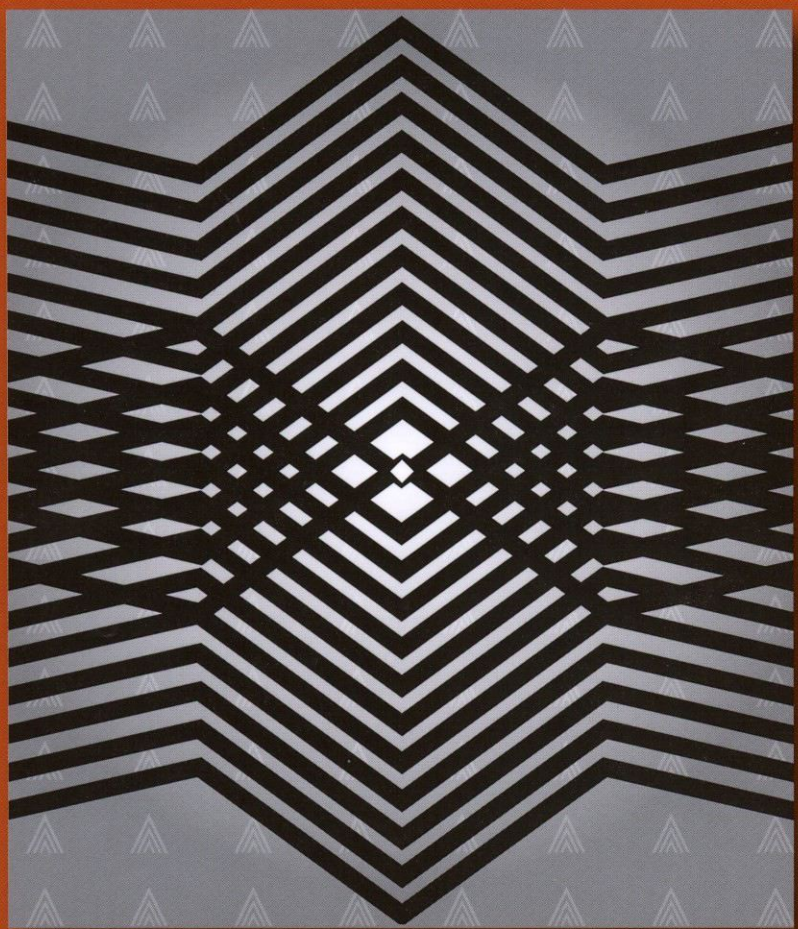
THE PEAK IN THE MIDDLE

Developing Mathematically Gifted Students in the Middle Grades



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THE PEAK IN THE MIDDLE

Developing Mathematically Gifted Students in the Middle Grades

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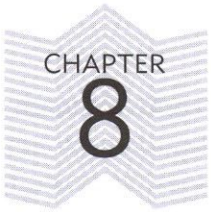

Equity

Max Warshauer, Terry McCabe, M. Alejandra Sorto, Sharon Strickland, Hiroko Warshauer, and Alex White

- All students should have opportunities to grow and develop, and equity means that this fundamental idea also applies to students who are gifted in mathematics.
- A set of guiding principles allows teachers to address the needs of all students in a variety of ways and create classrooms that provide all students with exciting learning opportunities in mathematics. Teachers should enable students to think deeply about seemingly simple ideas and processes, explore rich problems, develop a growth mindset, and gain early access to algebraic thinking.
- The needs of gifted students can be addressed outside the classroom in cost-effective ways; the key is to recognize that this effort is necessary and will benefit other students as well.

We use the word *equity* to mean the opportunity for all students to develop their mathematical abilities to their full potential. Equity is not the same as equality, in the sense of equality of services or resources. Although a physically challenged student may need the use of a wheelchair, equity does not imply that every student should have access to a wheelchair. Indeed, using a

The authors acknowledge the special contributions of Joanetta Ellis, principal of Fossum Middle School in McAllen, Texas, and Sam Baethge, retired high school teacher from Austin, Texas. Valuable insights were also shared by three Texas middle school teachers—Amy Warshauer from Austin, Sandra Saenz from McAllen, and Amanda Voigt from San Marcos—about their teaching and how they have provided opportunities for gifted students in mathematics.

wheelchair would deprive some students of necessary physical exercise. In the same way, equity for gifted students does not imply equality with others in educational environment. It does, however, imply equality of opportunity—every gifted child should have the same possibility as any other child of developing his or her capabilities to the full extent.

