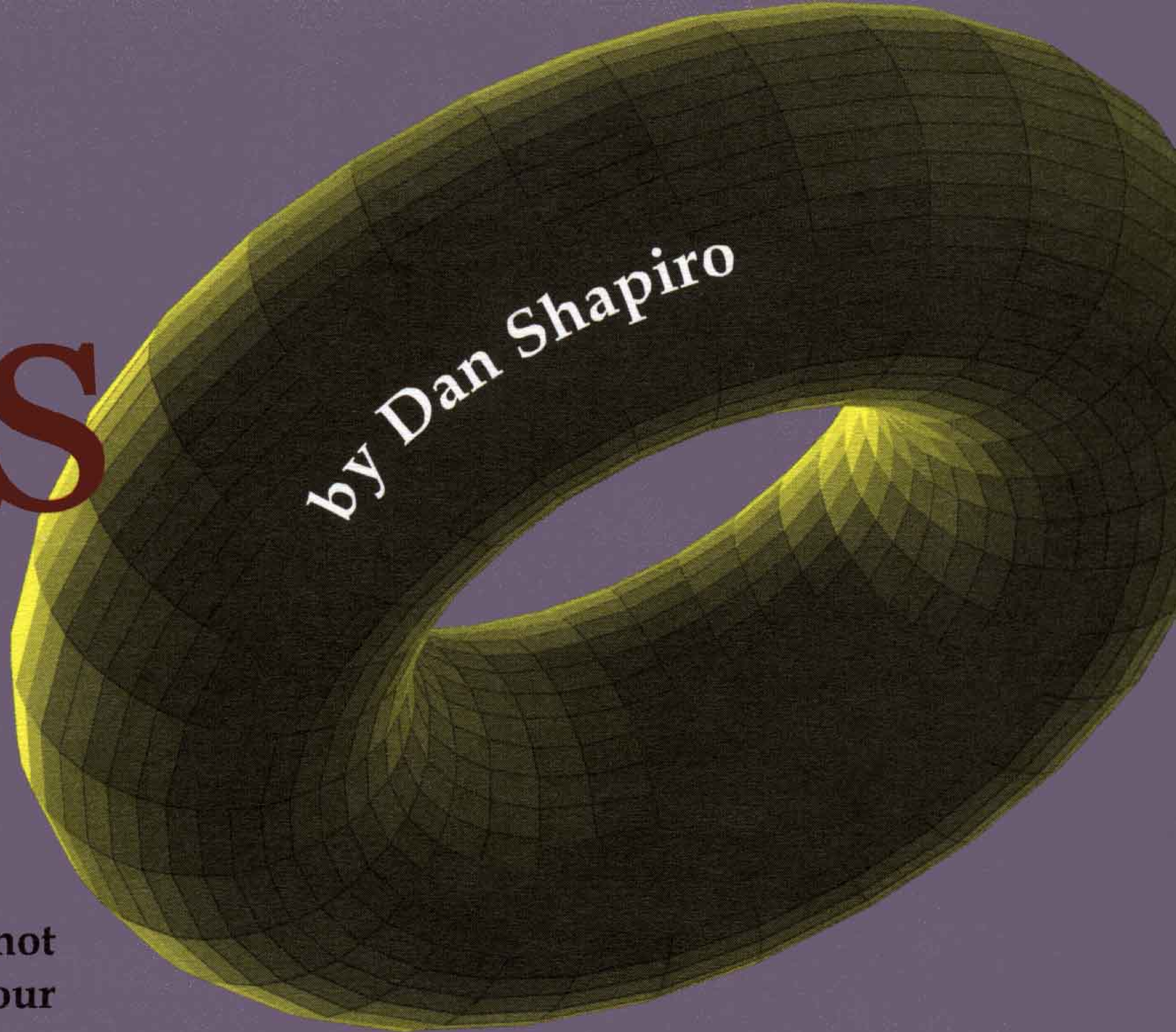
Torus Trek

by Dan Shapiro



In some old computer games (If you have not seen these ancient games, try to imagine your ancestors playing them.) there are a few space ships moving on the screen, along with various other objects. When an object (ship, bullet, asteroid, etc.) leaves the screen to the right it reappears on the left, as if the right and left edges of the video screen were connected together. In Figure 1, our ship is shooting out a stream of bullets, eventually hitting the asteroid.

