Math Explorer

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Sagrada Familia in
Barcelona, Spain
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Math Explorer

Contents

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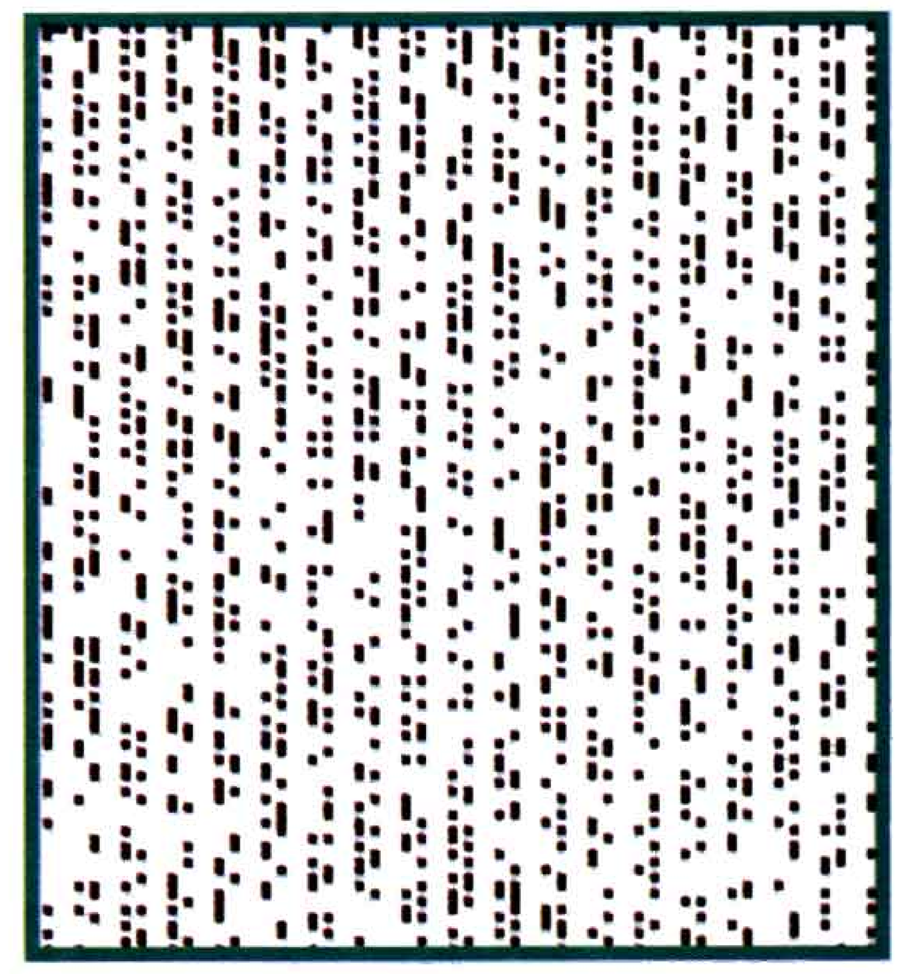
Eratosthenes

by Hiroko Warshauer



Pictured to the right is a 529 x 369 grid of numbers constructed using the sieve of Eratosthenes. To learn more about this "sieve," see the article **Prime Numbers** *on page 4.*

Eratosthenes of Cyrene, now Shahhat, Libya, was a learned man of many interests, including mathematics, poetry and geography. He lived in the third century B. C. in Alexandria, Egypt and was the contemporary of another famous mathematician, Archimedes, with whom he corresponded. Eratosthenes eventually became the head librarian at the Museum of Alexandria, considered the center of learning of the time.

One of Eratosthenes' most important contributions to mathematics was his method for computing the circumference of the Earth. By measuring the lengths of shadows at two different places on the same line of longitude, he was able to compare the different angles at which the sun's rays struck the Earth and calculate the Earth's circumference. It is said to be within 200 miles of the measurement made today by advanced technology. Not bad, given that the circumference is over 20,000 miles!