This point about equity versus equality brings up another distinct but related issue: equity is often associated with the education of students from disadvantaged backgrounds or whose families belong to groups that are underrepresented in science, technology, engineering, and mathematics (STEM) professions. As the National Mathematics Advisory Panel (NMP 2008) notes, "Unfortunately, most children from low-income backgrounds enter school with far less knowledge than peers from middle-income backgrounds, and the achievement gap in mathematical knowledge progressively widens throughout their Pre K–12 years" (p. xviii). In short, members of certain groups do not have the same opportunities as their peers.

Although equity for these students is an important issue, it is likewise important to recognize that gifted students, of whatever background, require certain resources to develop their abilities, simply by virtue of being gifted in the field of mathematics. Every student should have the opportunity to learn something new every school day.

We discuss both of these issues: the neglect of the growth of gifted students in general, and the additional neglect to which gifted students in certain groups (such as women, minorities, and economically disadvantaged students) are further subjected. We first describe elements of the kind of classroom that will equitably serve a gifted population. We next discuss some problems that arise in constructing such an environment, and we suggest some possible solutions. Finally, we address specific issues arising in identifying and serving gifted students in resource-poor communities.

Gifted education is a special branch of general education, so it should not be surprising that many of the points that we raise apply to students of all levels of ability and achievement. We have attempted to shape our remarks to emphasize their special significance for gifted students.

Likewise, many of the points that we make are valid at all levels of education and not just the middle school level. Again, we have tried to emphasize that the middle school years are critical. As the National Math Panel asserts, "The sharp falloff in mathematics achievement in the U.S. begins as students reach late middle school, where, for more and more students, algebra course work begins" (NMP 2008, p. xiii). International studies also show that middle school is precisely the time when the mathematics performance of many groups of students exhibits a precipitous decline (Schmidt et al. 2007; NCES 2000).

A Vision of a Middle-Grades Mathematics Classroom

Designing classroom learning environments that meet the mathematical needs of all students, including promising learners, is a daunting task for teachers but one that they confront daily (Chval and Davis 2008). Our concept of an ideal middle-grades mathematics learning environment rests on four fundamental principles, which we elaborate in this section. In some ways, these principles apply to any classroom and to work with any student. We stress specific ways in which they apply to the gifted student.

Principle 1: "Think deeply of simple things"—Arnold Ross

"Think deeply of simple things" was the often-repeated advice of scholar, teacher, and mathematician Arnold Ross and became the motto of the Ross Mathematics Program, a summer program that Ross founded for gifted precollege students at Ohio State University in 1957. Studies of gifted students point to the importance of creating a teaching and learning environment that nurtures their talents (Tomlinson et al. 2003). A classroom where every student is challenged academically, and, in particular, where gifted students are provided with opportunities to think deeply and creatively about problems and their solutions, helps students to become effective learners (Dixon et al. 2004). One danger in teaching is to stress procedures while neglecting the simple ideas that explain why they work and what they accomplish. To the average student, the study of mathematics as a set of procedures can seem complicated and mysterious. To the gifted student, this approach can render mathematics flat and lifeless—something to be merely tolerated while favoring more attractive and creative mental endeavors.

Teaching sometimes becomes so focused on mastering procedures that the simple underlying concepts are neglected. This hurts all students. Teachers often use warm-up problems to begin a mathematics class to ensure that the students are prepared for the standardized state assessment. When these problems are routine review and not clearly related to the main content of the day's lesson, the more proficient student may find the process boring and fail to engage in the important work still ahead. These students are not challenged.

According to the National Research Council, "The primary goal of advanced study in any discipline should be for students to achieve a deep conceptual understanding of the discipline's content and unifying concepts," and "schools and school districts must find ways to integrate advanced study with the rest of their program by means of a coherent plan extending from middle school through the last years of secondary school" (NRC 2002, pp. 197–98). In a later report (NRC 2005), the National Research Council recommended that the mathematics curriculum consist of a coherent plan to connect mathematical knowledge

organized around the foundational ideas of mathematics. Teachers need to create a classroom culture in which there is an expectation that all students will learn to think deeply about the mathematics.