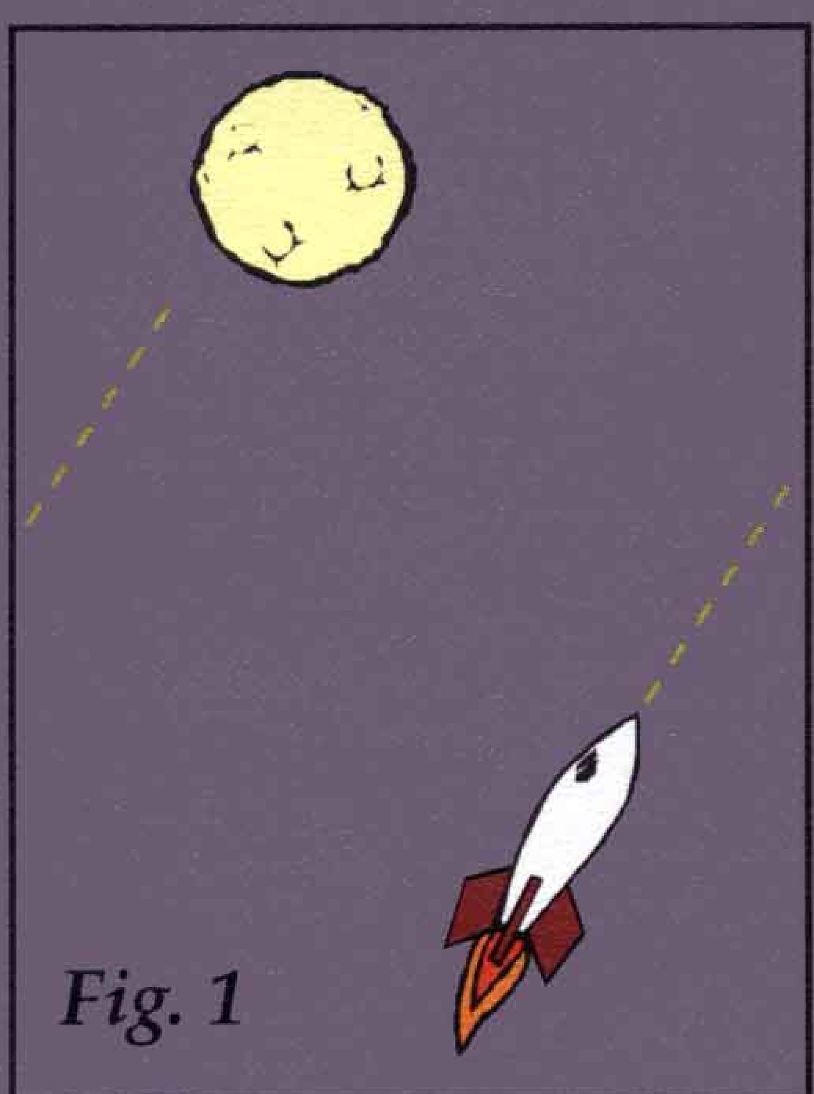


Fig. 1

Imagine a paper copy of the screen, and glue its right and left edges together to form a cylinder. This cylindrical version of the screen in Figure 2 explains the "wrap-around" effect for the left and right edges. On the cylinder there is no sudden jump of bullets from the right edge to the left: the bullets move naturally. This phenomenon reminds me of world maps hanging on school room walls. The western tip of Alaska on the far left of the map seems distant from the eastern tip of Russia on the far right, but looking at a globe we see that those spots are actually very close to each other.

