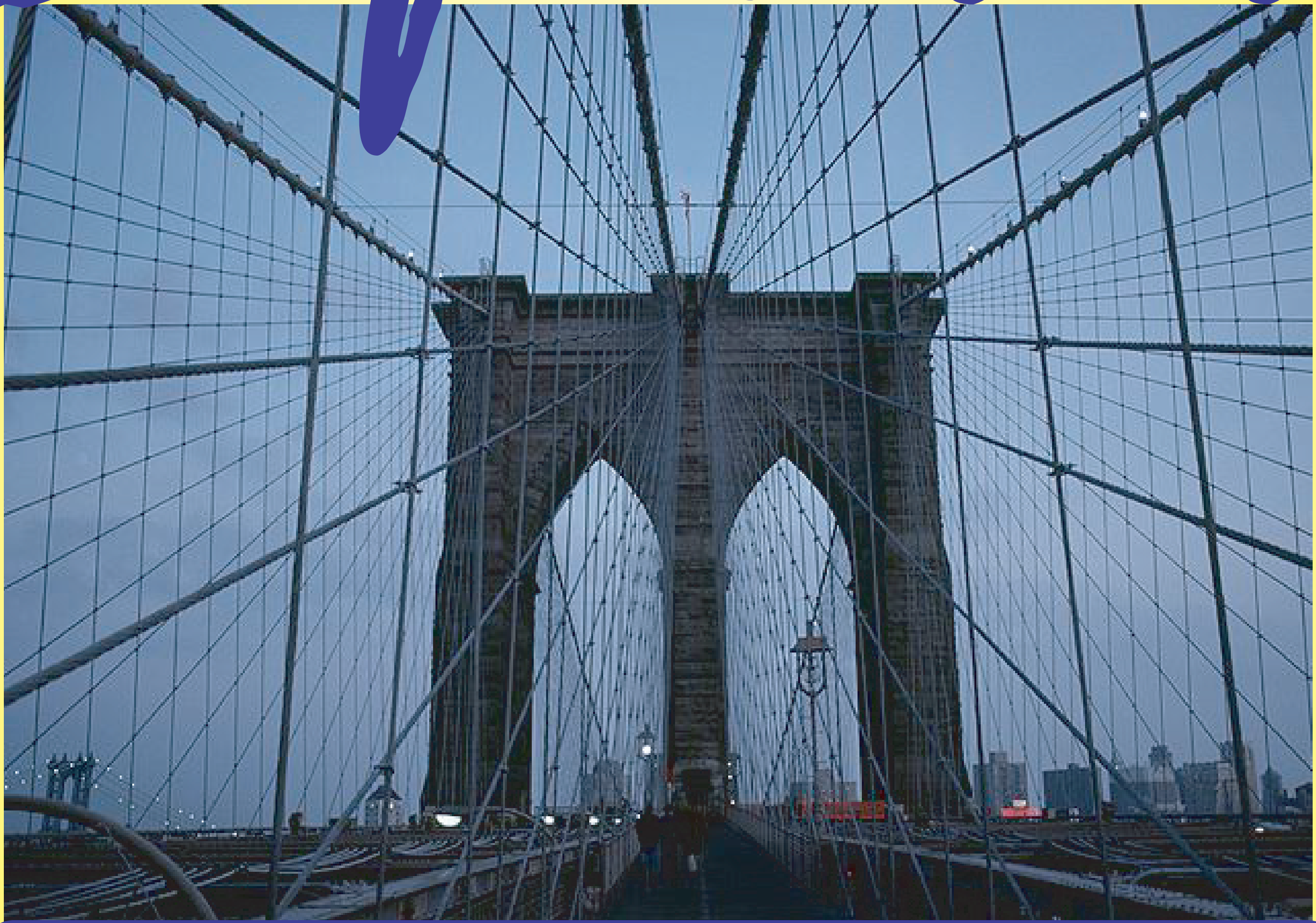
# Math Explorer



MATHEMATICS AND GRAPH THEORY

Those Fabulous Bernoulli Boys

A "plane" look at Euler's Formula

$V + F - E = 2$

*Valuing Face and Place*

Vol. 7.3 Spring 2005

# Math Explorer

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Jacob Bernoulli

# Those Fabulous Bernoulli Boys

by Jean Davis

Johann Bernoulli

In the 17th and 18th centuries, the Swiss Family Bernoulli produced a number of outstanding mathematicians. First and foremost among them was Jacob Bernoulli (1654 - 1705). Jacob was a bright young man who, under pressure from his parents, went to the University of Basel to study philosophy and theology. In spite of his parents' wishes, he was instead attracted to the study of mathematics and astronomy. In the best tradition of the rebellious child, his motto was "I study the stars against my father's will."

From 1676 to 1683, Jacob traveled through Europe, studying with the leading mathematicians and scientists of the day. In 1687, Jacob's little brother Johann became interested in mathematics and asked his big brother to teach him. Together the two brothers tackled one of the most difficult subjects of the day - the new mathematics of calculus.

Johann was 13 years younger than Jacob, and as the brothers worked on similar types of problems, their collaboration turned to competition and rivalry. A bitter and sometimes very public feud developed between the two. Both men were brilliant mathematicians. Jacob was the deeper thinker, but Johann was quicker intellectually.

Some of Jacob's important contributions were in the areas of probability, geometry, and the physics of moving objects. He worked on problems like this one: If you were to toss a coin 50 times, what are the chances of getting at least 25 heads?