A program such as the Texas State Honors Summer Math Camp (HSMC) (http://www.txstate.edu/mathworks/camps/hsmc.html) offers one model for this. In 2001, this program received the Texas Star Award for Closing the Gaps, and in 2008 it earned the Siemens Albert Hoser Founder's Award for its contributions to developing talented students in mathematics. Like the Ross Mathematics Program at Ohio State (http://www.math.ohio-state.edu/ross/) and the Program in Mathematics for Young Scientists (PROMYS) at Boston University (http://www.promys.org/), the Texas State summer program follows the Ross model of teaching students to "think deeply of simple things." Students explore numerical problems, make conjectures, and then justify their answers with proofs. The problems are given with instructions to "prove or disprove and salvage if possible." This places the burden on the students themselves to decide what is important, without simply giving them a procedure to follow.

Of course, there are other models. To build on prior knowledge, teachers need to recognize when students already know some of the curriculum (Page 2000). Amy Warshauer, a sixth-grade teacher at a magnet middle school in central Texas, suggests that a change in the culture of the classroom is necessary: "Before teaching how to do something, ask the students how they would do it.... The students usually will understand how to work through a problem after they are shown how, but it is even more exciting to see if they can do it on their own or working with their classmates. This allows them to share with others (and me) new ways of looking at and solving problems" (personal communication, March, 21, 2009). That is, this teacher assumes that the students can figure out how to do a problem, and then teaches only what needs to be taught. The technique can be used with all students, but acquires special meaning with gifted students: it acknowledges and uses their own contributions and makes the learning their own.

Many special needs of gifted students are different from those of more general students only in degree. Researchers and observers of classrooms have pointed out that all students must find the activity or task worthwhile and challenging. All students need a classroom environment that guides them to probe deeply, to risk making mistakes, and to take the task to a new level (Freiman 2008; NCTM 2000). Routine problems that superficially address concepts or provide little challenge or new knowledge for the students result in their disengagement from learning (Sheffield 1999). The classroom difficulty is that a problem that challenges the average student may be routine for the gifted student.

Consequently, we must be sure that the intellectual "ceiling" of our teaching is high enough that the gifted student isn't forced to stoop. One key is

differentiated instruction: the art of offering problems or environments that are at once meaningful to the average student and challenging to the gifted student. Chval and Davis (2008) provide ideas for successful differentiation in a middle school classroom, particularly for the gifted students, and Page (2000) addresses professional development needed to prepare and support teachers in implementing differentiation in their classroom practices.

When presented with the question, "How do you differentiate your instruction for these gifted students?" Sandra Saenz, a seventh-grade math teacher from McAllen, Texas, in the Rio Grande Valley, responded, "One way I differentiate instruction is by assigning students to groups by ability level (low, middle, high). They are assigned a basic problem and an algebra problem based on the same concept. As I assess them through observation and conversation, I am able to pinpoint those students capable of higher achievement. When they present their problems, the presenter(s) are usually those students that understood both the basic material and the algebra. To those students that are ready I assign algebra-based problems from their textbook" (personal communication, March 22, 2009).

Differentiation can occur in the areas of curriculum, practice, and assessment. It allows teaching practices that support students' participation, engagement, and mathematical learning at their own level. It also allows for assessment that gives students genuine feedback about their learning, while offering teachers feedback that informs their instruction (Winebrenner 2000).

Unfortunately, differentiating instruction can be very difficult, given the numerous demands on a teacher's time. The most challenging aspect of teaching in such a learning environment might well be how to orchestrate curriculum that engages all the students in the classroom. This leads to our second principle of an equitable education for mathematically promising students.

Principle 2: Challenge students with rich problems that encourage deep exploration

Problems that can be modified, extended, and differentiated can give all students opportunities for growth. Even struggling students need to engage in solving interesting problems that promote understanding and may require additional time and practice. For high-performing students, the need is greater still. Allowing students time to explore rich problems is critical to their developing the confidence to tackle new problems in the future. By discussing their solutions with one another, students will gain new insights and a much better understanding of their own ideas.

Amanda Voigt, who teaches seventh and eighth grades at a middle school in San Marcos, Texas, uses the routine task of adding and subtracting fractions

as an opportunity to differentiate her teaching. While most of Voigt's students work on numerical problems, she poses questions about adding and subtracting fractions with algebraic expressions to those students who are up to the challenge. This task gives her students an opportunity to extend their understanding of the process of operating with numerical fractions while weaving in algebra to generalize and extend the problem.

Other examples involve stretching or adding to the curriculum. For example, middle school students can explore problems involving graph theory, game theory, or combinatorics—topics not usually covered in their textbooks. The challenge is to find open-ended problems that can be investigated at multiple levels.

Successful experiences with such problems require that both the teacher and the students believe that through hard work any student can progress towards a solution. Too often mathematical problems are presented as puzzles that the gifted students "get" and the rest "don't get." It is our task as teachers to nurture and develop all students' abilities through hard work and careful training. Our third principle provides the key.