Another contribution of interest to us in this issue is his method for finding prime numbers. A **prime number** is a whole number (positive integer) that has exactly two positive factors, 1 and itself. For example, 3 is a prime number since its only positive factors are 1 and 3. Numbers with more than two positive divisors are called **composite numbers**. Notice that 4 has divisors 1, 2, and 4, so 4 is a composite number. Eratosthenes took the list of positive numbers in their natural order and sifted out the composite numbers one at a time from the list until only the prime numbers remained. This method of finding primes is called the **sieve of Eratosthenes**. Find out exactly how the sieve works in the pages to come.

[B] Bell, E. T., *Men of Mathematics*, Simon and Schuster, NY, NY, 1965.
[Ho] Hollingdale, Stuart, *Makers of Mathematics*, Penguin Books, NY, NY, 1989.
[L] Lasky, Kathryn, *The Librarian Who Measured the Earth*, Little, Brown & Co., NY, NY 1994.

PROBLEMS OF THE MONTH

Send your solutions to *Math Explorer*! We will publish the best solutions each month and send a free *Math Explorer* Pen to you and your math teacher if we print your solution.

1. What number between 1 and 100 has the most prime factors? (Count factors each time they show up in the number's prime factorization.)

2. What numbers between 1 and 1000 have the most distinct prime factors?

3. How many positive even numbers are there in which the sum of the digits is 8 and the product of the digits is 6?

4. If it is 2pm, what time is it 1000 hours later?

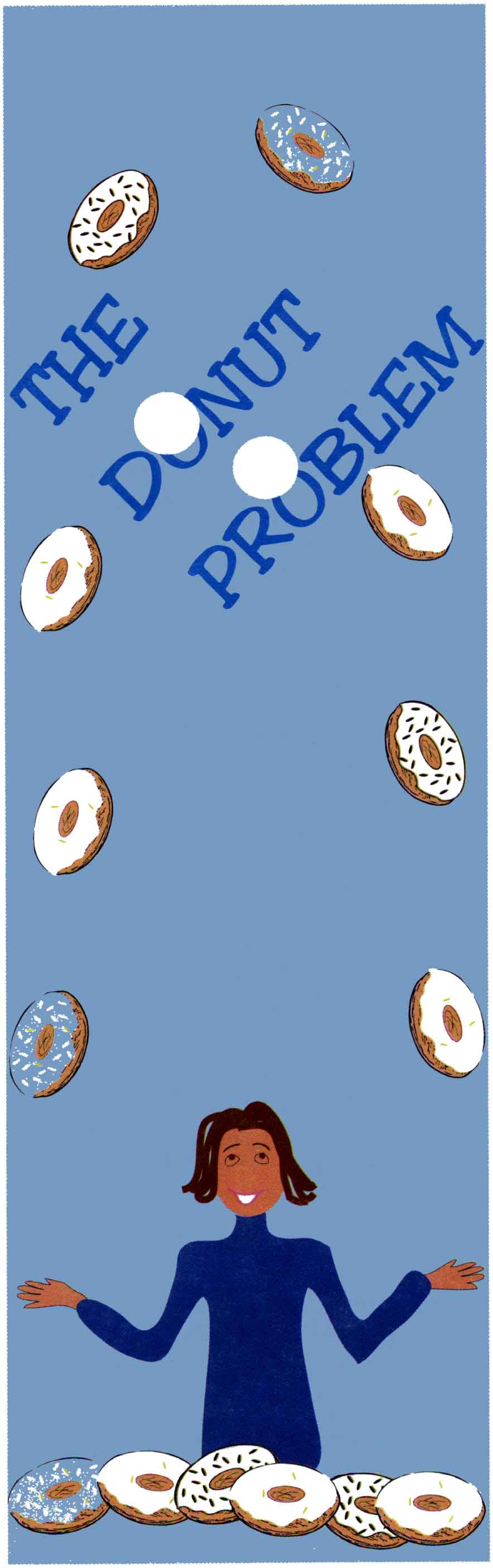
5. By increasing each side equally, you increased the perimeter of a square by 20%. By what percent did the area of this square increase?

6. A baseball league has 10 teams. How many matches will be played if each team plays each other team exactly once?

7. Numbers that do not change when read backwards are called palindromes. For example, 77 and 202 are palindromes. How many 4-digit palindromes are there?

