Math Explorer

MATH & DECISION MAKING

And the winner is...

RECOUNT!!!

It all Adds up!

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Math Explorer

Contents

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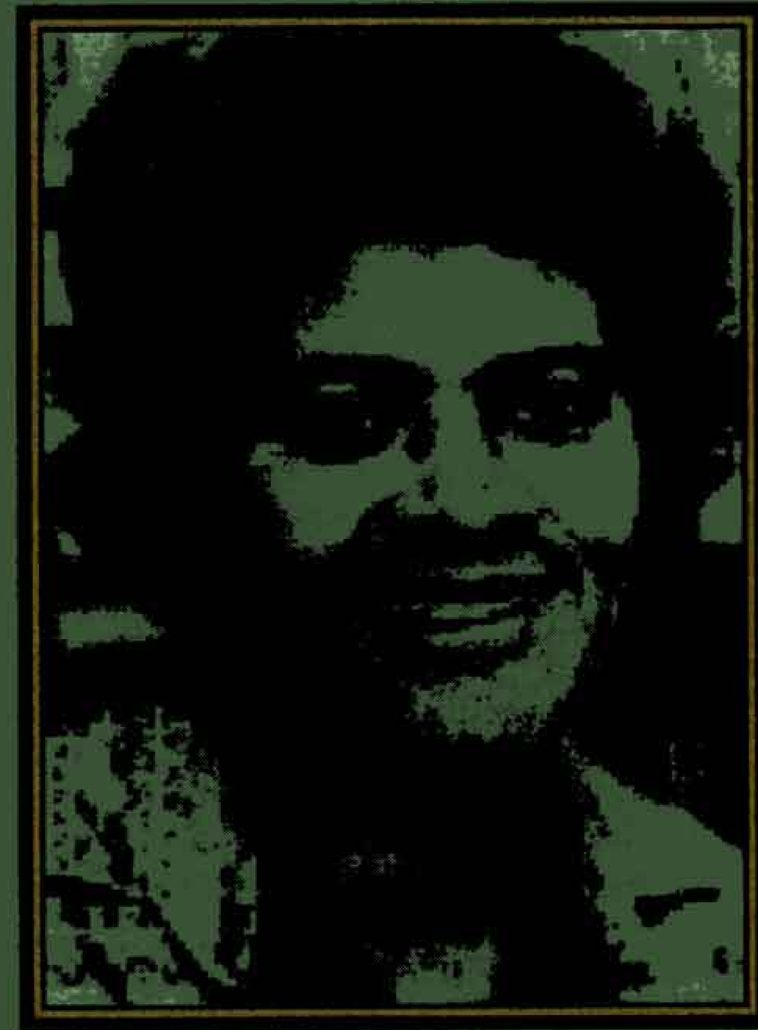
SWT

Evelyn Boyd Granville

by Jean Davis



Imagine yourself as a young African American girl growing up in Washington, DC in the 1930's. What would your life be like? Would you experience racial and gender discrimination? Would your opportunities in life be limited? Would your schools be inferior? Not if you were young Evelyn Boyd.

Evelyn Boyd was born on May 1, 1924, in Washington DC. She was raised by her mother and her aunt, who were both committed to her education. Her family and her teachers instilled in her the idea that education was the way to overcome racial prejudice. The schools Evelyn attended were segregated, but they were in no way inferior. They attracted well-trained teachers who demanded excellence from their students. She graduated high school as Valedictorian. Math was her favorite subject. After high school Evelyn attended the very prestigious Smith College in Massachusetts. She majored in math and physics. She fell in love with astronomy, and later was able to use her mathematics degree to get a job working in the space program. After graduating from Smith in 1945, she entered graduate school at Yale. In 1949, she became only the second woman in the United States to earn a PhD degree in mathematics .