Principle 3: More progress is possible if students develop a growth mindset

As social psychologist Carol Dweck (2006) elegantly observed, teachers' views of intelligence can lead to dramatically different outcomes of instruction. The National Math Panel summarized this idea:

> Children's goals and beliefs about learning are related to their mathematics performance. Experimental studies have demonstrated that changing children's beliefs from a focus on ability to a focus on effort increases their engagement in mathematics learning, which in turn improves mathematics outcomes: When children believe that their efforts to learn make them "smarter," they show greater persistence in mathematics learning. Related research demonstrates that the engagement and sense of efficacy of African-American and Hispanic students in mathematical learning contexts not only tends to be lower than that of white and Asian students but also that it can be significantly increased. Teachers and other educational leaders should consistently help students and parents to understand that an increased emphasis on the importance of effort is related to improved mathematics performance. This is a critical point, because much of the public's self-evident resignation about mathematics education (together with the common tendencies to dismiss weak achievement and to give up early) seems rooted in the erroneous idea that success is largely a matter of inherent talent or ability, not effort. (NMP 2008, pp. xx)

The view that mathematical ability is something that can be developed through hard work is what Dweck (2006) calls a "growth mindset." People with this view are more likely to persist when given a difficult problem and therefore

have the potential to make significant contributions. People who view mathematical ability as being static have what Dweck calls a "fixed mindset." They are much more likely to give up when faced with a difficult problem since they will conclude that they just don't have the ability to solve it.

In an interesting study of the mindset of gifted students, Assouline and colleagues (2006) reported on the factors to which gifted girls and boys attributed school success and failure, including success and failure in mathematics. Both boys and girls usually ascribed failure to effort—not ability—an outcome supporting their having what Dweck would term a "growth mindset." However, an important difference between the boys and girls in this sample of gifted students emerged in the factors to which they attributed success. More boys than girls attributed success to ability. These results would seem to indicate that more boys than girls had, in some sense, a fixed mindset, raising an important question: What is the impact of having a fixed mindset regarding success and a growth mindset regarding failure? Alternatively, the results may have had less to do with what the boys believed that it takes to succeed at mathematics and more to do with their desire to project an image of cool detachment toward the outside world. In their minds, to blame failure on a lack of effort may mean that they did not place enough value on the activity to succeed, and hence the failure reflects more negatively on the activity than on them. The critical issue is whether students with a fixed mindset related to success tend to give up more easily when confronted with a task that pushes them to the limits of their ability, or whether other variables that are equally important to understand and address determine one's persistence. More research in this area might prove fruitful.

It is important to stress that giving gifted students more difficult material—material with which they must struggle—is not inequitable. Indeed, it ensures equity for this group of students because it gives them the environment that they need to develop their abilities to the fullest. If gifted students are more likely to have growth mindsets than fixed mindsets, as Assouline and colleagues (2006) found, then it is very important for teachers to have growth mindsets when thinking about their students. Even the most promising students have significant room for growth and should be put in situations in which they must work hard to achieve that growth. Students who find that all the mathematics that they are given is easy are at risk of turning their minds to other, more intriguing intellectual endeavors. If they continue studying mathematics, at some point they will inevitably come to a time and place where hard work is necessary. They should learn this lesson when they are younger, or they will be discouraged when they are older.

Two questions arise: What level of mathematical content is appropriate, and how does one provide the appropriate level of challenge for all students, including the gifted student? For the most part, the actual content topics are of secondary importance to the three principles articulated so far. However, there is overwhelming evidence that one mathematical strand is essential: algebra (NMP 2008).

If students do not receive an early introduction to the use of variables to model problems, there is a risk that they will not develop the mathematical fluency to tackle difficult problems. Such a gap can result in their losing confidence later on in their ability to solve problems, since they will not have the basic tools to express themselves mathematically or generalize specific examples. Algebraic thinking provides much more than a powerful tool for solving problems; it provides a language for describing patterns and relationships. Its importance is at the heart of our fourth principle.

Principle 4: Early access to algebraic thinking is critical for all students

The National Math Panel reports, "Students who complete Algebra II are more than twice as likely to graduate from college compared to students with less mathematical preparation" (NMP 2008, p. xiii). Algebraic reasoning offers a key to thinking deeply about mathematical problems. The earlier students can use variables, the easier it will be for them to communicate and organize their mathematical thoughts. If students are not introduced to algebraic concepts when young, they often develop a fear of mathematics, thinking that abstract concepts, variables, and algebra are beyond their abilities. In fact, many students have never been given a chance to develop their potential.

