Strategies to Support Productive Struggle

Hiroko K. Warshauer

As students struggle over a mathematical task, teachers can often hear some students utter, "I don't get it" or "I don't know what to do." Some may even proclaim, "I know the answer, I just don't know how to explain it." As teachers, we want to help our students address their struggles and recognize that there are choices in how we respond. The kind of struggle referred to here is described by Hiebert and Grouws (2007) in this way: "Students expend effort in order to make sense of mathematics, to figure something out that is not immediately apparent" (p. 387).

The Rain Barrel problem in **figure 1** was a task designed and implemented in a research study that investigated students' struggle in middle school mathematics classrooms (Warshauer 2014). In one classroom episode, Sandy, a student in Ms. Hill's class, had difficulty getting started with the problem. The following interaction occurred:

Sandy: I don't know what to do.
Teacher: OK. How can we compare the amounts in the barrel to the jug?
Sandy: I'm not sure.
Teacher: Well, you have part and the whole for each, don't you?
Sandy: Yeah.
Teacher: What kind of numbers can we make with part and a whole?
Sandy: Fractions?
Teacher: Bingo! So, what do your fractions look like?
Sandy: Um . . . 24/48 and 3/5 . . . but. . . .
Teacher: Good. Now what do we do with fractions when they have different denominators?
Sandy: Get common denominators? Oh, OK. I think I can do that.

This series of teacher questioning alters a challenging problem, requiring students to think and reason deeply into a procedural problem structured by the teacher. Although this interaction moves this student toward the answer, the student's work is reduced to numerical manipulations.

An episode, however, in which students encounter difficulty while working on a challenging task can be viewed as an opportunity for them to

Edited by **Despina Stylianou**, dstylianou@ccny.cuny.edu. Readers are encouraged to submit manuscripts through **http://mtms.msubmit.net**.

Fig. 1 This problem was meant to elicit some mathematical struggle.

The Rain Barrel Problem
Suppose we have a 48 gallon rain barrel containing 24 gallons of water and a 5 gallon water jug containing 3 gallons of water. Which container is said to be fuller? If we drain a gallon of water from each container, does this change your answer about which container is fuller? Explain.

Source: McCabe, Warshauer, and Warshauer (2009)

grapple with important mathematical ideas. Teachers can use these instances to acknowledge struggle as a natural part of learning while providing appropriate guidance and support to maintain the mathematical goals and cognitive demand of the task (Smith 2000). Research suggests that a range of teacher support and responses are possible in episodes of student struggles to advance students productively toward a resolution of understanding (Warshauer 2014). Hiebert and Stigler add that such support and guidance "provide students with opportunities to think more deeply about mathematical concepts" (2004, p. 13).

A STRUGGLE INTERACTION: ONE EXAMPLE

Ms. George, a seventh-grade teacher, was one participant in the research study who implemented the Rain Barrel problem. She asked her students to work on the problem individually for about five minutes. When she noticed that most of the students had worked on it to completion, she asked them to share their work with their small group of three to four students.

She walked around the room, listening or asking questions. She came to Drew, one of the students in a small group. The following episode illustrates four strategies used by the teacher that we identified as productively supporting a student's struggle as a natural course of sense making and mathematical effort.

Strategy 1: Teachers ask questions that help students focus on their thinking and identify the source of their struggle. They encourage students to build on their thinking or look at other ways to approach the problem without solving the problem for them.

Teacher: OK. What do you think? [Going over to Drew's table of four students]
Drew: I put "no" because it would be the same as before because you have taken a gallon from both.
Teacher: OK. So show me what that would look like. Show me what your gallons would look like. If you take a gallon from each, what are you looking at?

The teacher began by asking Drew questions and giving him an opportunity to explain his solution.

Drew: [Writing on his paper]
Teacher: OK. So what you're telling me [pointing to Drew's work], you have 23 gallons out of 48 and 2 gallons out of 5, that you're still going to have that the 2 gallons out of 5 will be . . .
Drew: Lower, less full.
Teacher: But didn't you tell me 3/5 was more full?
Drew: Wait, wait.
Teacher: So would it change?
Drew: Oh, OK. Yes, yes because, see . . .
Teacher: Because why?
Drew: They would be the same as before because you're taking a gallon from both.
Teacher: OK, which one are you telling me is fuller?

Drew wavered with uncertainty in his explanation. He seemed to be basing his thinking on a misconception that removing an equal quantity would not affect the comparative fullness even if the sizes of the containers were different. While the teacher listened to Drew's attempt to explain, she asked him questions, first to have him clarify and make public his thinking, then to mathematically justify his claim on his paper.

Strategy 2: Teachers encourage their students to reflect on their work and support student struggle in their effort to explain their thinking and not just in getting correct answers.

Drew: 2/5 . . . but isn't that fuller now? [Looking up at the teacher questioningly]
Teacher: Hmmmm, why would that one be fuller now, do you think?
Drew: Because the other one isn't a half.
Teacher: OK, it's not half. That means it's not fuller?
Drew: It's more than 0.5 now.
Teacher: Tell me that one more time.