8. Using as many pennies, nickels, dimes, quarters, and half-dollars as you want, how can you make exactly one dollar using 5 coins? 6 coins? 7 coins? 8 coins? 9 coins? 10 coins?

9. **Ingenuity** (The Donut Problem)
   How many ways are there to select eight donuts from a collection of glazed and chocolate donuts? How many ways to select eight donuts from glazed, chocolate, and cream-filled donuts? Explain your answer.



Prime Numbers

by Janet Chen

The Mathematical BrainTrain Society (MBTS) was created by the late Doyle Coats, highly-successful teacher and coach of the A & M Consolidated High School math teams. Knowing that he was dying of cancer, Doyle formed MBTS to help talented math students develop their interest in mathematics. This article is the first in a series written by student members of MBTS. Janet is an undergraduate at Stanford University.

A prime number is a whole number (positive integer) that has exactly two factors, 1 and itself. For instance, 5 is prime because the only numbers it is divisible by are 1 and 5. The number 6 is not prime because it is divisible by 1, 2, 3 and 6. Although 1 is divisible by 1 and itself, it is not considered a prime because it has only one positive factor. Primes are the multiplicative building blocks for whole numbers; any whole number can be formed by multiplying the primes in its prime factorization together.

How to find primes is a problem that has engaged mathematicians since ancient times. One way is to pick a number and use division to check that it has exactly two factors. For example, we could divide 87 by every number from 1 to 87 to find all of its divisors. However this is a slow process, especially if we pick a large number. Computer programs used to encrypt and decrypt messages sometimes need primes with more than 100 digits. Picking and checking random 100-digit numbers would be extremely slow, even by computer; these programs use some very ingenious methods to generate primes.

The **sieve of Eratosthenes**, named for an ancient Greek mathematician, is a famous method for finding primes. In this method, you pick a number and then find all the primes between 1 and the number. We demonstrate this method with the number 99.

To use the Sieve of Eratosthenes, we first list all the numbers from 2 to 99, see the diagram that follows. Cross out 0 and 1 because they are not prime.

~~0~~	~~1~~	2	3	4	5	~~6~~	7	~~8~~	9
~~10~~	11	~~12~~	13	~~14~~	15	~~16~~	17	~~18~~	19
~~20~~	21	~~22~~	23	~~24~~	25	~~26~~	27	~~28~~	29
~~30~~	31	~~32~~	33	~~34~~	35	~~36~~	37	~~38~~	39
~~40~~	41	~~42~~	43	~~44~~	45	~~46~~	47	~~48~~	49
~~50~~	51	~~52~~	53	~~54~~	55	~~56~~	57	~~58~~	59
~~60~~	61	~~62~~	63	~~64~~	65	~~66~~	67	~~68~~	69
~~70~~	71	~~72~~	73	~~74~~	75	~~76~~	77	~~78~~	79
~~80~~	81	~~82~~	83	~~84~~	85	~~86~~	87	~~88~~	89
~~90~~	91	~~92~~	93	~~94~~	95	~~96~~	97	~~98~~	99

Circle the number 2 because it is prime. Every number that is a multiple of 2 (and greater than 2) has at least three factors, 1, 2, and itself and so is not prime. Cross out all the multiples of 2 (not including 2). What is the smallest number that is not crossed out? Is it prime? Circle it and cross out all of its multiples. If you continue doing this, the smallest number that is not crossed out will always be prime. Can you explain why?

Continue this process until all the numbers are either circled or crossed out. The numbers in the list that are circled should be all the primes between 1 and 99. We can also examine the patterns made by the numbers we crossed out. In the 529 by 369 grid on the cover, the dots represent the numbers that have been crossed out. Do you notice any patterns in this grid? To see an interesting discussion of such patterns, visit Peter Alfred's web page at http://*www.math.utah.edu/~alfred/Eratosthenes.html*

Looking at the prime numbers, what other questions come to mind?