Algebra means much more than simply using variables as unknowns. Saul (2008) provides examples of the many facets of algebra. Briefly, the use and understanding of variables on the middle school level allows for the generalization of arithmetic. As Hazlewood, Stouffer, and Warshauer (1989) suggest, algebraic thinking can be taught effectively to young students just as music can be taught to them successfully by use of the Suzuki method (Suzuki 1983).

Although all students should receive a thorough and early introduction to algebra, such an introduction is especially important for students with high potential in mathematics. These students can be stifled if they must wait until ninth grade to experience algebra. A delay in this critical development restricts their progress in ways that may be difficult for them to overcome.

Usiskin (2000) describes levels of mathematical talent. Level 1 allows students to achieve the basic mathematics education in arithmetic that the majority of U.S. students attain by tenth grade, if not earlier. Level 2 permits students

to complete "honors" classes in algebra (the work that most students do in high school). Usiskin's levels of talent continue all the way to level 7—the Fields medalist sort of talent. Usiskin points out that high school is not the only time when students can do mathematics associated with talent at level 2. To get to each new level of talent requires additional energy and effort. So a student wishing to get to level 3 not only needs to have talent at level 2, but also needs to exert more effort. Usiskin warns that because students need more and more energy for each new level, the longer they remain at one level, the less likely they are to continue to the next. The key is to begin doing mathematics when young—to learn the language of mathematics and algebra just as one learns to read. This is particularly important for the gifted, since each new level represents a challenge that requires more energy than the preceding levels.

The question of how to weave the simple ideas of variables and algebra into the curriculum is not a simple one. One approach is to encourage students with problems that are so elementary that they can be easily worked without variables. Although this approach is valuable as a first step, it is not powerful enough to generate the interest and excitement that come from using variables to solve problems that are not easily approached without them. This takes us back to our fundamental principle that all students need to be engaged in working on rich problems, not just routine calculations.

Fulfilling the Vision: The Challenge

We have described some of the ways in which we can support the learning of gifted middle school students. Why aren't these supports more commonly in place? Unfortunately, there are numerous obstacles. Some of these are structural. Recently, the No Child Left Behind Act (NCLB 2001) has focused the nation on addressing the needs of students who are having difficulty in meeting minimal standards. Although this is a worthy goal, one result—probably unintended—is that teachers are judged by how successful they are in bringing every student up to a minimal standard, rather than by how successful they are in raising the level of mathematics for every student. In a key report from the Fordham Institute, Duffet, Farkas, and Loveless (2008) examined data from the National Assessment of Educational Progress (NAEP) and showed that this rewards structure for schools and teachers has resulted in an erosion of the quality of education for gifted students.

Another structural problem stems from the evaluation of schools and teachers. It is difficult to find methods of evaluation that measure the progress of all students. One way to do so is to use a "growth model" that measures the increase in each student's performance. In 1991, William Sanders proposed such a system: the Tennessee Value-Added Assessment System (TVAAS; see

Sanders and Horn [1998]). This system tracks student performance over time. In analyzing the research findings from TVAAS, Sanders and Horn [1997, p. 252] point to research that indicates a pattern of higher-scoring students making disproportionately lower gains than average and lower-scoring students. Research indicates that there may be multiple reasons that the needs of the gifted are not being met, including a lack of challenging materials and accelerated course offerings, as well as a concentration of instruction on average or below-average students.

Another problem in working with gifted students, and a problem with significant implications for equity, is that of identification. How do we recognize which students need more challenging problems? Sandra Saenz describes her work with middle school students in McAllen: "I can recognize these students that are able to do more because these are students that are able to have a mathematical conversation with you. They get it. They are at a higher level. They think abstractly. They have a good handle on the basics. They use the vocabulary easily. Math comes naturally. They apply what they've learned with the basics to algebra. They have lots of tools in their tool belt and use them at the right time. They are problem solvers" (personal communication, March 22, 2009).

These comments aptly characterize the work of gifted students. Perhaps more important, they give us clues about how to identify them. Saenz used multiple ways to identify the gifts of students. Reliance on one test or one criterion will not show the various ways in which mathematical talent can reveal itself. This is particularly true of students in schools with limited resources that must be allocated for the mastery of basic skills. In such a context, the gifts that students exhibit in the classroom, in varied and subtle ways, can easily be overlooked.