Her first job was at New York University's Institute of Mathematics and Science. In New York she came face to face for the first time in her life with racial discrimination when she tried to find a place to live. A year later she took a teaching position at Fisk University in Nashville, Tennessee. Her education and achievements made her an inspiration to many of the women students. In the 1950's she worked at IBM and was involved with the Vanguard missile program. She worked on using computers to describe orbit trajectories. Evelyn liked the problem-solving aspect of computers. At last she was combining her mathematics with her beloved astronomy.

In 1960, she married for the first time and moved to Los Angeles, California. Her husband was a minister, and she frequently spoke at women's conferences at the church, inspiring and motivating her audiences. She is a wonderful example of what a person can achieve given opportunity and hard work. By the end of the 1960's she was divorced. She didn't want to move back east, so she took a job at Cal State University in Los Angeles teaching mathematics for teachers and computer science. She taught for many years and published a successful book in her field.

In 1970, Evelyn married Ed Granville and moved to Tyler, Texas where they bought a farm and built a house. In 1985 she began teaching mathematics and computer science at Texas College. Since that time she has enjoyed teaching, traveling, volunteering, and, in general, giving back some of what she had been given over the years. Now 78, she doesn't plan to retire any time soon. When asked to summarize her major accomplishments, she said, "First of all, showing that women can do mathematics." Then she added, "Being an African American woman, letting people know that we have brains too."

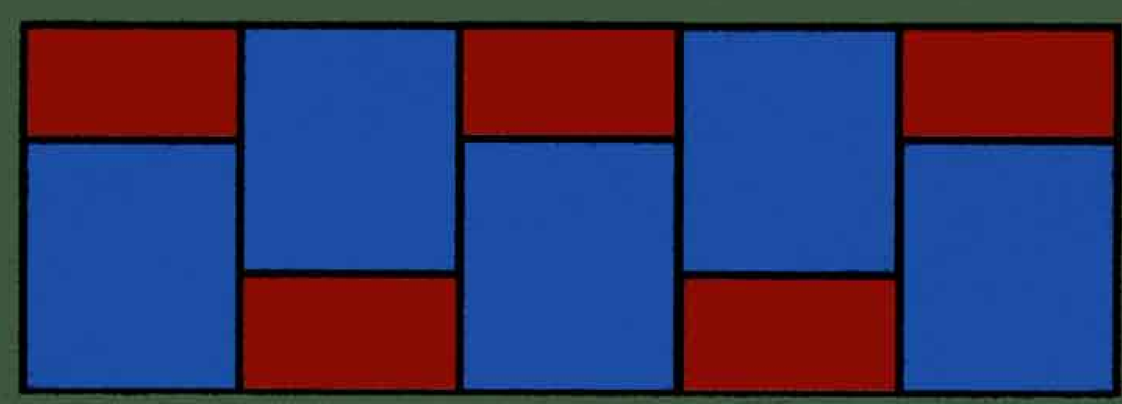
References: www.math.buffalo.edu/mad/PEEPS/granville_evelynb.
Women and Numbers, by Teri Perl

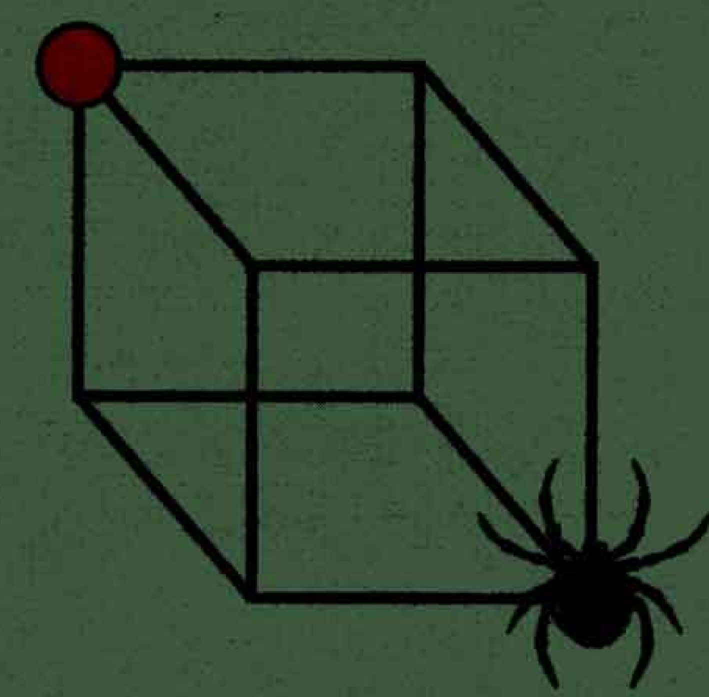
Jean Davis teaches mathematics at Southwest Texas State University