1. Are there infinitely many primes or is there a largest prime number?

2. Often primes come in pairs that differ by 2. These are called **twin primes**. For example, 3 and 5, or 17 and 19 are twin primes. How many twin primes pairs can you find? Are there infinitely many twin primes?

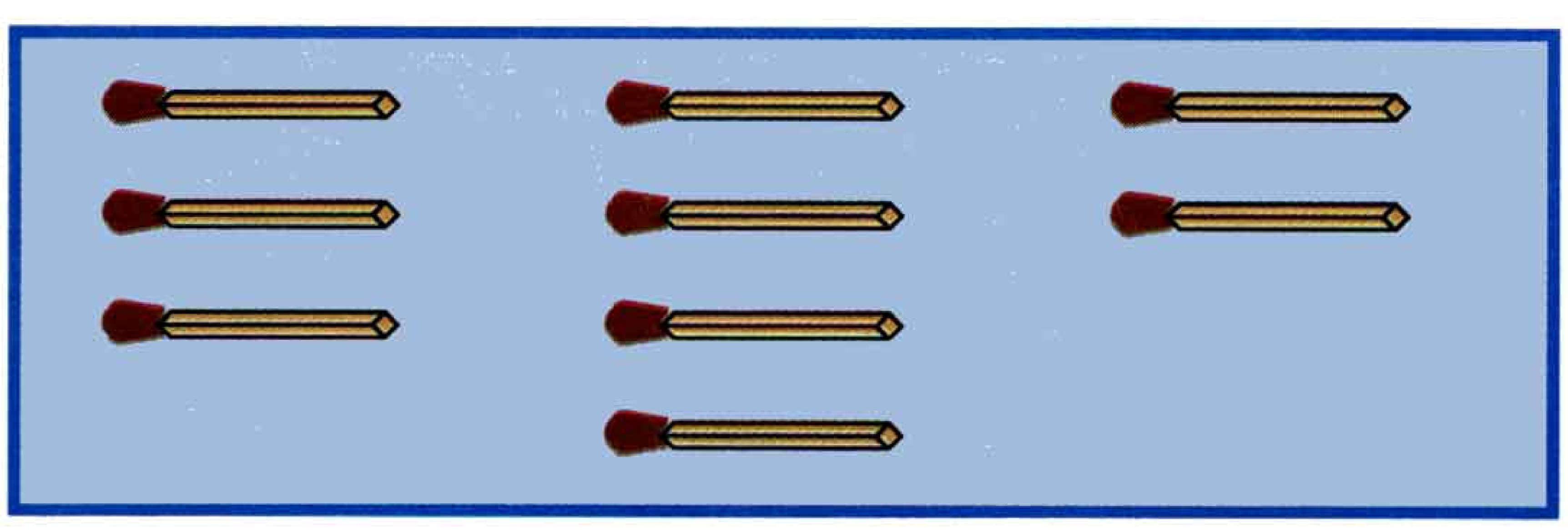
The Game of NIM

by Eugene Curtin

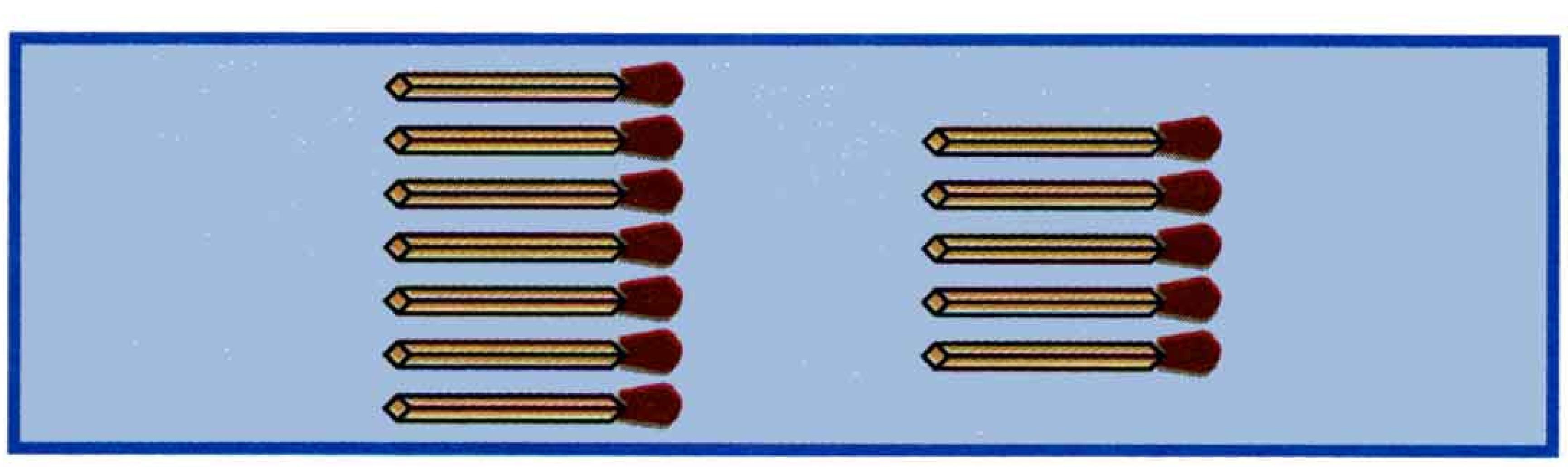
Nim is a game of strategy you can play using matchsticks or toothpicks, or even just by writing numbers on a piece of paper.

Number of Players: Two.
Equipment: Any number of matches, but you will need at least 10 or 12 to make an interesting game.
