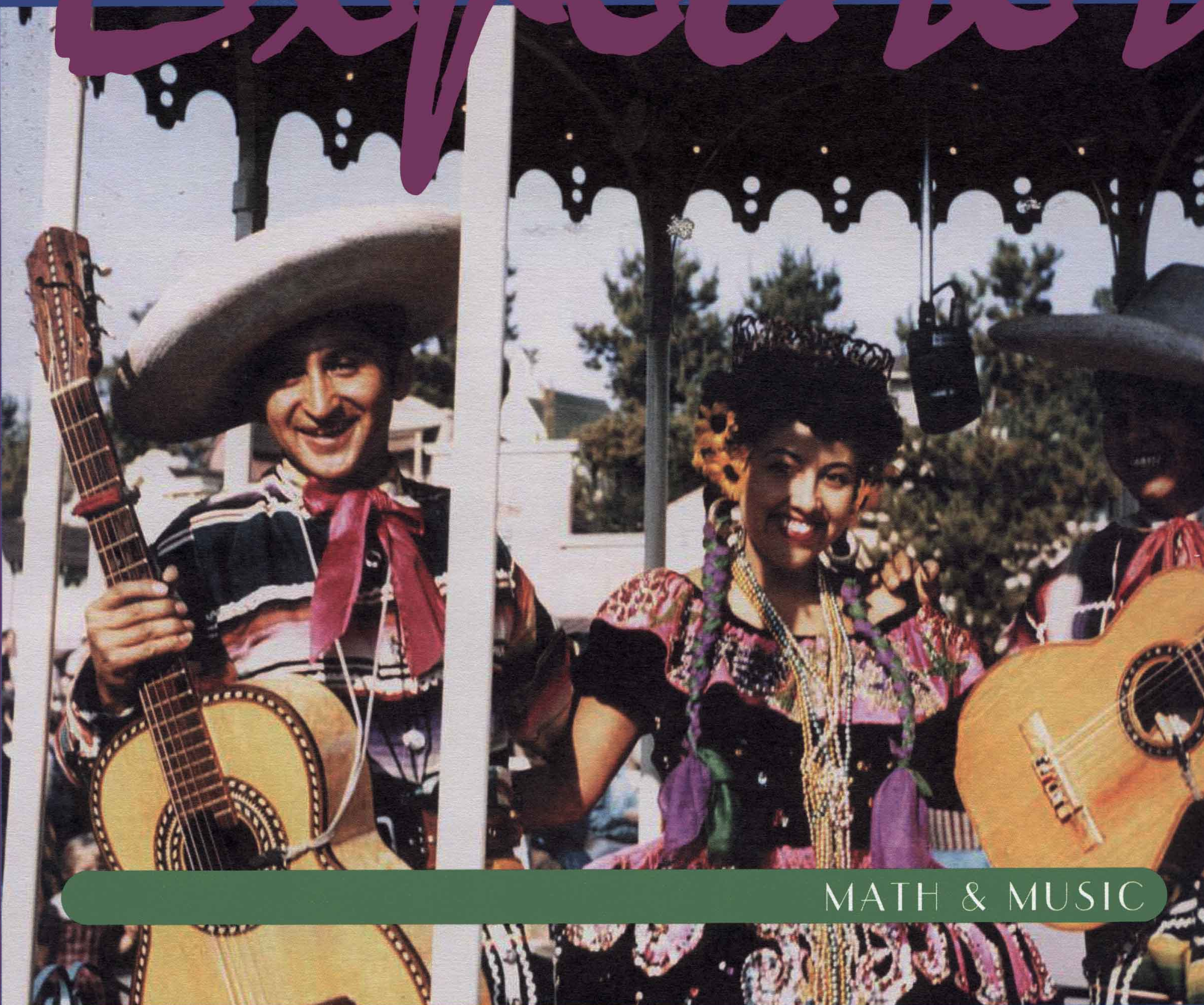
Math Explorer



MATH & MUSIC

Tune in!

Good Vibrations!!!

Which Bernoulli?