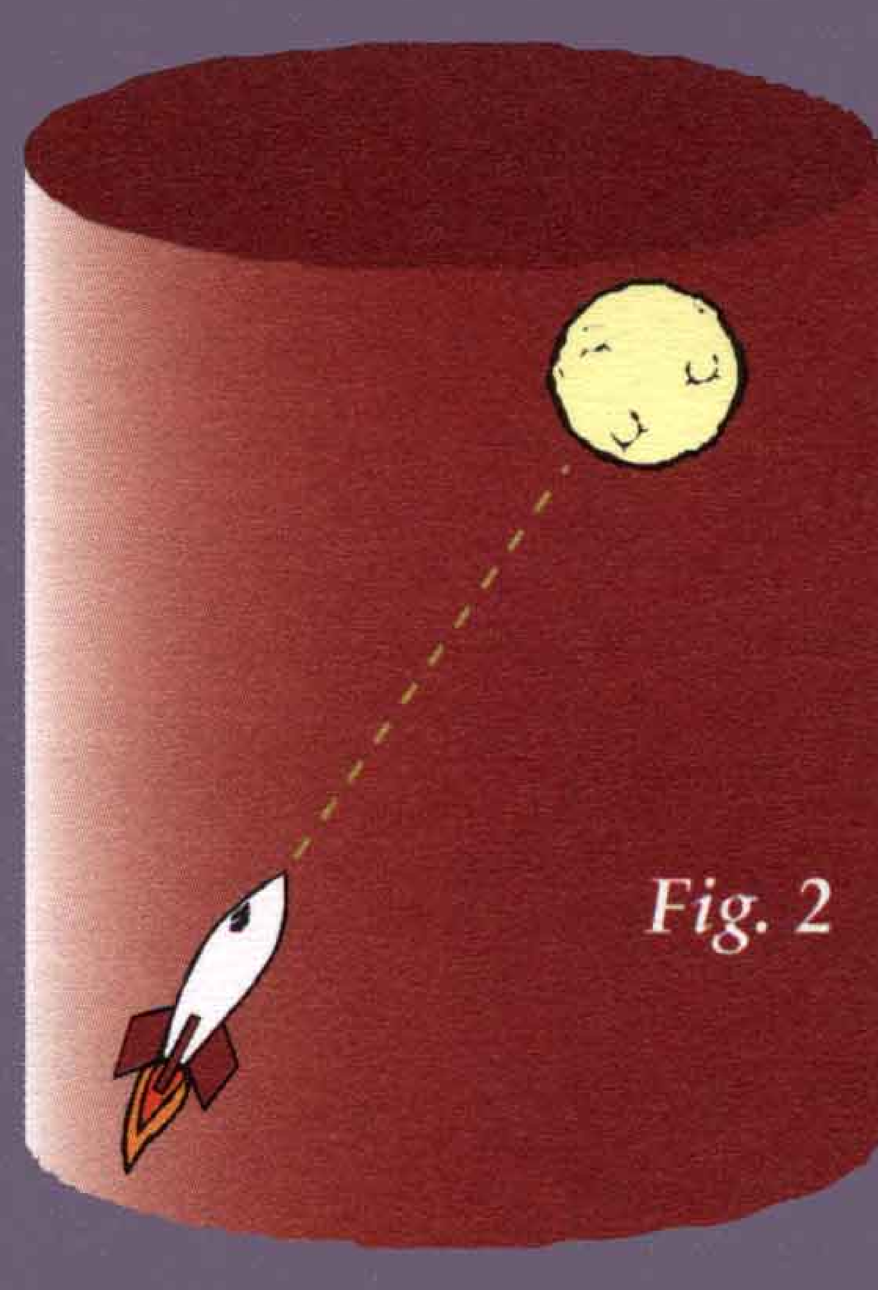


Fig. 2

Another way to understand the geometry of this computer screen is to "unroll" that cylinder over and over as in Figure 3, making a long strip containing many identical copies of the original screen. When the ship shoots a bullet, each of the identical ships on the strip simultaneously shoots a bullet, and those bullets all travel smoothly without any unnatural jumps. The boundaries of the original screen appear as evenly spaced vertical separators on this strip: they are an artificial choice, not part of the natural geometry. We could cut the cylinder along a different vertical line to get a somewhat different screen representing the same picture. Of course there is really only one ship there, represented as a long line of identical images.

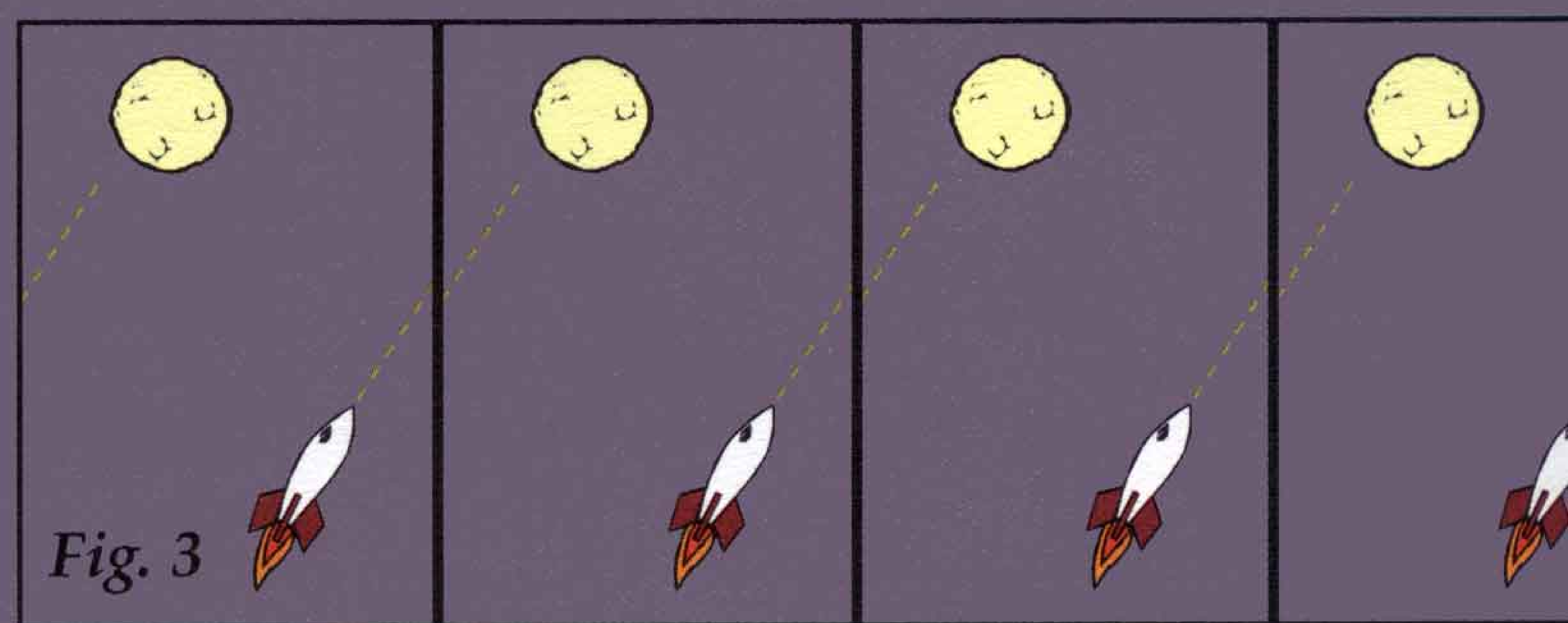


Fig. 3

Back in the original game, when a bullet leaves at the top of the video screen it reappears at the bottom, as if the top and bottom edges of the screen were connected together. In Figure 4 the stream of bullets has split the asteroid in two and continues on. (Luckily, those bullets missed the ship, but they came so close that the gunnery sergeant was punished.)