The teacher provided encouragement while Drew struggled with his uncertainty. She also restated Drew's justification without passing judgment and gave him an opportunity to share his thinking.

Strategy 3: Teachers give time and help students manage their struggles through adversity and failure by not stepping in too soon or helping too much and thus taking the intellectual work away from the students.

Drew: I said it's more than 0.5 now. Because this one is no longer 0.5

because [squints and thinks], you have to subtract; you have taken away a gallon.

Teacher: OK.

Drew: And this one is no longer 0.6 because you have taken a gallon. And this one would no longer be a half. . . . I know what I'm saying.

Teacher: Be patient [gives Drew time and listens intently].

Drew: [Prepares to compute long division of his fractions that he has on his paper]

Teacher: Get your percents and call me back over. Keep working.

Drew: Back off [said to another student in his group who is looking at his paper while Drew is doing his computation. Drew continues to work while the teacher checks on other students' work.]

George suggested that doing mathematics takes time, patience, and perseverance as she asked Drew to keep working and to write down his explanation as she left to respond to other students. Drew remained engaged as he persisted with his task, even asking his nearby classmates for quiet as he worked for approximately thirty seconds. This episode illustrated the teacher providing time for the student to keep working.

Strategy 4: Teachers acknowledge that struggle is an important and natural part of learning and doing mathematics.

Drew: OK. I have it. Ring, ring, ring [makes a bell like sound].

Teacher: What have you got? I'm here. Glad you called.

Drew: Now I have got the percentage, drum roll please.

Teacher: [Taps on Drew's desk] Go.

Drew: This one is now 47 percent [pointing to his work for the 48 gallon barrel]. This is 40 percent [pointing to his work for the water jug].

Teacher: Which one's fuller?

Drew: The 48 gallon barrel.

The teacher acknowledged that the effort Drew expended was something she valued. Drew now had a mathematical way to document his work, and he appeared more confident in the conclusion he had reached.

SUPPORTING STUDENTS' STRUGGLE

Studies suggest that struggling to make sense of mathematics is a necessary component of learning mathematics with understanding (Hiebert and Grouws 2007). Hiebert and Wearne stated that "all students need to struggle with challenging problems if they are to learn mathematics deeply" (1993, p.6). In the recent NCTM publication *Principles to Actions: Ensuring Mathematical Success for All* (2014), the writing team identified the support of productive struggle in learning mathematics as one of the eight important teaching practices.

In the moment-by-moment needs

Fig. 2 This table outlines both strategies and indications of productive struggle.

	Teaching Strategies	Student Indicators of a Productive Struggle
Question	Teachers ask questions that help students focus on their thinking and identify the source of their struggle, then encourage students to build on their thinking or look at other ways to approach the problem.	Students ask questions to identify the source of their struggle, write down their ideas, clarify ideas with others, and consider alternative strategies or representations to address their struggle.
Encourage	Teachers encourage students to reflect on their work and support student struggle in their effort and not just in getting the correct answers.	Students use their effort to solve problems and try to make sense of their work, not only satisfied with a correct answer or that they perceive themselves as smart or not.
Give Time	Teachers give time and support for students to manage their struggles through adversity and failure by not stepping in too soon or too much, thereby taking the intellectual work away from the students.	Students use their time to develop and follow through on their strategies, evaluate their progress, and understand what they can do and what still remains to be done.
Acknowledge	Teachers acknowledge that struggle is an important part of learning and doing mathematics.	Students persist in their work to make sense of and to solve their problem and not give up or get discouraged easily.

of the students in their struggle, the responses that teachers give may in one instance enhance and in other instances diminish the level of student learning. This often depends on the circumstances, including the goals of the task, the student's prior knowledge, and the student's willingness to attempt to do the problem. Teachers can incorporate into their practice explicit reminders to students that struggling to make sense of mathematics is an important and natural part of learning. Rather than avoiding this phenomenon, teachers can integrate struggle as part of doing mathematics by acknowledging students' consternation, encouraging perseverance, asking questions, and offering time to work through problems. **Figure 2** outlines teaching strategies that remind students of the positive aspects of struggle and student actions that indicate productive engagement toward a resolution.

In summary, when students appear confused, unable to make sense of an answer, or reach an impasse in working on their task, teachers can view these episodes not as impediments to learning but as an opportunity for deepening students' understanding of mathematics. Teaching must be adaptive and situated in the interaction that unfolds from the task activity. By incorporating instructional approaches that acknowledge student struggles and effectively support and guide the students' thinking toward a productive resolution, students are given opportunities to strengthen their disposition toward engaging in challenging tasks. Over time, they will persist through their effort to make sense of and understand important mathematics.

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Hiroko K. Warshauer, hw02@txstate.edu, is an assistant professor of mathematics at Texas State University, San Marcos. Her research interests include areas of teaching and learning that foster productive struggle and investigation of teacher noticing of student thinking at preservice and in-service levels.