Vol. 2.8 Summer 2000

Math Explorer

Contents

Executive Editor: Hiroko K. Warshauer
Senior Editors: Terry McCabe, Max Warshauer, Eugene Curtin
Special Writers: Tivadar Divéki, Jean Davis, Laura Chavkin, Joel Becktell, Kevin Jones
Design: Jennifer LeGrévellec, Robert A. Gonzalez, Amy Warshauer
Final Editing and Proofreading: David Nelson, Michael Kellerman, Dan Shapiro
Administration: Lydia Carbuccia
Circulation: Kristi Carter
Webmaster: Peng Wu

Math Explorer
Southwest Texas State University
San Marcos, TX 78666
Phone: (512) 245-3439, Fax: (512) 245-1469
e-mail: mathexplorer@swt.edu.
Visit our website: www.mathexplorer.com
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Daniel Bernoulli

by Laura Chavkin and Hiroko Warshauer

Daniel Bernoulli was born in Groningen, Netherlands in 1700 to a family of Swiss mathematicians. All three of his brothers as well as his father, Johann and his uncle, Jacob were prominent mathematicians. Daniel himself studied probability (how likely it was that an event would occur, such as that a coin would land heads up). He also invented devices such as an hour glass (a clock that measures time by the amount of sand that trickles from one compartment to another) to be used at sea, where the sand flows smoothly despite any rough waves.

One of the major contributions by Daniel Bernoulli was in the field of hydrodynamics. His work, called "Hydrodynamica", was published while he was working in St. Petersburg, Russia. As the title suggests, his work analyzed patterns of water flow in different situations as well as the basic laws for the theory of gases. During these years, Daniel won prizes from the Paris Academy for his mathematical and scientific work and studied closely with another mathematician, Euler.

The most famous discovery made by Daniel Bernoulli was about the strings of musical instruments. He studied their vibrations and showed that the movements (or oscillations) of the strings of musical instruments are composed of a large number of harmonic vibrations all superimposed on the string. He defined the simple nodes and the frequencies of the oscillation of a system. Even after leaving St. Petersburg, Bernoulli continued to work with Euler on ideas about vibrating systems.

Daniel's father competed with him all his life and as Daniel won more prizes, his father grew more angry. One time, Daniel and his father tied for a mathematics award from the Paris academy. Daniel's father was insulted by the idea of his son as his equal and banished him from his house. Daniel's father even tried to claim credit for Daniel's book by publishing a similar book and dating it before that of Daniel's book. He wanted Daniel's work to

(continued on page 7)

by Laura Chavkin, who attends Yale University, and Hiroko Warshauer, who teaches mathematics at Southwest Texas State University.

PROBLEMS OF THE MONTH

1. How many numbers between 1 and 500 have the digit 2 or 7 in them? How many have the digits 2 and 7?

2. Which four numbers should you pick from the following table to get the largest possible sum? You are not allowed to pick two numbers from the same row or the same column.

4	2	9	7
7	8	5	6
3	3	4	1
7	5	2	6

Send us your solutions! Every month, we will publish the best solutions on our website: www.mathexplorer.com. If we print your solutions, we will send you and your teacher free *Math Explorer* pens!

3. Which number is greater: 7/15 or 6/13 ? Can you draw picture to explain your answer?

4. Find the total: (1– 1/3) + (1/2 – 1/4) + (1/3 – 1/5) + ... + (1/98 – 1/100) = ?

5. What is the 100th digit to the right of the decimal point in the decimal form of the fraction 1/13?

Do problems 6-8 after reading the main article on pages 4 and 5.

6. Suppose the note G is tuned to the frequency 390 Hz.
   a. Find the frequency of the G an octave higher than G (we'll call it G′).
   b. Find the frequency of the note a fifth above G. If we go up the scale that note is D.
   c. What is the frequency of the note D′?
   d. What is the frequency of the note A′?

7. If we tune A to a frequency of 440 Hz, what is the name of the note that has frequency
   a. 880 Hz?
   b. 660 Hz?

8. Use the following table to answer these questions.
   a. What is the frequency ratio of D to G?
   b. Find the frequency ratio of A to D.