PROBLEMS PAGE

1. How many ways can the number 8 be written as the sum of 3 positive integers? Note that 2+2+4 and 2+4+2 are counted as different ways.

2. Patrick has 5 identical 2x1 red tiles and 5 identical 2x2 blue tiles, and he has to use these to tile a 3x10 region. How many ways can he tile the region? Which way is your favorite?





3. A spider is at one corner of a cube-shaped room, and wants to walk to the furthest corner, always walking along the edges. How many routes does she have to choose from, assuming she always walks towards her destination?

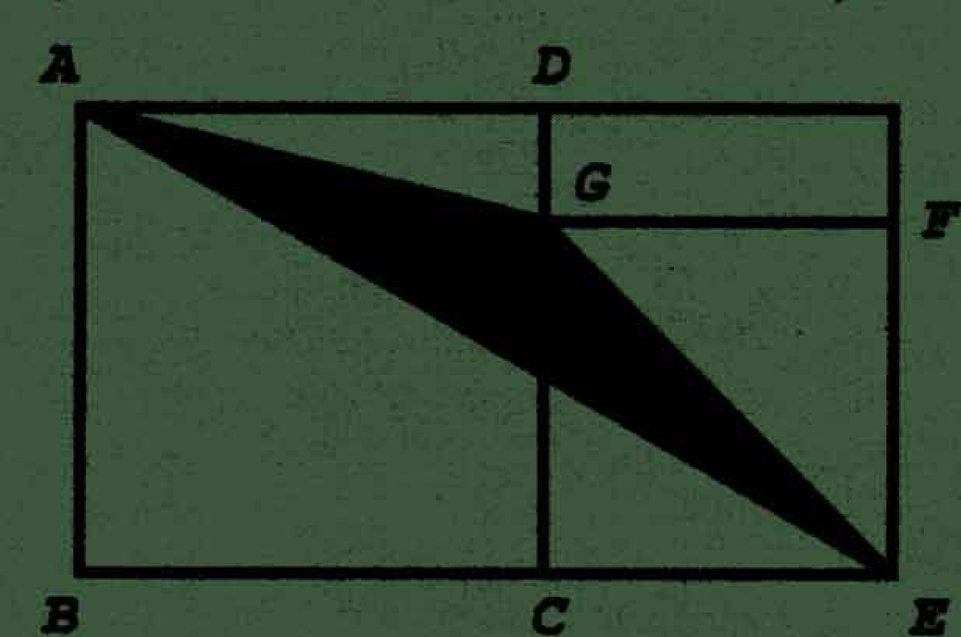
4. a) Find the sum (1+100)+(2+99)+(3+98)+ . . . +(50+51).
   (Hint: How many pairs of numbers are there? What does each pair add up to?)
   b) Find the sum 1+2+3+...+100. (Hint: This has something to do with part a)
   c) Now find 1+2+3+...+1000. (Maybe you have a bright idea to save some work.)

5. Susan and Jim go running around a track, and they start at the same time. Susan runs at 10 miles per hour and Jim runs at 8 miles per hour. When Susan finished 5 laps, how many laps had Jim finished?

6. How many of the integers between 1 and 200 are divisible by 3? by 7? by both 3 and 7? by either 3 or 7? by neither 3 nor 7?