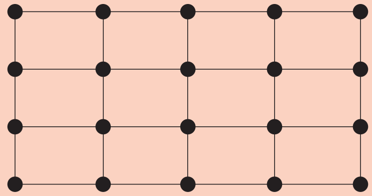
Johann applied calculus to the solutions of many real-world problems. What shape is formed when a flexible cable is held up by its ends? What path should an object follow to move from point A to point B in the shortest amount of time?

When Jacob died in 1705, guess who took over his job as professor of mathematics at the University of Basel? You guessed it - little brother Johann!

*Jean Davis has been a member of the Mathematics faculty at Texas State since 1979. She loves telling people about mathematics and is particularly interested in the history of mathematics.*

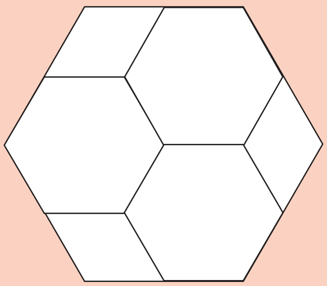
## PROBLEMS PAGE

**Here are some mathematics from "Down Under". These problems come from the Australian Mathematics Competition for the WestPac Awards given in July 2004. More coming your way in our Summer Issue.**



1. How many rectangles can be formed in this grid, using the vertices and edges of the grid?

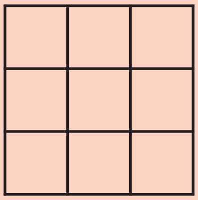
2. Alana had an unusual experience while shopping: she found that every time she bought something, she spent exactly 20% of the money she had in her purse. She bought three items and finished her shopping with $64 in her purse. How much did she have at the beginning of her shopping trip?



3. A regular hexagon is divided into three smaller hexagons and three equal rhombi as shown. If the area of the larger hexagon is 360 square centimeters, find the area of each rhombus.

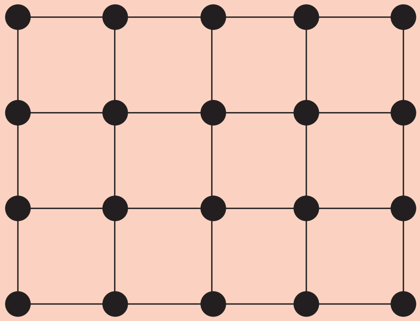
4. Isaac has 6 sticks, all of different lengths, from which he can make an equilateral triangle with two sticks along each side. Five of his sticks measure 25, 29, 33, 37, and 41 centimeters. How many different lengths are possible for his 6th stick?