D	E	F	G	A	B	C	D′	E′	F′	G′

Mathematics and M

by Joel Becktell and Hiroko Warshauer

"Music is the hidden arithmetical exercise of a soul unconscious that it is calculating"

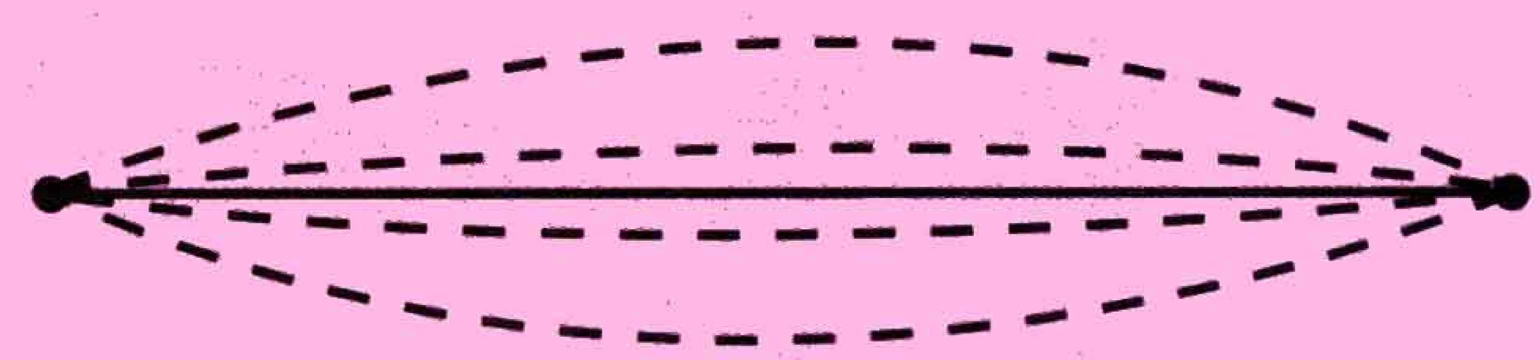
Gottfried Leibnitz

When you listen to music, what sort of differences do you hear between the various sounds? Some sounds are louder or softer than others, some are longer or shorter, some have different sound qualities, and some are higher or lower.

In music, we refer to the loudness or softness as the *dynamics* of sound. When the notes are played, some longer or shorter than others, a *rhythm* is created. The tone quality or *timbre* of the sound distinguishes the sounds of different instruments such as the saxophone from the violin. And when we talk about how high or low the sound is, we are referring to its musical *pitch.* When composers write music, they take these four elements into consideration. These four elements of music all have characteristics that can be described using mathematics. Let's look at the pitch when we pluck a string.
Sound originates from a string when it is plucked, sending vibrations through the air until it reaches our ears and we hear it.

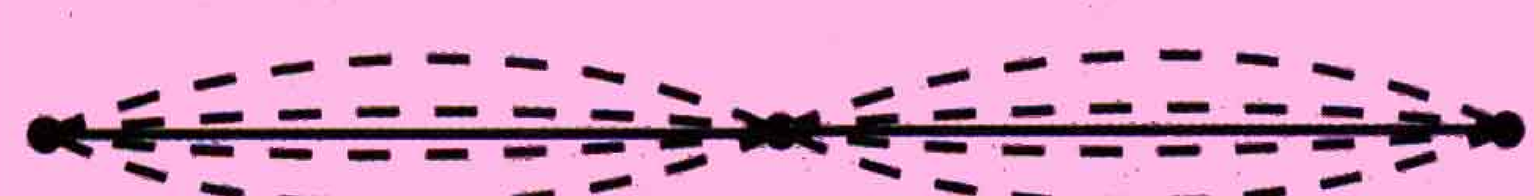
There are several factors that affect the pitch of the plucked string. The length, the tension and the total mass of the string all affect the pitch. For this article, let's look at how the length of a certain string affects the pitch.

You'll notice that when you pluck a string that is attached at two ends, it vibrates, that is it moves back and forth, as in the figure below.