7. Heather has an ordinary 6-sided die with sides numbered 1, 2, 3, 4, 5 and 6 as usual. She also has a special 6-sided die numbered so that when she rolls the ordinary die and the special die and adds the numbers showing, she could get any integer from 1 to 36. What numbers are on the sides of her special die?

8. What is the 2002nd decimal digit when 1/14 is expressed in decimal form? (From the 6th PMWC test)



9. In the figure to the right, ABCD and CEFG are both squares. If EF=12 cm, find the area of triangle AEG. (From the 6th PMWC test)

10. Sarah has 6 coins in her pocket. Each coin is a penny, nickel, dime or quarter. Sarah could have as little as 6 cents or as much as $1.50. How many different possibilities are there for the sum of money in Sarah's pocket ?

Answers: 1) 21 2) 32 3)6 4)a.5050 b. 5050c. 500500 5) 4 6) 66, 28, 9, 85, 115 7) 0,6,12,18,24,30 8) 4 9) 72 10)84

The Mathematics of Voting

by Kevin Jones

For me, one of the most interesting parts of mathematics is when it relates to areas that are completely unexpected. One of those areas is the idea of a group of people making a group decision -- a social choice -- what we refer to as "voting." Who has the best football team this year? Who is our favorite actor? Who do we elect to represent us in the student council or in Washington D.C? What kind of pizza do we want delivered?

You might think all this is pretty easy, but it isn't. People have been puzzling over this for a very long time. The Roman statesman, Pliny the Younger wrote 2000 years ago about a case in the Roman Senate where the decision in a murder case was in doubt because there were different outcomes depending on the way the votes were collected. More recently (but still 200 years ago), with the emergence of the American and the French republics, there was renewed interest in how to vote for representatives, and voting has been examined very closely ever since. So let's explore this topic a bit and see what can happen. Plus, you might be hungry right now, so let's order some PIZZA!

Suppose for the end-of-school celebration your math class has talked your teacher into ordering pizza. Suppose it all has to be the same kind of one-topping pizza, and your class must choose among hamburger, pepperoni, sausage, and anchovy. There are 21 students in the class (the teacher doesn't get to vote!) I'll pretend to be a student and I am hoping for my favorite: anchovy!

Dave suggests that each person vote on the kind that they like, and whichever gets the most votes wins. That sounds reasonable. A show of hands finds the following votes:

Hamburger	Sausage	Pepperoni	Anchovy
7	6	5	3

So it looks like the plurality (most votes -- whether a majority or not) goes to hamburger pizza. I notice that 7 votes for hamburger is only 1/3 of the class, so 2/3 of the class is NOT getting their favorite. Maybe 2/3 of the class hates hamburger pizza! (Of course I am secretly hoping that anchovy will win.) I propose the following: "I think each of us should rank the 4 kinds of pizzas and have a "runoff" between the top two vote getters. This is how a lot of local elections are done."

That sounds reasonable (especially since 2/3 of the class did not get their top choice in pizza), so the class (to the regret of the "hamburger group") decides this is a fairer way. Suppose the voting came out this way:

7 students voted: hamburger, anchovy, sausage, pepperoni
6 students voted: sausage, anchovy, pepperoni, hamburger
5 students voted: pepperoni, sausage, anchovy, hamburger
3 students voted: anchovy, pepperoni, hamburger, sausage
(I'm one of those three)

If we look at the top two vote getters for first place votes, they are hamburger with 7 first place votes, and then sausage with 6 first place votes. If we have a runoff between hamburger and sausage, we're asking , "how many people prefer hamburger to sausage," and, "how many people prefer sausage to hamburger?" There are 7+3 = 10 students who prefer hamburger to sausage, and 6+5 = 11 students who prefer sausage to hamburger, so by this method (called the "plurality with runoff" method), it looks like the pizza should be sausage. The sausage group is delighted, and the hamburger group is disappointed. So am I, because my favorite -- anchovy -- did not win here either.