5. The fraction 1/4 is tripled by adding the same number to both numerator and denominator. What is that number?



6. How many ways can an X be placed in the cells of the grid shown so that each row and each column contains exactly two cells with an x?

7. Aaron tells the truth on Monday, Tuesday, Wednesday and Thursday. He lies on all other days. Jacob tells the truth on Monday, Friday, Saturday and Sunday. He lies on all other days. On what day do they both say, "Yesterday I lied."



8. In Euler county there are exactly 20 cities and 31 roads connecting neighboring cities as shown in the diagram. Unfortunately, all the roads are in bad condition and need repair. What is the maximum number of roads that can be closed for repair at the same time so that is is still possible to travel from each city to any other along roads?

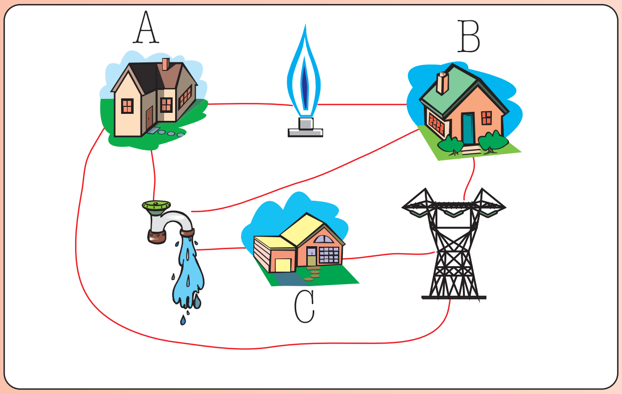
Answer: **1)** 60 **2)** $125 **3)** 30 sq cm **4)** 3 **5)** Friday **6)** 6 **7)** 60 **8)** 12

# Euler's Formula
## and the Three Utility Problem

by Eugene Curtin

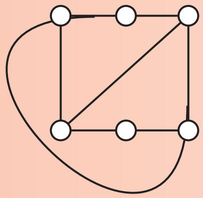
When I was about 12, I was challenged with the following puzzle: Given a sheet of paper with 3 houses and 3 utility stations (water, electric and gas), draw a connection from each utility to each house without having the connections cross on the page. You might like to try this.

In one unsuccessful effort below I have all the connections except from gas to house C, and I can't draw it without crossing one of the other connections.



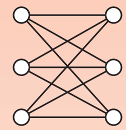

After a while I suspected that there was no way to do it, but I did not understand why. Several fundamental mathematical ideas arise in analyzing this. Let's start!

A *graph* is a collection of *vertices* (points), and *edges* (line segments) which connect the vertices. In our puzzle the vertices are the houses and utilities, and the edges are the connections.

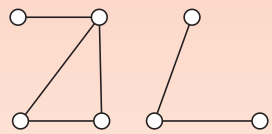


A *plane graph* is one which is drawn without having any edges cross. A graph is *connected* if you can go from any vertex to any other vertex by travelling along edges. Here are some examples to illustrate the concepts:

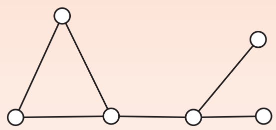
Connected but not plane.



Plane but not connected.



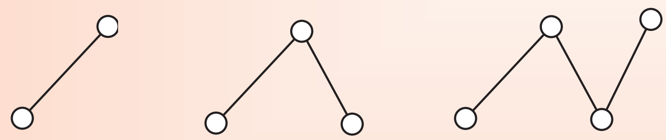
A connected plane graph.



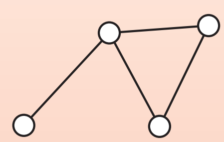
The great Swiss mathematician Euler (pronounced "Oiler") made a very interesting observation about connected plane graphs. First notice that when we draw a connected plane graph, the page is divided up into regions. These regions are called *faces*.

We start with one vertex.

Keep our drawing connected and add a piece at a time. If we draw a new vertex we must also draw a new edge to connect it.