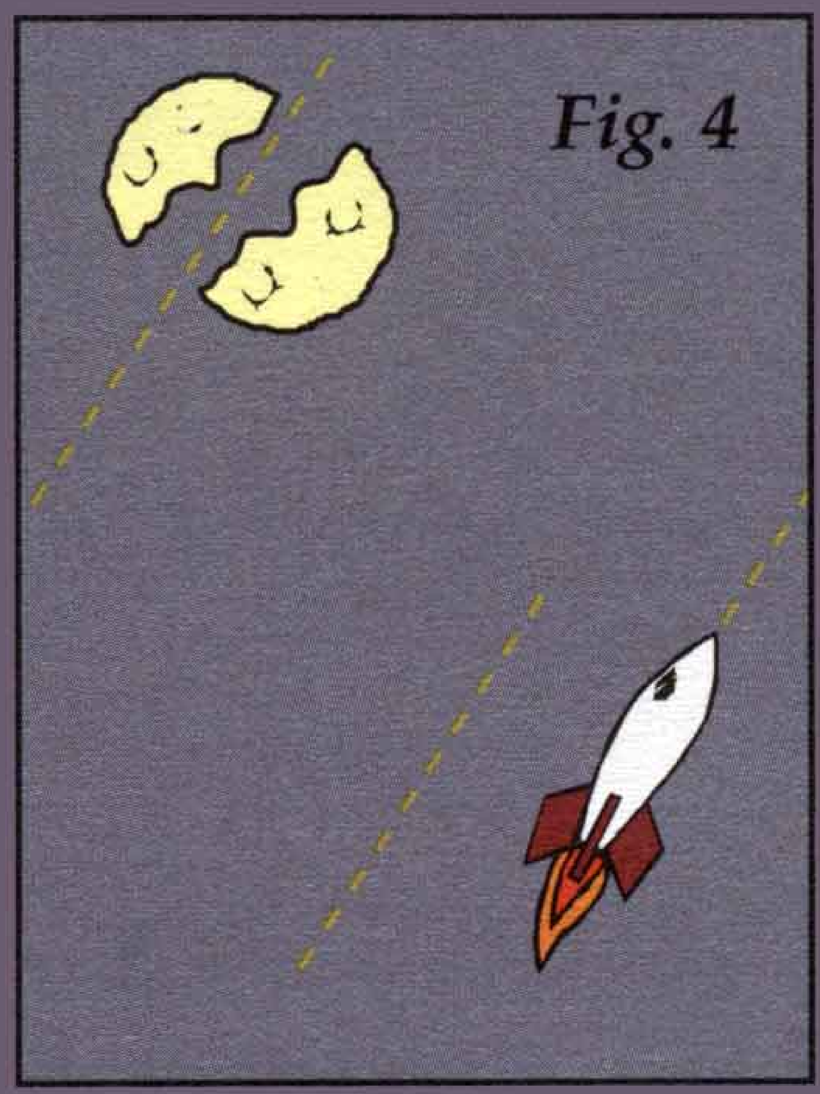


Fig. 4

As before, let's consider a paper copy of the video screen, this time gluing the top and bottom edges together. This cylindrical version of the screen in Figure 5 shows the "wrap-around" effect for the top and bottom edges. Now there is no sudden jump of bullets from the top to the bottom (although there is still a jump from left to right in the cylindrical picture). This cylinder could also be unrolled into a long vertical strip of identical copies of the screen.