Since I suggested this method, I am reluctant to offer another one, so Pat (a member of the pepperoni faction) makes the following suggestion, "Why take just the top two first place vote getters in the runoff? If you are going to have a runoff, then I think you should eliminate the weakest first place vote-getter and keep having runoffs until there is only one left. This is like that "Weakest Link" show on TV!" Actually this is called the "Hare Method" after an Englishman named Thomas Hare who proposed it in the 1860's.) Well, that seems reasonable too. Applied to the previous votes, it goes like this:

Round 1

7 students voted: hamburger, anchovy, sausage, pepperoni
6 students voted: sausage, anchovy, pepperoni, hamburger
5 students voted: pepperoni, sausage, anchovy, hamburger
3 students voted: anchovy, pepperoni, hamburger, sausage

Least number of first place votes: anchovy with 3. Anchovy is eliminated.

Round 2

7 students voted: hamburger, ________, sausage, pepperoni
6 students voted: sausage, _______, pepperoni, hamburger
5 students voted: pepperoni, sausage, _______, hamburger
3 students voted: _______, pepperoni, hamburger, sausage

Least number of first place votes: sausage with 6. Note that hamburger has 7 and pepperoni now has 5+3 = 8. Sausage is eliminated.

Round 3

7 students voted: hamburger, ________, _______, pepperoni
6 students voted: _______, _______, pepperoni, hamburger
5 students voted: pepperoni, _______, _______, hamburger
3 students voted: _______, pepperoni, hamburger, _______

Least number of first-place votes: hamburger, with 7. Pepperoni now has 6+5+3 = 14 first-place votes. Hamburger is eliminated, and pepperoni wins! The pepperoni faction is overjoyed.

My friend Earl is a football player and a fellow anchovy-lover. He stands up and says, "I think we ought to do this like they do in football polls. They give a certain number of points for a first place vote, fewer points for a second place vote, and so on. Then, they add up the points and whoever has the most points is the number one team in the country! I think we should vote like a football poll." Well, this logic is hard to refute (and Earl IS the star of the team), so the class goes along with it (this method is known as a "Borda count" after Jean-Charles de Borda, a French mathematician who worked in the late 1700's). They decide that a first place vote is worth 4 points, second place is 3 points, third place is 2 points, and fourth place is 1 point. Here are the calculations for the pizza types:

hamburger: 7 x 4pts + 0 x 3pts + 3 x 2pts + 11 x 1pt = 28 + 0 + 6 + 11 = 45 points
pepperoni: 5 x 4pts + 3 x 3pts + 6 x 2pts + 7 x 1pt = 20 + 9 + 12 + 7 = 48 points
sausage: 6 x 4pts + 5 x 3pts + 7 x 2pts + 3 x 1pt = 24 + 15 + 14 + 3 = 56 points
anchovy: 3 x 4pts + 13 x 3pts + 5 x 2pts + 0 x 1pt = 12 + 39 + 10 + 0 = 61 points

Finally, anchovy wins!! Yum! ...... continued on page 7

Puzzle Page

Math Explorers: We want to print your work! Send original math games, puzzles, problems, and activities to:
swtMathworks, 601 University Dr., San Marcos, TX 78666