If we draw a new edge without a new vertex, then the edge must connect vertices which are already in the picture.



This will divide one of our faces in two, and so we get an extra face.

Use V for the number of vertices, E for the number of edges and F for the number of faces. We start with 1 vertex, no edge and 1 face,

$V - E + F = 1 - 0 + 1$

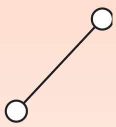
$V - E + F = 2$ to begin with.



*Eugene Curtin is a professor of Mathematics at Texas State University-San Marcos.*

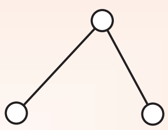
We will see that this sum V - E + F never changes as we draw! Notice, every time we draw an edge, this increases E by 1 and subtracts 1 from V - E + F, but when we do this we always increase either V by 1 (new vertex) or F by 1 (new face,) adding the 1 back.

Notice for each of the figures we have the corresponding relationship.

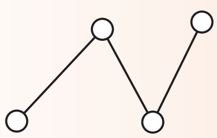


$$\begin{aligned} V - E + F &= 2 - 1 + 1 \\ &= 2 \end{aligned}$$

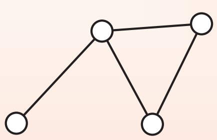
The formula V - E + F = 2 is called *Euler's formula*, but is not the only formula with this name! Let's look at more examples:



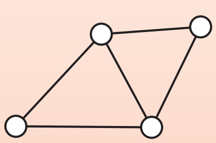
$$\begin{aligned} V - E + F &= 3 - 2 + 1 \\ &= 2 \end{aligned}$$



$$\begin{aligned} V - E + F &= 4 - 3 + 1 \\ &= 2 \end{aligned}$$



$$\begin{aligned} V - E + F &= 4 - 4 + 2 \\ &= 2 \end{aligned}$$

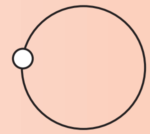


$$\begin{aligned} V - E + F &= 4 - 5 + 3 \\ &= 2 \end{aligned}$$

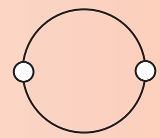
Suppose we could draw the graph for the utility puzzle without crossings. Then we would have a plane graph with V = 6 vertices (the 3 houses and 3 utilities) and E = 9 edges. Now V - E + F = 2 so 6 - 9 + F = 2 and we must have F = 5.

We say a face and edge meet if the edge is part of the boundary of a face. Lets count all the face-edge meetings too. Each edge meets 2 faces, and there are 9 edges, so we get a total of 18 edges.

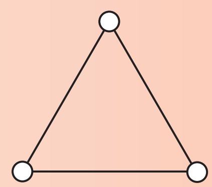
Now let's count in another way. As we walk around the boundary of a face we follow a path that goes vertex-edge-vertex-edge and so on. If a face meets only 1 edge there it must be connecting a vertex to itself.



If it meets only 2 edges then it meets 2 vertices also, and we have a double connection between some house and some utility.



If it meets only 3 edges it meets 3 vertices, and 2 must be houses or 2 must be utilities.



Then one of the edges connects 2 houses or connects 2 utilities. All these cases don't occur because every edge connects a house to a utility and each house has exactly one connection to each utility. So every face has at least 4 boundary edges, and with 5 faces we get at least 5 x 4 = 20 for the number of edge-face meetings. But we already saw this number is 18. This is impossible! 18 is not greater than or equal to 20, and our mistake must be assuming we could solve the puzzle!

We have used some important ideas. The quantity V - E + F is an example of what mathematicians call an *invariant,* as it does not change even as our picture does. We saw that it can be useful to solve a problem 2 different ways. The final argument, where we assumed we could do something and were led to an impossible conclusion, is called a *proof by contradiction*.