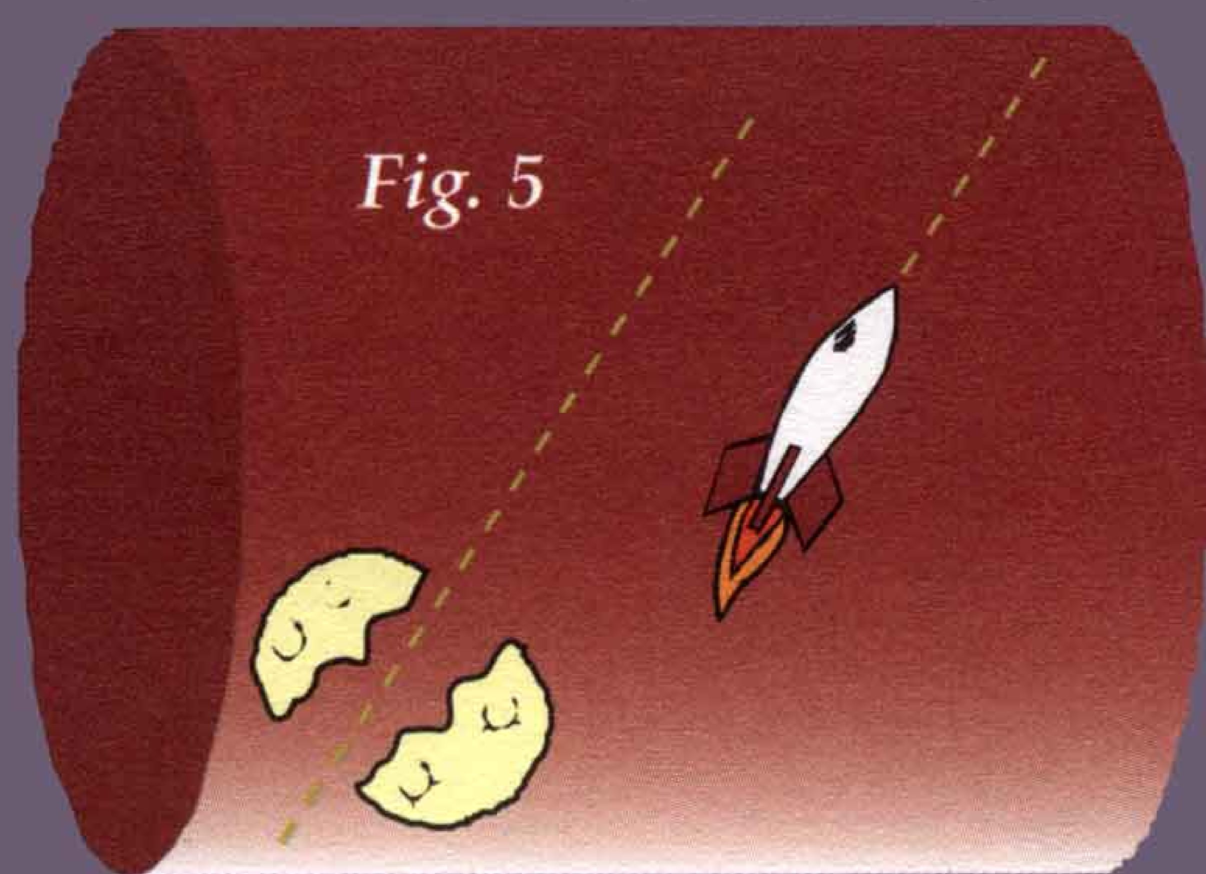
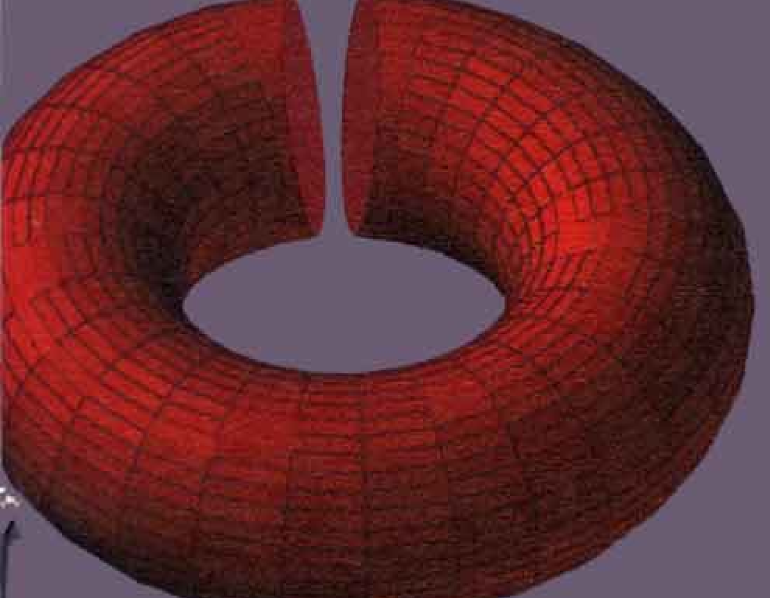


Fig. 5

We glued vertical edges one time and horizontal edges the other time. Now we want to glue both pairs of edges at the same time. Some stretching of the surface seems required to actually do this. Let's start with the cylinder in Figure 5, which has two circular edges, one on the left and one on the right. Stretch that cylinder and smoothly bend it around so that those two circular edges are facing each other, as in Figure 6.


g. 6

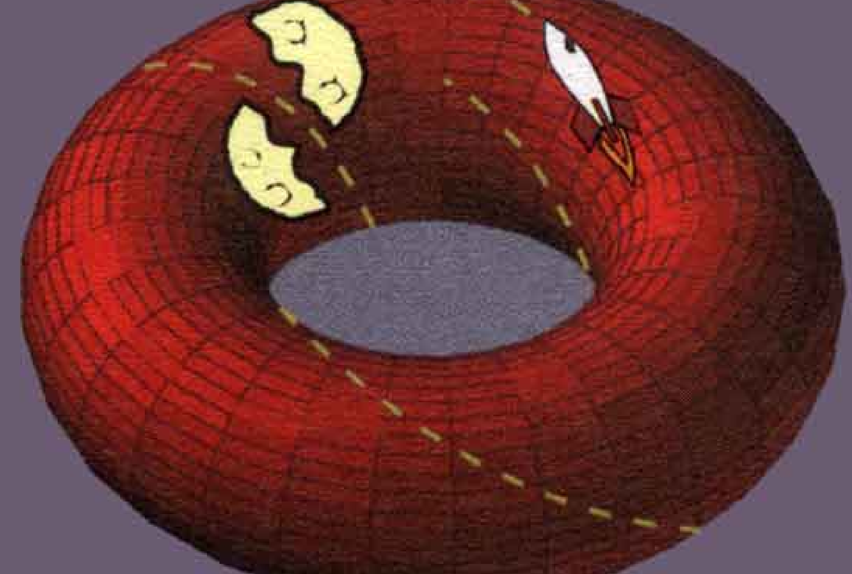


Fig. 7

Move those circular edges together and attach them to form a closed donut-shaped object as in Figure 7, with our space ship and asteroid picture drawn on its surface. That surface is called a torus. (The plural form of torus is "tori", like cactus and cacti and octopus and octopi.) The bullets don't make unnatural jumps across any edges: the torus has no edges at all!