Indeed, the efforts of the teachers on the classroom level are central to our success in serving gifted students. With all the demands on teachers' time and attention, providing special care for gifted students can easily fall to the bottom of the list. Equitable education for gifted students requires training, support structures, and incentives for teachers, along with the time, tools, and desire to create a positive and challenging learning environment.

In addition, teachers' evaluations of their students must place equal emphasis on all students if we truly want equity in our teaching. It must no longer be acceptable to say, "This student is gifted and will reach a high level no matter what we do." Rather, we need to say, "This student is promising, and it is critical that we nurture and develop the abilities of all of our students." In particular, teachers need to understand the special challenge of providing rich problems that nurture the creativity and imagination of all students. As teachers learn

to address the special needs of their gifted students, they will simultaneously create an environment that will offer additional stimulation to all of their students—an environment where students are not afraid of failure but share in the joy of mathematical exploration and discovery.

Doubly at Risk: The Gifted Student in a Resource-Poor Environment

An uncomfortable fact remains for us to examine: in many schools and communities, students who are gifted do not receive the same opportunities as similar students in more affluent schools and communities. These typically underrepresented students (including, but not limited to, women, minorities, rural and some urban students, and economically disadvantaged students) often find themselves in school contexts where there are few resources to encourage the discovery or development of their mathematical gifts.

Consider, for example, the students in Texas who are selected each year to compete on the state's American Regions Math League (ARML) team in a national math competition. The thirty-two to forty-five students who compose this team come predominantly from only a handful of schools in the Houston, Dallas, and Austin areas. According to Sam Baethge, head of the ARML team from Texas, "Over the past five years, the team has been approximately 88 percent boys, 12 percent girls; and is 57 percent Asian American, 42 percent European American, and 1 percent Hispanic. Over the past fifteen years, only one or two students on this team have come from the Rio Grande Valley [a heavily Hispanic area of Texas]" (personal communication, March 15, 2009).

In addition to their regular mathematics classes, the schools that are the source of many of the Texas ARML team members provide special classes for advanced students. A well-funded school might be able to afford such special classes, expenses related to sending students to competitions, or teachers to oversee student math clubs outside of classroom time. In certain environments, the parents of these students might have extra free time to work with students, volunteer at school, or coordinate trips related to mathematics.

These sorts of resources are not available everywhere. In many locations across the United States, schools are doing well if they can fill teaching positions without resorting to under-qualified teachers (for example, non-mathematics majors or teachers without the standard certifications) and can meet the minimum needs of staffing, textbooks, and basic materials. In these schools, finding specialized teachers to work with gifted students and finding the money to pay them to teach extra, elective-style courses would be a luxury. For rural or inner-city schools, filling teacher positions can be a difficult task. To require those schools to add extra staff and offer courses especially for gifted students

(aside from fairly common honors-style courses) can seem impossible. Yet, talented students live in and grow up in communities without these resources. What can be done to help students and families in communities with fewer resources?

Fortunately, mathematics education for gifted students is not as expensive as training programs for gifted athletes and musicians. Mathematics does not require large fields for practice, expensive musical instruments, workout machinery, or large band halls. The basics of mathematics instruction are readily available in most schools—a room to learn in, some paper, books, pencils, calculators, and possibly computers. With the exception of computer software and specialized textbooks, the tools of mathematics are reasonably inexpensive and well within most budgets. Money begins to be a larger problem when personnel issues become factors. Some suggestions follow for creating a gifted program without much additional money:

1. Analyze the current school schedule. If a school makes use of a special time each day (or every few days in a block schedule) for homeroom or study hall, talented mathematics students could be assigned to the classroom of a particular teacher—say, from the math department—where students could spend that time working on interesting problems, contest preparation, or a general enrichment curriculum. In San Marcos, Texas, additional time each day is devoted to homeroom and tutorials. During this time, some students attend a class where they prepare for MATHCOUNTS competitions. This approach requires creative scheduling but offers promise as a way to engage talented students constructively while the rest of the school continues to use the same time in regularly scheduled homeroom activities. It is important to look for time in the schedule to offer a challenging elective course. Assuming that such an extra course offers even thirty minutes of instruction daily, students will receive two-and-a-half extra hours a week, or ten hours a month. In a school that offers six six-week units, thirty minutes daily adds ninety extra hours for students to develop their talents in mathematics.

