Geometrical Explorations

Max Warshauer

Southwest Texas State University, San Marcos, TX 78666, U.S.A.

In many ways, mathematics at all levels is the study of patterns. Teachers try their best to make clear to their students that mastering a small number of patterns will bring order to a large number of seemingly disparate facts. For students, the "turning on of the light" is often no more that the recognition of a pattern, and the search for patterns drives research among professional mathematicians and university professors. The mathematical mind, even subconsciously, tends to sift through large stores of observations trying to find and uncover latent patterns.

Patterns imply structure, and nowhere in mathematics are patterns and structure more clearly evident than in geometry. Herewith, we introduce a new column which has to do with patterns exhibited by figures in plane Euclidean geometry. Think, kind readers, how many times you may have drawn figures to test whether or not a conjecture seems sensible before you have been willing to invest the time necessary to prove or disprove that it is so.

We are not talking about compass and straight-edge construction but about the art of drawing figures. Recall the advice that you may have given or been given: "Draw exaggerated figures." If your triangles look to be equilateral, things may appear to be true that are not in general true. Draw long skinny triangles to convince students that angle bisectors, medians, and altitudes are all different. Avoid special cases. Exaggerated figures expose false patterns.

Psychologists tell us that we learn through seeing, hearing, and touching. In geometry, a major component of "touching" is the drawing of figures. It seems as though there is something about the coordinated motions of the hand and eye required in drawing lines, circles, triangles, trapezoids – and so on – which, through physical sensation, enhances and deepens our perception of space.

In our new column, we shall offer problems each of which will begin with a series of instructions for drawing a figure. Inspection of each figure which has been correctly drawn should suggest a conjecture about all correctly drawn versions of the figure. Readers will then be invited to prove or disprove the conjecture. They will need only pencil, paper, and imagination. However, we believe that our exercises will also offer the opportunity to use technology to extend the notion of learning through drawing and "touching" by bringing into play such powerful and efficient software as the *Geometric Sketchpad*.

We note that it is sometimes quite a challenge to give precise instructions for drawing complicated figures in reasonably few words. Therefore, we also invite suggestions from our readers for improvements in our instructions and in the statements of our problems.

Finally – and most important – have fun!

GE 1. *Draw a convex quadrilateral $ABCD$ with the following properties:*

1) $AC = BD$;
2) *AC and BD intersect at P which is not the midpoint of AC;*
3) *the segment joining the midpoints E and F of AD and BC, respectively, intersects AC and BD at Q and R, respectively.*

Denote the point at which AC and BD intersect by P.

If the drawing is closely done, it will appear that $\angle PQR = \angle PRQ$. Show that such is the case.

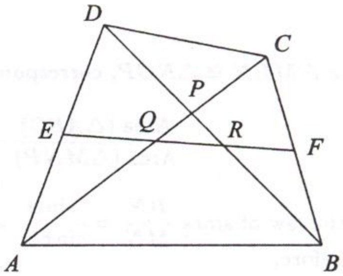

Figure 1. Quadrilateral $ABCD$.

GE 2. *Draw a circle with center O and diameter AD. Then draw the line tangent to the circle at D and denote by P another point on the line. Draw a second line through P which intersects AD between O and D. This second line intersects the circle at the two points. Name the point nearer to P as C and the other point as B.*

Now draw AB and AC to form inscribed triangle ABC. Finally, draw line PO intersecting AC at N and AB at M.

If you have been careful, O should look very like the midpoint of NM. Make and then prove the conjecture that $OM = ON$.

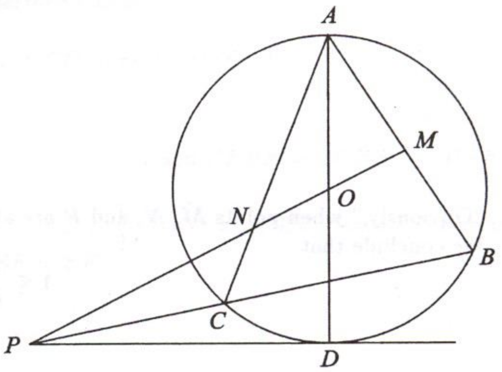

Figure 2. Circle O with Tangent PD.