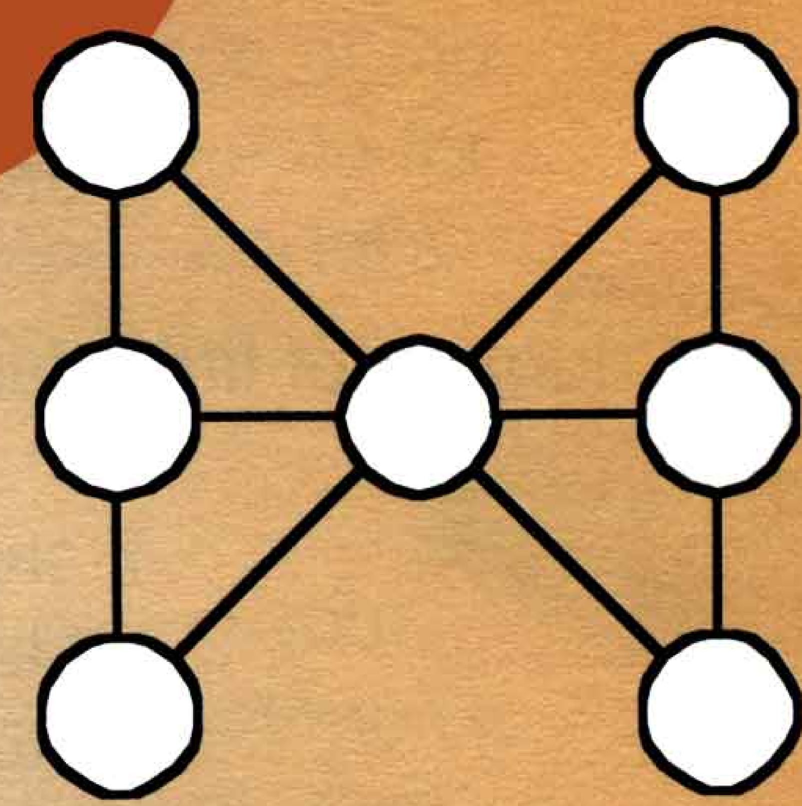
Word Scramble

Unscramble the letters below and discover words that relate to:

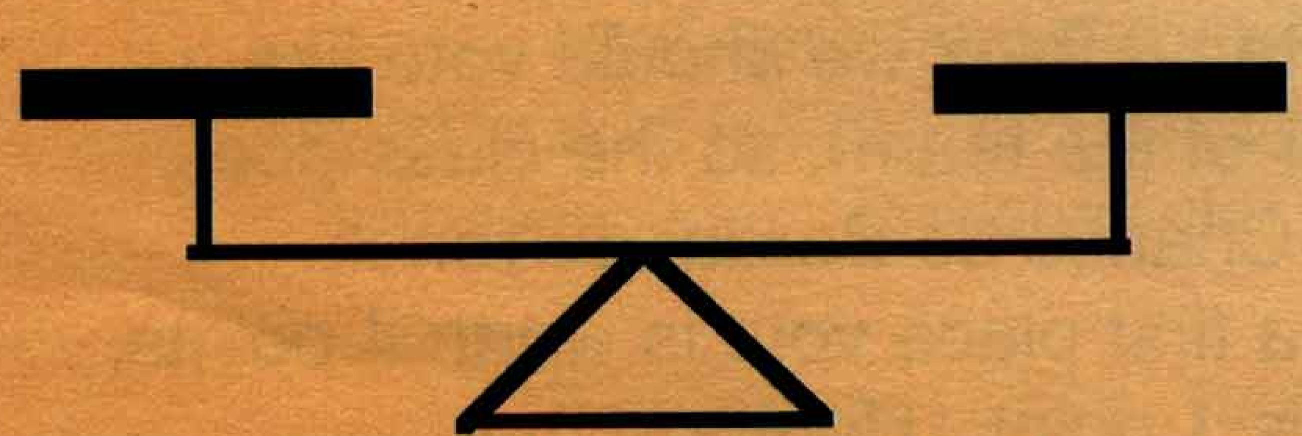
Voting

1. ROAJMYIT
2. EVTO
3. LAUIRPTLY
4. OMUSIMTAN
5. NENIRW
6. AONOOTINT
7. MEHSEC
8. OCUTMEO
9. NEOLIETC
10. RFUFON

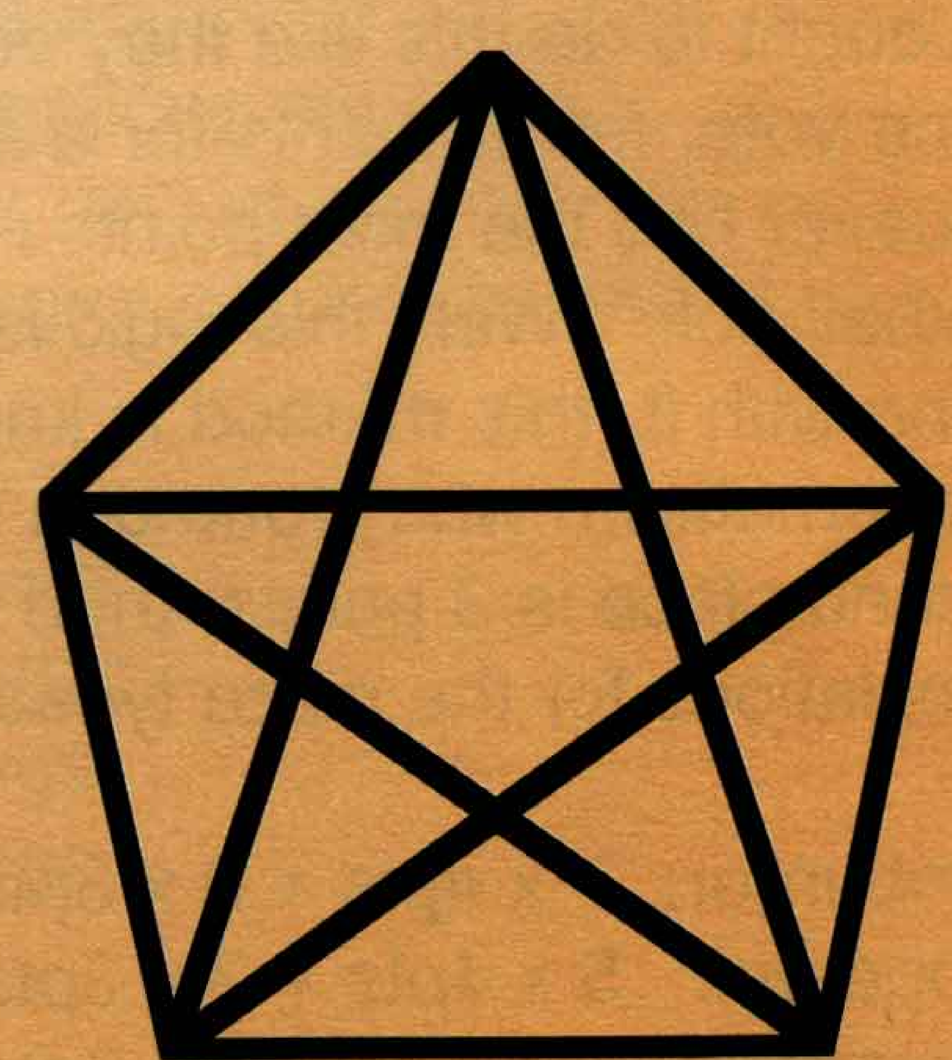
Using each number 1 to 7 only once, arrange them so that each row adds up to 12.



Of 9 coins of the same denomination, 8 weigh the same and 1, a counterfeit, is lighter than the others. Find the counterfeit coin in 2 weighings on a balance without using any weights.



How many triangles can you find in the diagram below?



Answers to Word Scramble:
1.majority 2. vote 3. plurality 4.summation 5. winner 6. notation 7. scheme 8. outcome 9. election 10. runoff

Bulletin Board

swtMathworks team goes to the Hong Kong Primary Mathematics World Contest!

