Mystical Magic Squares

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"Math is so boring!" How many times have we heard this statement from our students? As teachers, we are constantly looking for different activities to interest our students in mathematics and to convey the true richness and diversity of the subject.

Mathematical puzzles, like magic squares, have generated tremendous interest among children and adults alike for thousands of years. They also represent one way that cultures throughout history have ascribed mystical properties to numbers. A *magic square* is an $n \times n$ matrix, or grid, in which every row, column, and diagonal sums to the same number; this number is known as the *magic sum*. An example of a 3×3 magic square with a magic sum of 15 is shown in **figure 1**.

Magic squares first appeared in China (approximately 2200 B.C.). According to legend, Emperor Yu of China was standing on the bank of the Luo River when a turtle appeared with a curious and mystical symbol on its shell (see **fig. 2**). In this image, the groups of dots represent the numbers that comprise the magic square. Thus, the square is equivalent to that shown in **figure 1.**

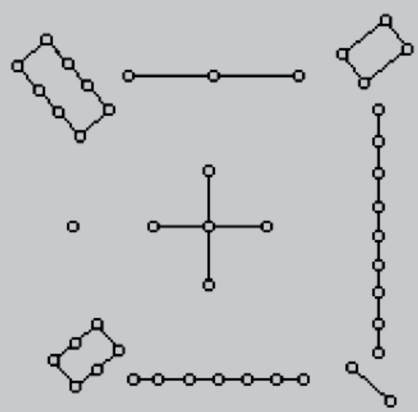
Magic squares were also known to Arab mathematicians in the ninth century and used by Arab astrologers to tell fortunes (PBS 2014). They appeared in the work of sixteenth century German artist Albrecht Dürer (see **fig. 3**). The 4×4 magic square in Dürer's engraving *Melancholia I*, in addition to having rows and columns that sum to 34, contains five 2×2 squares that also sum to 34 (located in the four corners of the larger square and in the center). (See **fig. 4**.) This particular type of magic square is called the *Gnomon magic square*. Note also that 1514, the date of the engraving, is featured in the bottom row of the magic square.

In the classroom, magic squares present a great opportunity to make historical connections and explore problem solving in a fun way. Magic squares can also support many of NCTM's Principles and Standards (NCTM 2000), including the Connections and Number and Operations Standards. *Principles to Actions: Ensuring Mathematical Success for All* (NCTM 2014) includes a call to implement tasks that promote reasoning and problem solving by engaging students in solving and discussing tasks

Fig. 1 This 3×3 magic square has a magic sum of 15.

8	3	4
1	5	9
6	7	2

Fig. 2 According to legend, Emperor Yu, standing on the bank of the Luo River, saw these symbols on a turtle.



(PBS 2014)

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Fig. 3 Dürer's engraving *Melancholia I* contains a particular type of square called the *Gnomon magic square* (it appears in the upper-right-hand corner).

Fig. 4 This magic square is found in the upper right in Dürer's *Melancholia I.*

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

that promote mathematical reasoning and problem solving with varied strategies. The Common Core's Standards for Mathematical Practice (CCSSI 2010) also call for students to be given opportunities to apply appropriate strategies and reflect on their strategies and those of their classmates. In this activity, students develop number sense, explore operations, practice their problem-solving skills, investigate the distributive and commutative properties, and learn about mathematical history in a fun and engaging manner.

TEACHER NOTES

The **activity sheet** is suitable for middle school students since it requires customary background knowledge. The activity explores magic squares and their properties, so teachers should have extra copies of blank 3 × 3 and 4 × 4 magic square grids available for student use.

Students are given a definition of a magic square, but must determine the magic sum for a 3 × 3 magic square by themselves (question 1). They will most likely use many different strategies. Allow students to present the squares they created and their methods for determining the magic sum because such presentations provide a rich opportunity for students to articulate their thinking and compare their methods. One efficient strategy for determining the magic sum of a 3 × 3 magic square that teachers could suggest (if students do not discover it for themselves) is to add all the numbers used in the square, 1 through 9, then divide the sum by 3, the number of rows (or columns). This method can be generalized to larger magic squares. An $n \times n$ magic square contains the whole numbers 1 to n^2. Summing these and dividing by n, the number of rows (or columns), gives the formula for the magic sum: $n(n^2 + 1)/2$.

Comparing the students' magic squares can lead to a discussion of symmetry, rotation, and reflection (see questions 3 and 5).

Questions 6 through 10 explore the foundations of matrix addition and scaling. They also provide the foundation for a discussion about distributive and commutative properties. These questions are designed to give students the opportunity to look for patterns, investigate the results that different operations have on the magic square, and practice the logical reasoning that they need to succeed in mathematics.

MAGIC SQUARES PROMOTE MATHEMATICAL THINKING

This activity was designed to be a challenging investigation that provides a glimpse into the vibrant history of mathematics and the enjoyment that working on a mathematical task can provide. Magic squares offer a rich environment in which students can practice mathematical thinking, discuss and build foundations for advanced mathematical topics, and build historical connections. After completing this activity, students should have a new appreciation for mathematics and problem solving.

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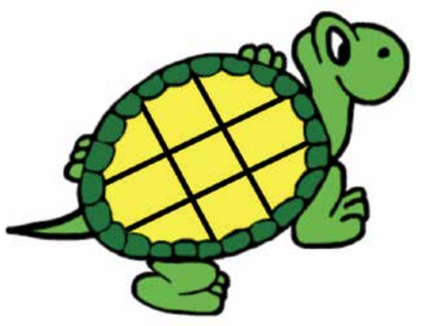
THE HISTORY OF THE MAGIC SQUARE



Once upon a time in ancient China, there was a huge flood. According to legend, the people of the village prayed to the god of one of the flooding rivers, the Luo River, to stop the flooding. The god listened to their prayers, and lo and behold, there appeared a magic turtle by the name of Sunzi with a magical-looking shell. *See the image at right*. Sunzi said, "Place the numbers 1–9 in such as way that each row, column, and diagonal sums to the same number—the magic sum. Once you've done this, the flooding will stop." This 3×3 grid is called a *magic square.*

Can you help the villagers stop the flooding?

1. What is the magic sum?

2. Arrange the numbers in the way the turtle requested.

3. Keeping the same number in the center square, what other magic squares can you find? How did you find them? Remember to use the numbers 1–9 and the magic sum you found in question 1.

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THE HISTORY OF THE MAGIC SQUARE (*continued*)

Several years later in 1206 A.D. in neighboring Mongolia, Genghis Khan was founding his empire when he encountered a magic cave that was guarded by Sunzi's cousin, Yang Hui. Yang Hui demanded that Genghis Khan solve the 4×4 magic square before he would allow entry into the magic cave.

4. What is the magic sum? Can you help Genghis Khan enter the magic cave by solving this 4×4 magic square?

16		2	13
5			
	6		
	15	14	1

5. Can you find different 4×4 magic squares? Remember to use the numbers 1–16 and the magic sum you found in question 4.

activity sheet

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THE HISTORY OF THE MAGIC SQUARE (*continued*)

6. What happens when you add corresponding entries of two magic squares? Is the result a magic square? If so, what is the new magic sum?

7. What happens when you multiply corresponding entries of two magic squares? Is the result a magic square? If so, what is the new magic sum?

8. What happens when you double every number in a magic square? Is the result a magic square? If so, what is the new magic sum?

9. What happens when you add a constant, like 5, to every number in a magic square? Is the result a magic square? If so, what is the new magic sum?

10. Can you make a 2×2 magic square? Try to understand why or why not, and then explain your reasoning to a neighbor.