2. Look for volunteers. If the school cannot find time or staff for a modified homeroom or elective period, it may be possible to find community members who are willing to work with students. This strategy requires an adult volunteer and space for students to work beyond the school day (or during lunch or some other available time). Mathematics teachers are obvious candidates as volunteers. They are already at the school, know the students, and have access to classroom resources

such as computers, pencils, and copiers. If teachers are unavailable, community leaders can sometimes help. Volunteers sometimes come from church and civic organizations or are retired seniors, parents, or members of the business community. Meetings can also be planned creatively, such as brown-bag lunch days where everyone meets at a special table or room, or weekend sessions.

3. Keep parents informed. Some families may be able to volunteer time or space. An available parent might host weekly meetings of a math club. Prepare a newsletter or an e-mail list that notifies parents of competition dates and special activities. This newsletter can also include samples of competition-style problems that the family can try at home and links to web resources and materials. A family might not have the means to volunteer time, space, or goods to a program, but might be able to fit in extra at-home learning opportunities.
4. Create a summer math club or camp. Sometimes a school has resources that are available during the summer but not during the year, making a summer math camp or club possible. A summer math club might be similar to an after-school club held during the school year. A camp could take the form of a one- to six-week day program offering interested students either enrichment or an accelerated curriculum.
5. Consider outside funding. Many local, state, and national organizations, as well as private foundations and local businesses, offer money for schools to develop and run extra educational programs. To be successful in obtaining funding, conduct research to see what sort of program might fit your school's needs and then write a proposal highlighting those needs and detailing a coherent plan of action for using the funds. Questions to consider are what students your program will serve, how you will locate those students, how you will advertise the program, what resources you can offer (space, materials, etc.), and what resources you will need. Consider ways to credit the donors publicly for their support. You might be able to solicit volunteers to research and write the proposals.
6. Think outside the school. If there is a local university that offers a teacher preparation program, maybe an arrangement could be made for these preservice teachers to do internships with gifted mathematics students at the school, either during regular school hours or outside of them. Similarly, professors of mathematics or education might

be able to coordinate their work with the school. Many university educators are in constant need of opportunities to work with students and teachers for research or curriculum design. A possible arrangement might be for professors to have access to relevant data from the school in exchange for helping the school's math program, or even generating such data from their work with the gifted students.

7. Explore the possibility of launching a "math circle" that brings together university faculty to work with middle and high school students on interesting math problems. This could be particularly effective for schools that are located in close proximity to a university.

Conclusion

Gifted students of mathematics compose a group of students with special needs that are often neglected. It is not a question of elitism to insist on serving this group but rather a question of equity. Meeting the needs of students in this group is vitally important if our country is to be competitive in the twenty-first century. This problem can be addressed in multiple ways, ranging from in-school adjustments of expectations for what students can learn, to early algebra, to creative ways of using local resources.

As we have seen, the needs of gifted learners often closely parallel those of other learners. In fact, the issues of equity—both the problems and the solutions—for this group of students are not really different in quality from similar issues for other groups of students. So it is perhaps no surprise that schools making an effort to address the needs of their gifted students often raise the level of learning for all their students.

The most important step in solving any problem is to realize that it is a problem. Then, as Pólya (1945) suggests, we need to make a plan, carry out that plan, and continually check to be sure that what we are doing is effective. By using the same ideas that mathematicians use in problem solving, we can provide a rich learning environment that will develop our next generation of students in math and science, enabling them to be competitive with the best students anywhere.

References

Assouline, Susan G., Nicholas Colangelo, Damien Ihrig, and Leslie Forstadt. "Attributional Choices for Academic Success and Failure by Intellectually Gifted Students." *Gifted Child Quarterly* 50, no. 4 (2006): 283–94.

Chval, Kathryn B., and Jane A. Davis. "The Gifted Student." *Mathematics Teaching in the Middle School* 14 (December 2008): 267–74.

Dixon, Felicia A., Kimberly Prater, Heidi Vine, Mary Jo Wark, Tasha Williams, Tim Hanchon, and Carolyn Shobe. "Teaching to Their Thinking: A Strategy to Meet the Critical-Thinking Needs of Gifted Students." *Journal for the Education of the Gifted* 28, no. 1 (2004): 56–76.

Dweck, Carol. *Mindset: The New Psychology of Success.* New York: Random House, 2006.

Duffet, Ann, Steve Farkas, and Tom Loveless. *High-Achieving Students in the Era of No Child Left Behind.* Washington, D.C.: Thomas B. Fordham Institute, 2008. http://www.edexcellence.net/doc/20080618_high_achievers.pdf.