# Puzzle Page

**Math Explorers:** We want to print your work! Send your original math games, puzzles, problems, and activities to
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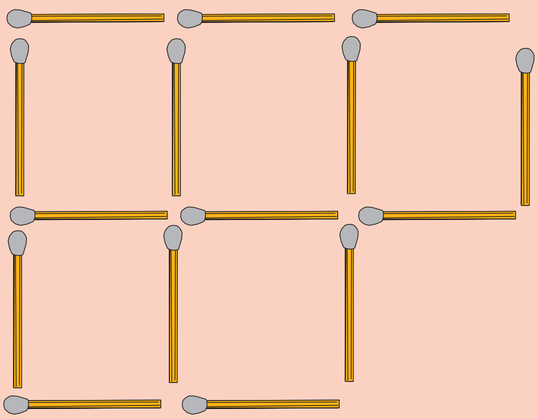
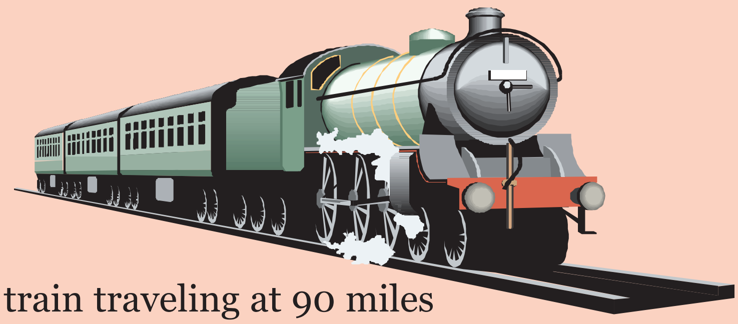
Arrange 15 matches on a table as shown in the illustration below. Now remove just 3 of these matches to leave only 3 squares.





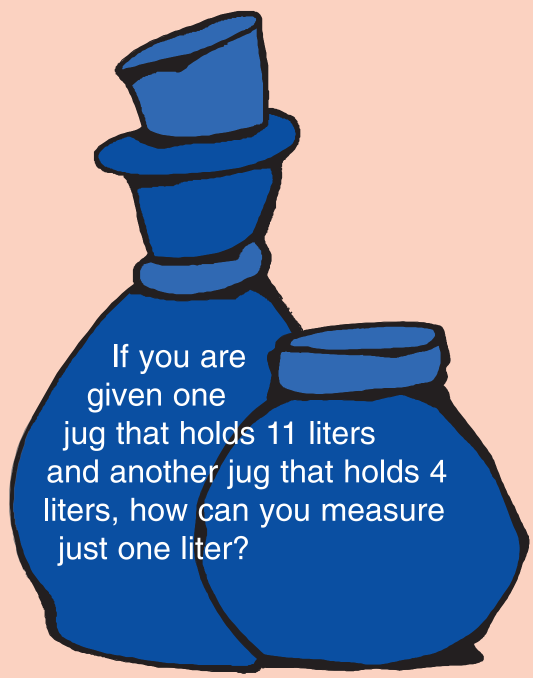
A train traveling at 90 miles per hour takes four seconds to completely pass a tree. In the next 40 seconds, the train completely passes another tree. How long is the train? Find the distance between the two trees?

## Word Search

Forwards or backwards, up, slanted, or down.
Where can the words in this puzzle be found?

**Bernoulli**
**Euler**
**Plane**
**Connected**
**Face**
**Proof**
**Edge**
**Graph**
**Vertex**

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| R | I | T | C | M | S | L | F | T | S | P | C | B | B |
| H | X | N | B | O | E | A | S | A | K | C | Z | P | J |
| P | Z | H | B | Y | C | G | N | D | P | L | A | N | E |
| A | P | M | K | E | S | X | N | C | Q | O | E | J | G |
| R | G | R | I | L | L | U | O | N | R | E | B | N | E |
| G | P | H | O | Q | S | N | U | N | U | D | V | Z | S |
| X | S | Y | C | O | N | I | Z | O | F | G | L | M | R |
| X | W | E | B | E | F | N | J | Y | G | E | P | E | H |
| A | E | P | C | Y | L | K | I | Q | C | N | L | Y | X |
| E | X | T | D | O | U | P | T | V | V | U | E | F | H |
| A | E | U | R | A | I | C | Q | P | E | H | D | Q | X |
| D | M | D | I | E | X | O | I | Z | M | A | I | Y | W |
| J | C | N | X | N | V | E | C | A | U | U | J | E | L |
| V | N | S | I | D | A | E | I | G | O | C | E | D | E |



# Bulletin Board

## April was Math Awareness Month

www.mathaware.org has information on this year's theme, "Mathematics and the Cosmos" including a poster that you can print and display to celebrate mathematics and its importance in our study, work and lives.