Cutting it along a vertical circle leads to Figure 6 and the horizontal cylinder of Figure 5. We could instead cut a horizontal circle around the hole of the torus and then straighten the torus out vertically to obtain the cylinder in Figure 2 (with a split asteroid).

Another way to understand the geometry of the torus is to "unroll" it over and over in both directions, making a plane covered by many identical copies of the flat screen in Figure 4. Every point of this screen is represented in Figure 8 by a whole two dimensional array of points equally spaced across the plane. (The six copies of the picture here are supposed to repeat indefinitely in all directions.) There is "really" only one ship and one (split) asteroid and one bullet stream in this unrolled view.

We "unrolled" the screen into those multiple copies in order to demonstrate that the bullets travel in a straight line without any jumps at all. Those jumps in the original game from right to left and from up to down are not really jumps at all. They result from the artificial way the torus was represented as a flat video screen.

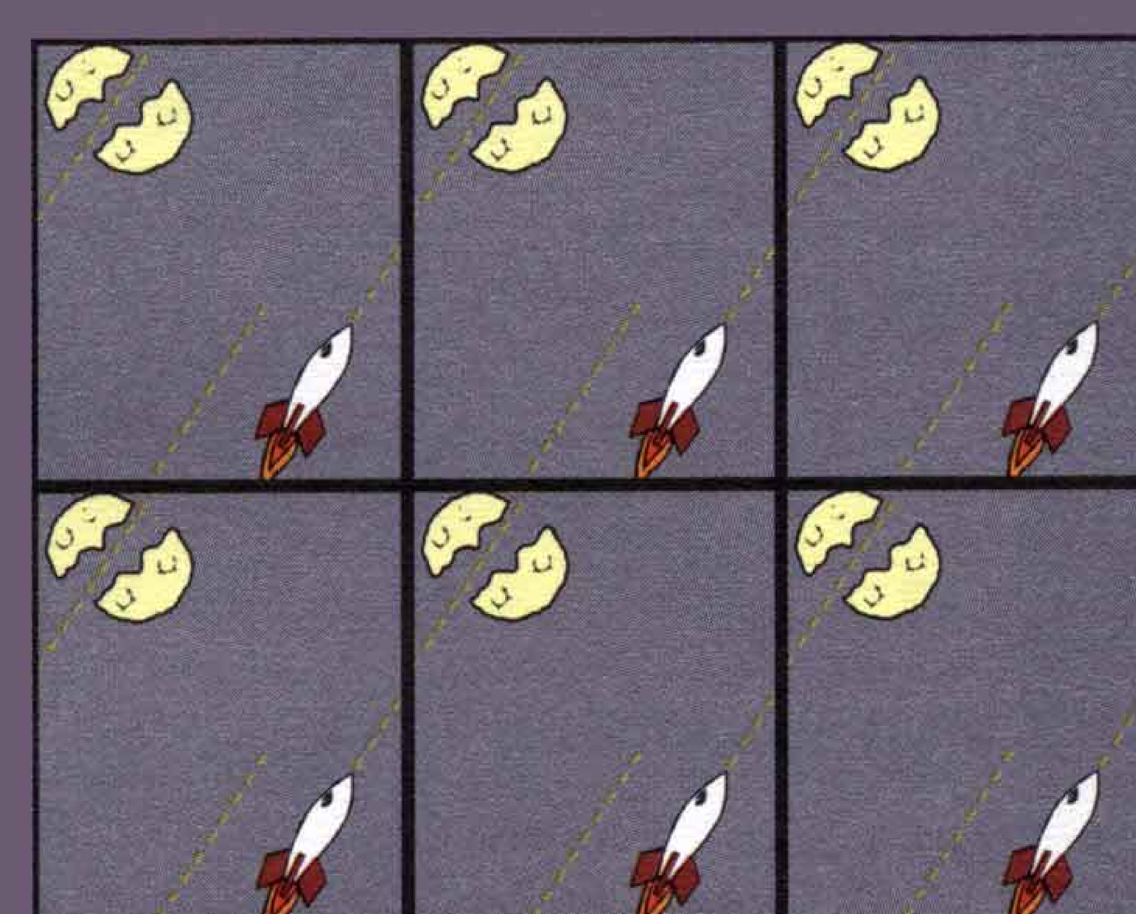


Fig. 8

This article explains that the original computer game is actually played on a torus, not on a plane. Now whenever you see a rectangular wallpaper pattern repeated over and over you can say to yourself: "That's really an unrolled version of a torus."