Freiman, Viktor. "Problems to Discover and to Boost Mathematical Talent in Early Grades: A Challenging Situations Approach." In *Creativity, Giftedness, and Talent Development in Mathematics,* edited by Bharath Sriraman, pp. 155–84. Missoula, Mont.: Information Age Publishing, 2008.

Hazlewood, Donald G., Sandy Stouffer, and Max Warshauer. "Suzuki Meets Pólya: Teaching Mathematics to Young Pupils." *The Arithmetic Teacher* 37 (November 1989): 8–10.

National Center for Education Statistics (NCES). *Pursuing Excellence: Comparisons of International Eighth-Grade Mathematics and Science Achievement from a U.S. Perspective, 1995 and 1999.* NCES 2001-028. Washington, D.C.: U.S. Department of Education, 2000.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, Va.: NCTM, 2000.

National Mathematics Advisory Panel (National Math Panel; NMP). *Foundations for Success: The Final Report of the National Mathematics Advisory Panel.* Washington, D.C.: U.S. Department of Education, 2008. http://www.ed.gov/about/bdscomm/list/mathpanel/report/finalreport.pdf

National Research Council. *How Students Learn: History, Mathematics, and Science in the Classroom.* Committee on How People Learn: A Targeted Report for Teachers. Edited by M. Suzanne Donovan and John D. Branford. Washington, D.C.: National Academies Press, 2005.

———. *Learning and Understanding: Improving Advanced Study of Mathematics and Science in U.S. High Schools.* Edited by Jerry P. Gollub, Meryl Berthenthal, Jay Labov, and Phillip Curtis. Washington, D.C.: National Academies Press, 2002.

No Child Left Behind Act of 2001. Public Law 107-110. 107th Cong., 1st sess. 8 January 2002.

Page, Sandra W. "When Changes for the Gifted Spur Differentiation for All." *Educational Leadership* 58, no. 1 (2000): 62–65.

Pólya, George. *How to Solve It.* Princeton, N.J.: Princeton University Press, 1945.

Sanders, William L., and Sandra Horn. "The Tennessee Value-Added Assessment System (TVAAS): Mixed-Model Methodology in Educational Assessment." *Journal of Personnel Evaluation in Education* 8 (1994): 299–311.

Sanders, William L., and Sandra P. Horn. "Research Findings from the Tennessee Value-Added Assessment System (TVAAS) Database: Implications for Educational Evaluation and Research." *Journal of Personnel Evaluation in Education* 12 (1998): 247–56.

Saul, Mark. "Algebra: The Mathematics and the Pedagogy." In *Algebra and Algebraic Thinking in School Mathematics,* Seventieth Yearbook of the National Council of Teachers of Mathematics (NCTM), edited by Carole E. Greenes, pp. 63–79. Reston, Va.: NCTM, 2008.

Schmidt, William H., Maria Teresa Tatto, Kiril Bankov, Sigrid Blömeke, Tenoch Cedillo, Leland Cogan, Shin Il Han, Richard Houang, Feng Jui Hsieh, Lynne Paine, Marcella Santillan, and John Schwille. *The Preparation Gap: Teacher Education for Middle School Mathematics in Six Countries.* East Lansing, Mich.: Center for Research in Mathematics and Science Education, Michigan State University, 2007. http://usteds.msu.edu/MT21Report.pdf.

Sheffield, Linda Jensen. "Serving the Needs of the Mathematically Promising." In *Developing Mathematically Promising Students,* edited by Linda Jensen Sheffield, pp. 43–56. Reston, Va.: National Council of Teachers of Mathematics, 1999.

Suzuki, Shinichi. *Nurtured by Love: The Classic Approach to Talent Education.* Miami: Warner Bros. Publishing, 1983.

Tomlinson, Carol Ann, Catherine Brighton, Holly Hertberg, Carolyn M. Callahan, Tonya R. Moon, Kay Brimijoin, Lynda A. Conover, and Timothy Reynolds. "Differentiating Instruction in Response to Student Readiness, Interest, and Learning Profile in Academically Diverse Classrooms: A Review of Literature." *Journal for the Education of the Gifted* 27, no. 2–3 (2003): 119–45.

Usiskin, Zalman. "The Development into the Mathematically Talented." *Journal of Secondary Gifted Education* 11, no. 3 (2000): 152–62.

Winebrenner, Susan. "Gifted Students Need an Education, Too." *Educational Leadership* 58, no. 1 (2000): 52–56.