The ***swtMathworks*** team received the championship cup for the top score among the non-Asian teams. They were ranked 7th overall. This year's team of Katy Gonzales of Corpus Christi, Anna Malagon of Austin, Edward Schmerling of Austin and Samson Zhou of College Station along with the adult team leaders Sam Baethge, coach for the Texas American Regional Math League team and Hiroko Warshauer, mathematics faculty member at Southwest Texas State University participated in Po Leung Kuk's Primary Mathematics World Contest held in Hong Kong on July 12-18. Forty teams from around the world, including Japan, China, Malaysia, Singapore, India, South Africa, Mexico, Bulgaria and others spent 5 days in Hong Kong sharing their common interest in mathematics and learning about their different cultures.

For information regarding screening tests and applications contact ***swtMathworks*** at (512)245-3439.

Voting (Cont'd)

Of course by now the class is in an uproar: according to four different but reasonable voting systems, (in fact, each of these four methods is used in some way even today), we get four different answers. There is no "right answer" unless you select a particular voting method. In hindsight, we should have decided on a voting method FIRST, and then we would have a single answer. But how are you going to decide which method to use? Vote on it? What voting method are you going to use to determine the voting method? This is sure getting complicated! Please pass the anchovy pizza.

For more information, look at these websites:

http://www.pbs.org/teachersource/mathline/concepts/voting/activity1.shtm
http://www.wikipedia.org/wiki/Voting_system
http://www.ctl.ua.edu/math103/Voting/4popular.htm
http://www.princeton.edu/~matalive/VirtualClassroom/v0.1/html/lab6/lab6.html
http://www.math.vanderbilt.edu/~bruff/voting/

Kevin Jones is an associate professor of mathematics at SWT. His major interests are math, statistics and science of all kinds. He especially enjoys seeing mathematics crop up in unexpected places...like voting!

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by Hiroko K. Warshauer

It all Adds Up!

Mathematics is often thought of as a language to communicate ideas very precisely and concisely. As a written language, mathematics uses symbols along with words. If we didn't know what the symbols meant, they would seem like code. In mathematics we find simpler ways to express ideas.

Let's look at one of the symbols that commonly occur in mathematics. The Greek letter Σ, read sigma, is referred to as the summation notation in mathematics. It means, "add" or "find the sum". For example, we can write 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 or using the summation notation, we can write,

$$\sum_{i=1}^{10} i$$

where i = 1 tells us where we start and the 10 above the summation sign tells us the last number we use for i. When we add, we use only whole numbers and no fractions between 1 and 10. You can see how handy this notation can be: instead of writing, find 1 + 2 + 3 +...+ 99 +100 we can write, find

$$\sum_{i=1}^{100} i$$

See if you can write the summation notation that means the same as 2 + 4 + 6 + 8 + 10 + 12 + 14 + 16

Did you get $\sum_{i=1}^{8} 2i$? Notice that notation is telling us to add the numbers 2 times each i, starting with i =1. The first number is 2x1 = 2. Next we use i =2 and we get 2x2 =4. the third number uses i =3 and we have 2x3=6 and so on. Now what is our last i? Since we need 2 x i =16, did you use i =8 as the last i? Correct.

What if the above addition continued past 16 to 20? What changes occur in your summation notation that mean the same as 2 + 4 + 6 +...+ 16 + 18 + 20?

Now try writing the summation notation for some odd numbers: 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15.

Here are a few others to try. 1) 3 + 6 + 9 +... + 36 + 39 2) 100 + 200 + 300 + ... + 2000

As an extension problem, can you find several ways in which to actually find the sums?
Good luck with your exploration.

Dear Math Explorers:

Welcome readers to a new season of Math Explorer! The fall season often brings elections in many areas of the country. The focus of our main article is voting. Our author uses mathematics and logic to analyze various voting methods and shows us how different the outcomes can be! You might find it interesting to see what voting schemes are used in your local elections.

Each issue contains challenging problems, puzzles , a biography and articles that relate mathematics to intriguing topics. During the year, please feel free to share with us any mathematical activities or problems you and your math class may be working on. We look forward to a fun year of exploring mathematics with you!

Sincerely,

Hiroko K. Warshauer

Hiroko K. Warshauer, editor