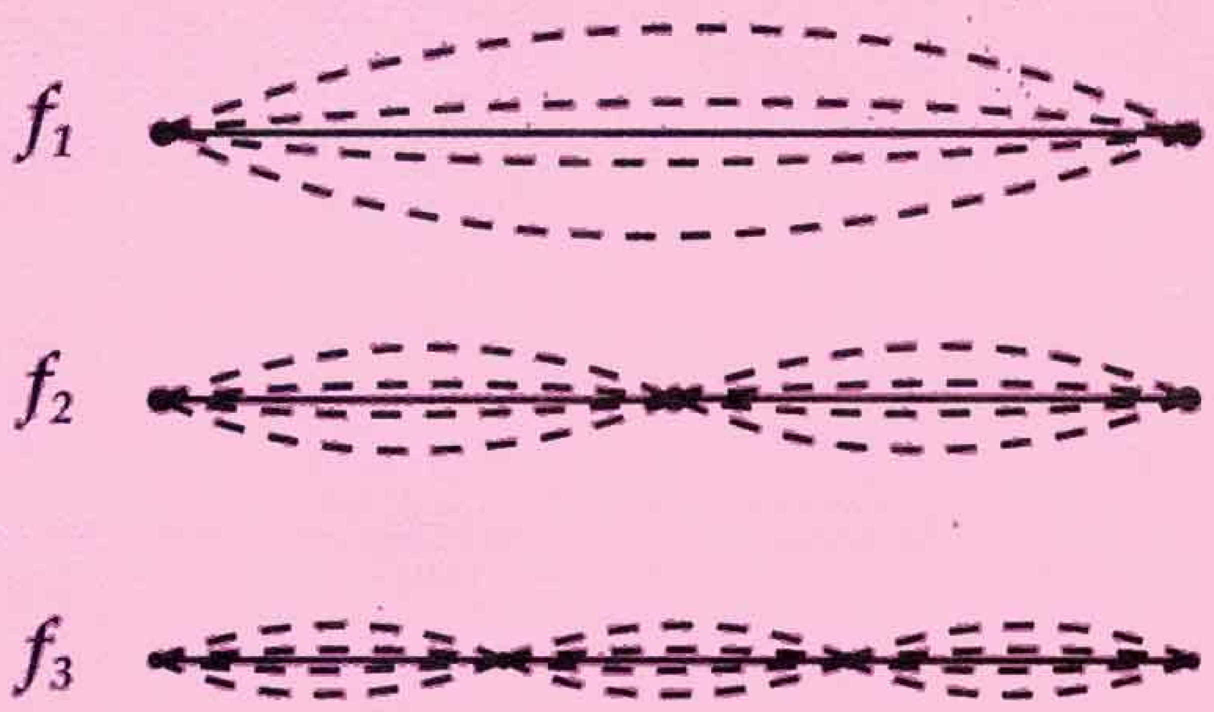
The vibrating string has two nodes, which are the attachments at the ends. In science, sound is measured in frequencies, which measure the number of impulses within a given time. A common unit of frequency used for audible sounds is hertz (Hz). It is the mathematical way of describing pitches by counting the number of vibrations of the plucked string in cycles per second.

The lowest frequency at which the string vibrates is called the *fundamental pitch*, or the first harmonic. Simply plucking an open string on a violin, for instance, would produce the fundamental. If we put a finger down on a violin string at the half-way mark, we create a third node. The sound that we hear is now higher. In fact, the note is an octave higher than the fundamental and is sometimes referred to as the second harmonic. An *octave* is called that because a standard musical scale has eight (oct is the Latin root of 8) notes and the first and the last notes have the same name and similar sound.



Music in Harmony

The frequency of the second harmonic is twice the frequency of the fundamental, but notice the length of the string for the octave is half the length of the fundamental. In frequencies, if the fundamental is 100 Hz, the second harmonic is 200 Hz, the third harmonic 300 Hz and so on.



Now that we see how scientists show pitch using frequencies, let's examine the language used by musicians as they write the pitch of a particular note. The musical alphabet consists of the letters A though G. The seven letters are used over and over again to correspond to different pitches. One set of pitches is called a *scale*.

Pythagoras, the Greek mathematician and philosopher who lived in the 500's BC, considered music a branch of mathematics. He was one of the first to examine how to tune notes precisely and mathematically. He observed that when he plucked two strings, one twice as long as the other, it produced a pleasing sound. It was the octave he was hearing! Using ratios and fractions, Pythagoras not only measured the intervals of the octave to its fundamental as 2:1, he also looked at another interval called the *fifth*, which is in the ratio of 3:2. The fifth looks at the interval between a note and the fifth note above it in the scale.

For example, on the C major scale, what is the fifth note?