Game Set-Up: The matches are divided into any number of piles, with any number of matches in each pile. Players take turns making moves.
Moves: To make a move, a player chooses a pile and removes one or more matches from that pile, even removing the whole pile if she wants.
Winner: The player who removes the last match wins.

We will represent the situation in a game as follows: (3, 4, 2) means there are three piles, one with 3 matches, one with 4 matches and one with 2 matches. Pictorially, this looks like:
Lets look at some example situations. Suppose we

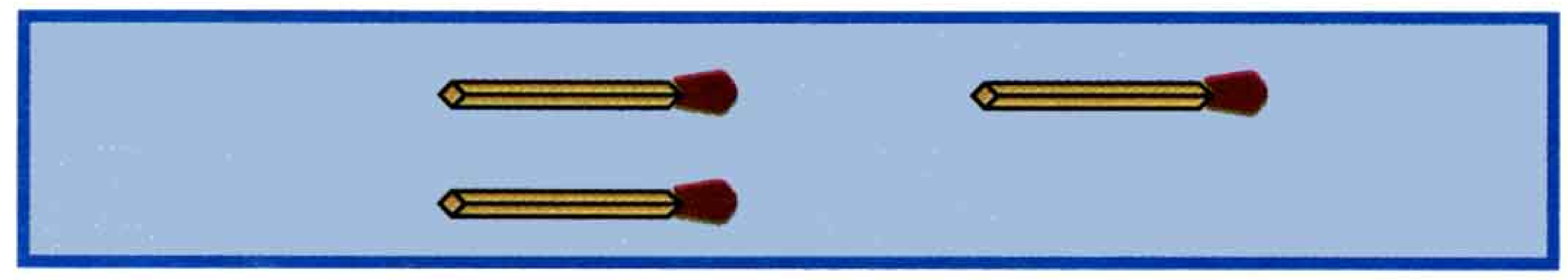


have the position (7,5). This means we have two piles, one with 7 matches and the other with 5 matches.