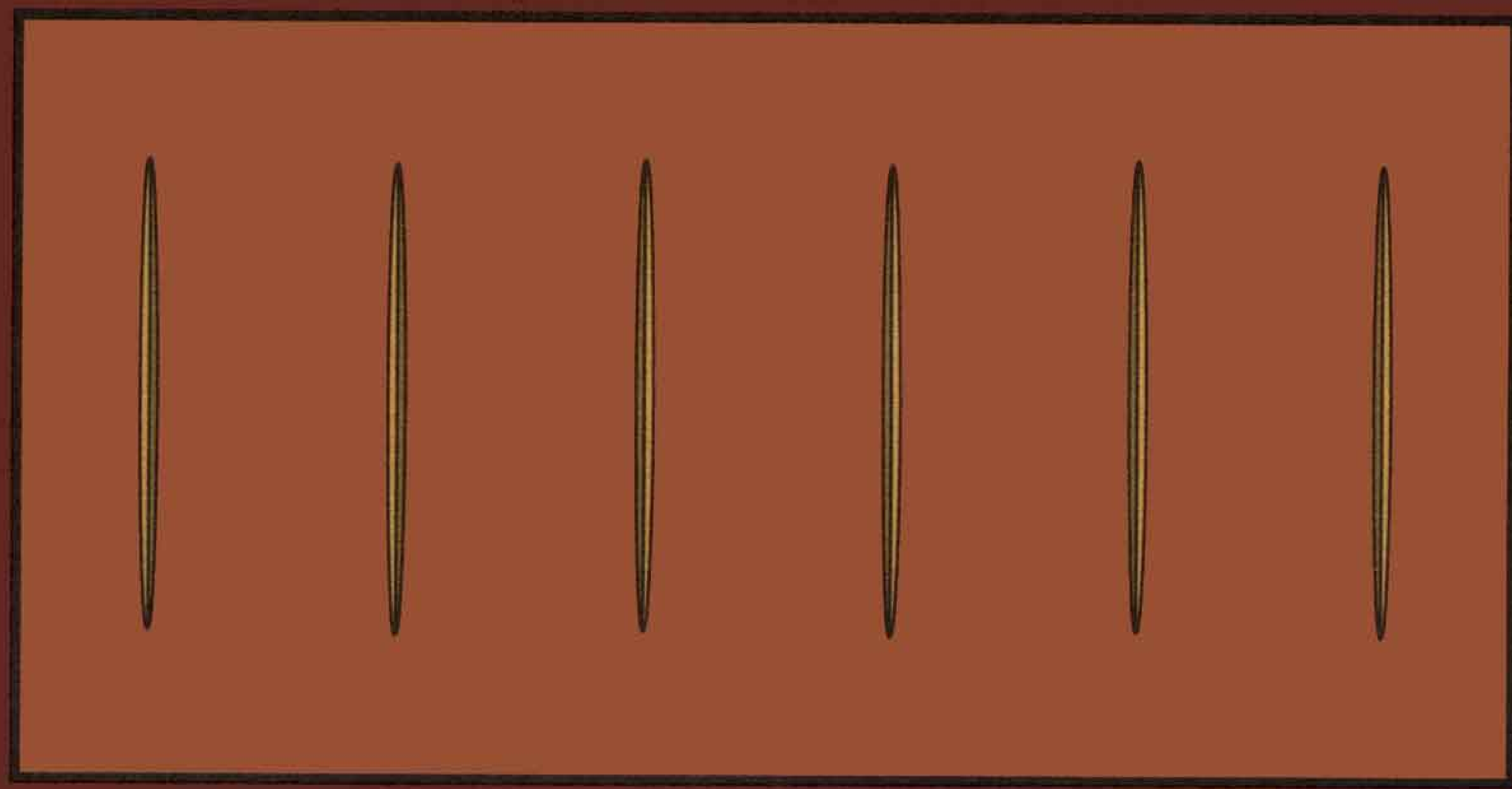
Our author and frequent contributor to Math Explorer, Dan Shapiro, is a professor of mathematics at Ohio State University

Puzzle Page

Math Explorers:

We want to print your work! Send us original math games, puzzles, problems, and activities. If we print them, we'll send you and your math teacher free *Math Explorer* pens.

How can you make four equilateral triangles from these six toothpicks?



FOUR 4's

Make the following true by writing in +, - , x, /, ( )

(4 +	4) -	(4 +	4)	=	0
4	4	4	4	=	1
4	4	4	4	=	2
4	4	4	4	=	3
4	4	4	4	=	4
4	4	4	4	=	5
4	4	4	4	=	6
4	4	4	4	=	7
4	4	4	4	=	8
4	4	4	4	=	9
4	4	4	4	=	10

Each letter below represents a different digit. What numbers make this problem true?

```
  HOCUS
 +POCUS
 ------
 PRESTO
```