1	2	3	4	5	6	7	8
C	D	E	F	G	A	B	C

If you said G, you would be correct. The interval between C and G is called a major fifth and the frequency of G : frequency of C is 3:2. So if C has frequency N, then G has frequency 3/2 N. G′ an octave higher has frequency 3N.

Now what is a fifth above G? D′ is correct. Let's compute 3/2(3/2)N which equals 9/4N. If D′ is 9/4N, then D which is an octave lower is half of 9/4N or 1/2(9/4)N = 9/8N.

C	D	E	F	G	A	B	C′	D′	E′	F′	G′
N	$\frac{9}{8}$N			$\frac{3}{2}$N			2N	$\frac{9}{4}$N			3N

Can you fill in the rest of the frequency chart using more fifths? You'll notice C′ is not an exact fifth above F. This is a famous historical problem in making musical scales. Over 2000 years after Pythagoras, J.S. Bach came up with the tempered scale that avoids the problem Pythagoras had. How do you think the frequency chart would look in another key such as in the G major scale?

Next time you are at an orchestra concert, listen carefully for the sound of 440 Hz. That's the frequency of the musical note *A* used to tune the instruments in the orchestra.

Source: Kennedy, Paul, ***Modeling Techniques in Mathematics***

Joel Becktell is the principal cellist with the Austin Symphony Orchestra.

Puzzle Page

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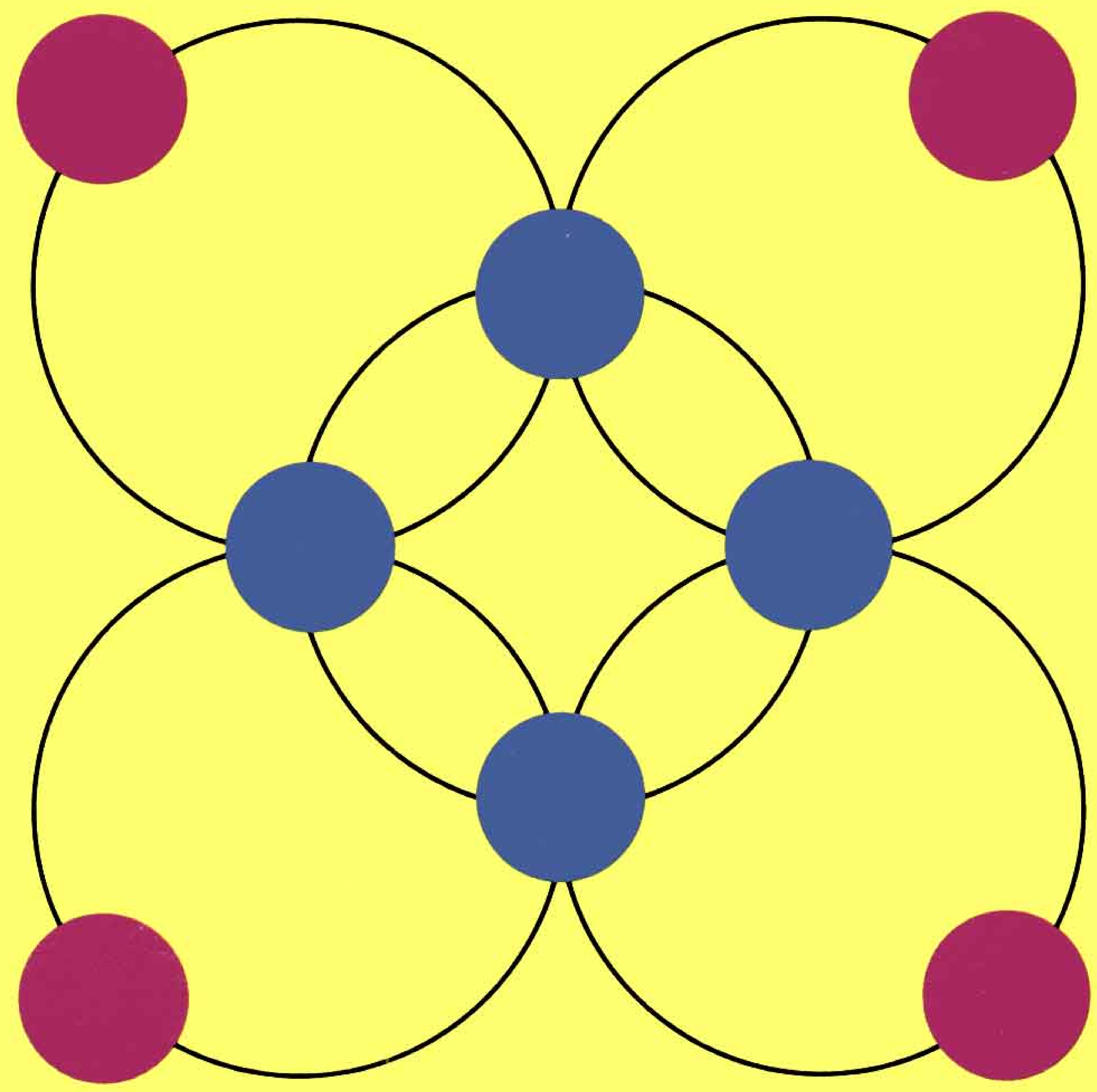
The King and the Thief