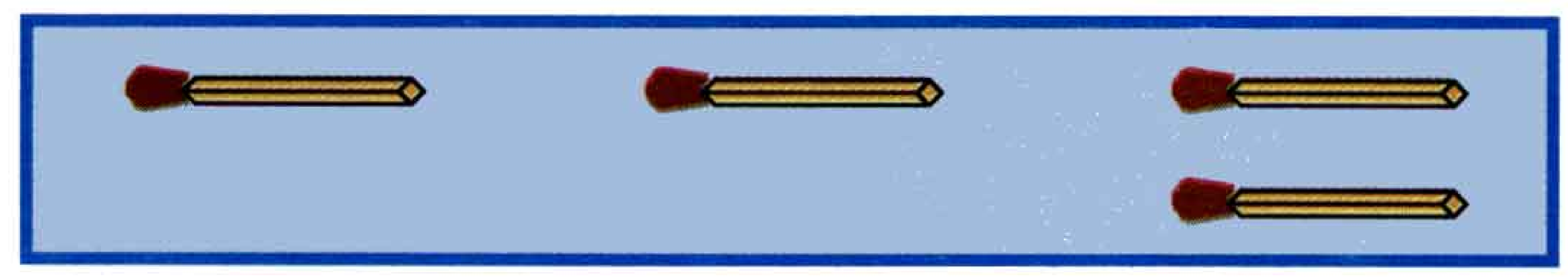
It would be a mistake to take all of one pile. Can you see why? So we have some bad moves. Let's see if we can find a good move. Here are some puzzles to get you started.

Puzzle 1
Suppose it is your move and the situation is (2,1)

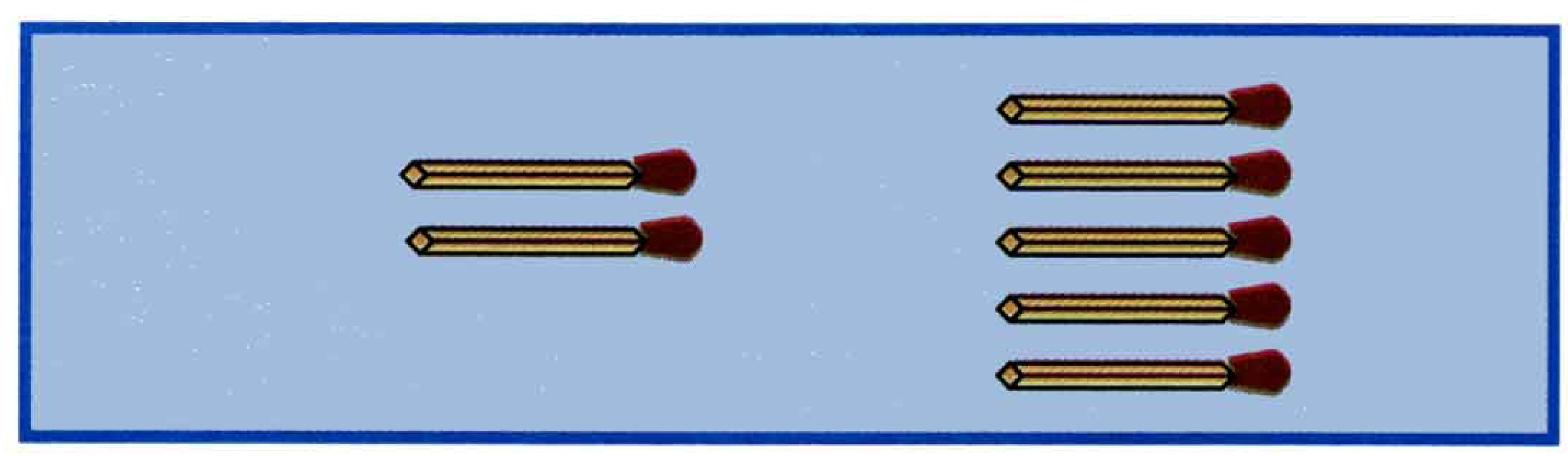


What is your best move? You could take the 2 matches from the first pile, one from the first pile, or one from the second pile. Did you find this one easy? Try to find the best move in these situations:

Puzzle 2 What move should you make for the (1,1,2) puzzle?