Bulletin Board

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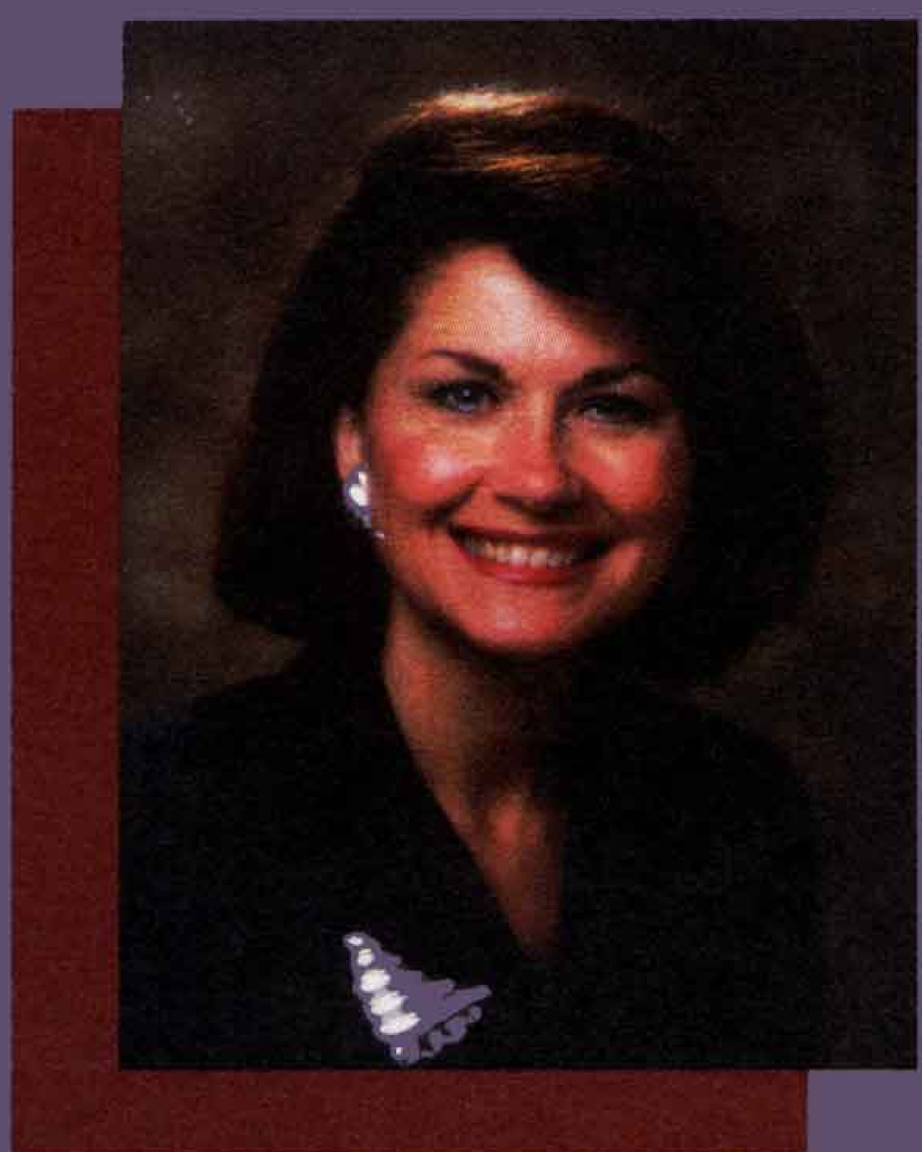

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-- Karen Jennings
Sr. Executive VP - Human Resources
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Words of Wisdom

"It is impossible to be a mathematician without being a poet in soul...The poet has only to perceive that which others do not perceive, to look deeper than others look. And the mathematician must do the same thing."

-- Sonya Kovaleskaya

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Riemann (continued)

problem of mathematics, called the **Riemann Hypothesis,** is a result of one of his many investigations. His health declined from tuberculosis and he died in 1866, barely 40 years of age.

Reference:http://www.groups.dcs.st-andrews.ac.uk/~history/Mathematicians/Riemann.html

Hiroko K. Warshauer teaches mathematics at Southwest Texas State University.

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MATH ODYSSEY

Torus Word Search

by Dan Shapiro

In this rectangle of letters we use the wrap-around methods described in the main article. The right connects to the left and the top connects to the bottom. The hidden words are listed below. All except one of the words listed appear in the rectangle in straight lines. However, these "lines" can wrap around the rectangle, with right connecting to left, and top connecting to bottom. These words can appear in different directions: left, right, up, down or diagonal.

You may find it useful to make a copy of the grid of letters on a square piece of paper and roll the paper to see how the letters "line up". To get you started, the word "wraparound" is already highlighted.

Word list:

wraparound	*cylinder*
torus	*mobius*
bullet	*color*
loop	*pain*
asteroid	*appeal*

L	N	W	N	M	O	B
I	L	P	R	D	A	O
N	U	E	O	A	R	A
D	M	S	T	O	P	S
E	P	P	A	P	L	A
R	U	S	T	E	T	O
C	O	C	I	M	B	E
Y	O	U	O	C	O	U

Remember, one of those words does not appear in the grid.

Math explorations take us on many different paths. In our winter issue of *Math Explorer*, we take a closer look at how points on flat and curved surfaces connect. Graph theory and topology are some of the most fascinating areas of mathematics. We hope this is just the beginning of your exploration.

Happy Holidays and Good Wishes for the year $2 \times 10^3 + 1$!

Sincerely,

Hiroko K. Warshauer

Hiroko K. Warshauer, editor