## 2005 Junior Summer Math Camp

The 10th Annual Summer Math Camp will be held in San Marcos, Texas June 6-17. The two-week camp is for students in grades 4 through 8. For an application and more information, visit: http://mathworks.txstate.edu/ or call the Texas Mathworks office at 512-245-3439.

## Thank You

Texas Mathworks received a generous donation from the Jeff and Gail Kodosky Foundation. Their gift helps provide scholarships for Junior Summer Math Camp students and develop Mathworks curriculum. Our deep appreciation to the Kodosky's for their generous support over the years.

## Check It Out!

Visit http://mathforum.org/yeargame/2005/ to find math games to challenge you including Problems of the Week.

*FleetKids* uses money situations in games to do math. Visit www.fleetkids.com

## Chuckle Bytes

What do you call one million phones?

*A Megaphone*

What do you call one million aches?

*Megahertz*

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## Considering bases

We often take our base 10 numeration system for granted. Let's look closely at some of the important characteristics of our base 10 (or decimal) system. We use only the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. We call these the *face values* for our system. Notice that we have numbers with more than one digit. Each digit then is in a particular *place value*. For example, we read 123 as "one hundred and twenty three" and mean that the numeral represents 1 hundred plus 2 tens plus 3 ones. If we used the same face values but in the order 312, then we read this as "three hundred and twelve" and mean 3 hundreds plus 1 ten plus 2 ones.

Let's consider a parallel world in which we use only the digits 0, 1, 2, 3, and 4. Whereas in the base 10 world we grouped by tens, let us now group by fives. In other words, the numeral 23 in base 5 means 2 fives and 3 ones. This is like having two nickels and three pennies (not 2 dimes and 3 pennies as in the base 10 system.) What would 32 mean in base 5?

What about 123? Notice in base 10 the place value occupied by the face value 1 is the 100's place or ten 10s. So in base 5, the natural position to the left of the 5 place value is five 5s or the 25's place. For 123 did you get 1 twenty-five plus 2 fives plus 3 ones? What would 312 represent in our base 5 system?

In this numeration system for base 5, what should be the place value to the left of 25? Determine what 3142 represents in base 5.

Rather than 5, what if we chose to use base 2, which uses only the face values 0 and 1. The right-most place value is one and to its left 2. What is the place value to the left of the 2's place? 4 would be correct since it is 2 twos. And to the left of the four's place is 8 since it is 2 fours.

What does 101 represent in base 2? Well let's see. 1 four plus 0 two plus 1 really represents our base 10 number 5. What does 110 represent in base 2? Did you determine it to be 6 in our base 10? When we write 10 and read it as "ten" in base 10 it is very different from seeing 10 in base 2. Can you determine what the numeral 10 in base 2 represents? You would be correct if you said 2. Can you see why?

Using digits from our familiar base 10 system, we can write numerals in other bases. Be careful to note the place value carefully. 1,000,000 is not as much as you think if this is written in base 2. In fact, what does this numeral represent in base 10?

Dear *Math Explorers*,

The study of mathematics is an opportunity to investigate both numbers and geometry and their connection. Our main article looks at Euler's Formula and its connection (no pun intended) to graph theory. The Math Odyssey explores numbers represented in numerals of bases other than our familiar base ten. We at Math Explorer hope these articles lead to expanding your understanding of mathematics. Enjoy doing the challenging problems from Australia and reading about the amazing Bernoulli family.

Sincerely,

*Hiroko K. Warshauer*

Hiroko K. Warshauer, Executive Editor