Far, far away, where the sun rises, lived an old king. He had a smart thief in his jail for such a long time that the king wasn't even angry at him anymore. One day he ordered to see this thief and told him:

"Listen to me, today is my birthday, so I give you a chance to win your freedom. Here are two identical pitchers, and four identical-size balls: 2 of them are white and the other two are black. Put all the balls in the pitchers so that none of the pitchers is empty. Then we call the guard in who will pick one ball from one of the pitchers. If this ball is white I will set you free, but if it is black then you will stay here in jail."

The thief thanked the king his kindness, and started thinking how he should place the balls in the pitchers.

How would you place the balls in the pitchers so that your chance to win your freedom is the greatest?



Place the numbers 1, 2, ..., 7, 8 in the little circles so that the sums of the numbers on each big circle are 12.

Point Taken

Draw 6 points and connect some of them with segments of a straight line in such a way that out of every point 4 segments should start, and the connecting segments should not intersect each other.

I Feel a Song Coming On!

by Cindy Boyd Tune: Sixteen Tons

Some people say a square is made outta sides.
The four are congruent with nothin' to hide.
Nothin' to hide: congruent, all.
A simple parallelogram, a polygon call.

You have four congruent sides, what do you get?
A really BAD rhombus and a rectangle yet.
Say, Olive, don't call me, 'cause I can't go,
I owe study time to the square you know.

A square has right angles, four of 'em there.
Opposite angles, two congruent pair.
These angles are halved by di-ag-noles.
So Olive said, "Well, bless my soul!"

If you see the diagonals, what will you gain?
They bisect each other without any pain.
Congruent diagonals are one more plus.
This special parallelogram I'd like to discuss.

You have four congruent sides, what do you get?
A really BAD rhombus and a rectangle yet.
Say, Olive, don't call me, 'cause I can't go,
I owe my study time to the square, you know.

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Cindy J. Boyd teaches mathematics at Abilene High School and presents workshops nationally. Contact Ms. Boyd at cjboyd@tenet.edu for more information.

Daniel Bernoulli (continued)

appear to be based on his own. Despite this family rivalry, Daniel was a mathematician who discovered much about the world around him. He was interested in the sea, the stars and musical instruments and made fascinating discoveries.

Source:
http://www-history.mcs.standrews.ac.uk/history/Mathematicians/Bernoulli_Daniel.html

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Math Explorer

Welcome to *Math Explorer*!

The fascinating relationship of mathematics and music is the theme of this last issue of *Math Explorer* for 1999-2000. From counting the beats to looking at intervals between notes as ratios, mathematics is an important part of music.

We at *Math Explorer* have enjoyed exploring, learning, and growing with you over this past year. Next year we hope to continue bringing you exciting mathematics and relating them to the world around us.

Have a wonderful summer and we hope you'll join our *Math Explorer* family again next fall.

Sincerely,

Hiroko K. Warshauer

Hiroko K. Warshauer

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Next year's magazines will be quarterly -- an issue each season -- bringing you biographies, puzzles, problems and articles that connect mathematics to the world around us. Be sure to subscribe now so you don't miss the fall issue!