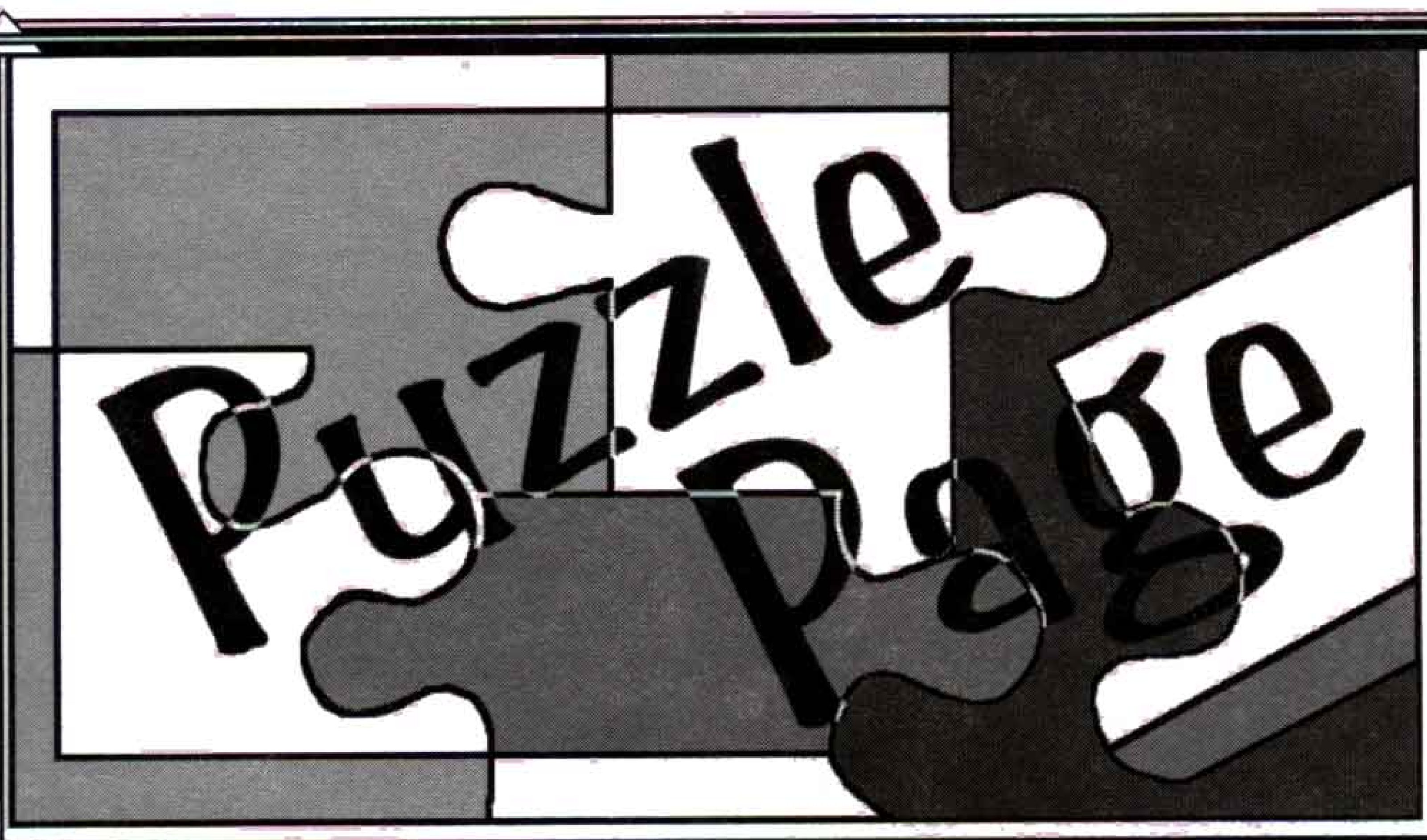
Puzzle 3 What move should you make for the (2,5) puzzle?



Have you figured out a strategy for when there are only two piles left? Can you figure out who wins if the situation is a (3,2,1) puzzle? How about a (5,2,1) puzzle? Or a (9,4,3) puzzle?

Here are some other questions which you can ask about any Nim position, but let's use the example (3,4,7,2).
Question 1: How many moves can a player choose from in this position?
Question 2: What is the largest number of moves this game could last?
Question 3: What is the smallest number of moves this game could last?
Don't assume that the players only make good moves!

Challenge: Find a Nim position where the player has more than one winning move available!



Math Explorers,

We want to print your work! Send us your own math games, puzzles, problems, and activities. If we print them, we'll send you and your math teacher free *Math Explorer* pens.

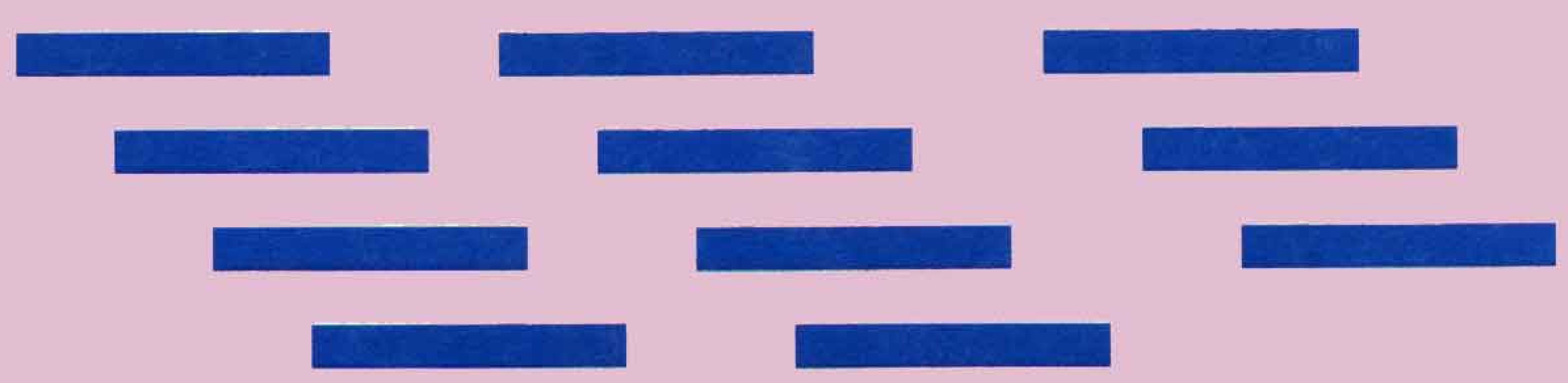
A MAGIC SQUARE

Make a magic square by placing the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 in the square below so that the numbers in each row, column and diagonal add up to 15. How many solutions can you find? Can you find a magic square on the cover? What is the sum of the numbers in each row, column, and diagonal?

	5	

FOUR SQUARES

Arrange 11 toothpicks to make four equal squares.
Hint: If you cannot do this on a flat surface, what else might you try?



FOUR 3'S

Use the operations

$+ \; - \; \times \; \div$

and parentheses to combine the four threes and make each equation below true. For example, we could use four threes to make 0 like this:

$(3 - 3) \times (3 + 3) = 0$

3 3 3 3 = 2

3 3 3 3 = 3

3 3 3 3 = 4

3 3 3 3 = 5

3 3 3 3 = 6

3 3 3 3 = 7

3 3 3 3 = 8

3 3 3 3 = 9

3 3 3 3 = 10

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Summer Math Camps in Texas

School districts in San Marcos, Port Lavaca, McAllen, Rio Grande City, Donna, Progreso, and Mission are planning summer math camps in 1999. Check for a

Rice Math Contest

is scheduled for Feb. 27. For details, see the website http://www.ruf.rice.edu/~eulers/RMT.shtml

Math Riddle:

What did 0 say to 8? (See Math Notes for the answer!)

Students in Progreso, Texas enjoyed the first annual Progreso Junior Summer Math Camp in the summer of 1998. Pictured on the left are Stephen Jackel and Esteban Salinas (teachers) and their students. Stephen and Esteban plan to expand the program in 1999.

Pictured on the right are Elaine Hernandez from South Texas Community College, Stephen Jackel, Esteban Salinas, Rachelle Meyers from San Marcos, and students at the 1998 graduation ceremony for the Progeso Junior Summer Math Camp.

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Math Notes is our Reader's Showcase. Write us with news from your school; about math events you've enjoyed; or with your own puzzles, activities, and problems. Please include:

- **Your name • Your teacher's name • Any related pictures.**

We'll publish as many letters as we can each month. I hope to hear from you soon.

Sincerely,

Max Warshauer

Max Warshauer

Answer to Riddle: Nice belt.