Abacus Interactive

Ellen Robinson, Xiaowen Cui, Nama Namakshi, Hiroko K. Warshauer, Sonalee Bhattacharyya, and Christina Koehne

Calculators are often efficient in finding the answer to an addition or subtraction problem, but they do not reveal the process by which the answer is obtained. Developing students' fluency in addition and subtraction using strategies and algorithms based on place value, composing and decomposing numbers in base 10, and reading and writing numerals in expanded form are important teaching and learning standards not only for the elementary grades but for middle school students as well (NCTM 2000; CCSSI 2010; TEA 2015). We introduced the Chinese abacus to our students as a hands-on tool to illuminate the meaning of a number in expanded form in terms of place values and to strengthen students' conceptual understanding of the standard algorithms of addition and subtraction. "Students' understanding of the base 10 number system is deepened as they come to understand its multiplicative structure" (NCTM 2000, p. 143). This activity will let students explore the mathematical properties of the base 10 system in a creative and interactive way. Students develop a deeper meaning of why the standard algorithms work and how they relate to a number in expanded form. This activity is best suited for elementary and middle grades.

Edited by **Marilyn Howard**, marilyn-howard@utulsa.edu, University of Tulsa, Oklahoma; **Christine Kincaid Dewey**, dewey@wcskids.net, Macomb Mathematics Science Technology Center, Warren, Michigan; and **Hiroko K. Warshauer**, hiroko@txstate.edu, Texas State University, San Marcos. Readers are encouraged to submit classroom-tested activities through **http://mtms.msubmit.net**.

THE CHINESE ABACUS AND HOW IT WORKS

The Chinese abacus, or *suanpan*, in Chinese, was first described in a 190 CE book of the Eastern Han dynasty (Yoke 2000). It is a calculating tool that was in use centuries before the adoption of the modern, written numeral system, and it is still used by merchants and bookkeepers (Heffelfinger 2010). Below we present a brief introduction on how to use the Chinese abacus for representing numbers and performing the basic arithmetic operations of addition and subtraction. A more detailed description on how to use this tool can be found at http://www.tuxgraphics.org/giniandkarlsworld/mathblog/abacusexplained.shtml.

Fig. 1 This standard Chinese abacus is shown in the starting position.

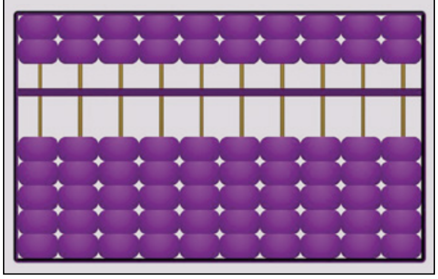
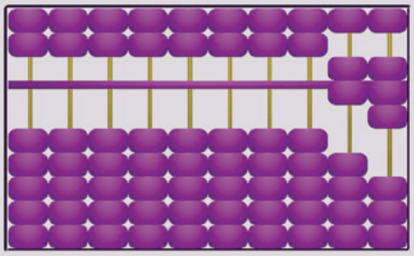


Figure 1 illustrates a standard Chinese abacus in the starting position, which represents a value of zero. The abacus consists of seven beads on each vertical rod with one horizontal beam referred to as the *center beam*. Of the seven beads, two beads are located above the center beam and five beads are located below, called the upper and lower beads, respectively. The columns (vertical rods) represent the place values of the beads. The rightmost column represents the units place, the second column from the right represents the tens place, and so forth. When a bead is moved toward the center beam, it is given a value.

In the rightmost column, each bead below the center beam is worth one unit; each bead above the center beam is worth five units. In the second column, each bead below the

Fig. 2 The number 67 is shown.



center beam is worth one unit in the tens place, and each bead above the center beam is worth five in the tens place. As an example, we show 67 in **figure 2**.

Whenever all five lower beads are moved toward the center beam, they can be substituted for one upper bead. Also, whenever all five lower beads and one upper bead are moved toward the center beam on a single vertical rod, these six beads can be substituted for one lower bead on the rod to the left of the current rod. Using these steps, students may notice that the second upper bead is not necessary for counting on the abacus. In fact, a Japanese abacus, called a *soroban*, has only one bead above the center beam and four beads below the center beam (Heffelfinger 2010). Therefore, when counting from four to five on a Japanese abacus, the four lower beads are shifted down and an upper bead is brought toward the center beam, thus increasing the value from four to five.

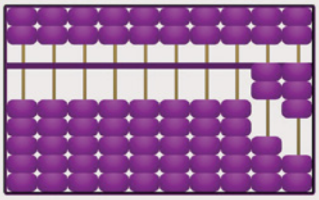
When adding on the Chinese abacus, we can begin on the leftmost column and move toward the right or begin on the rightmost column and move toward the left. This differs from the standard addition algorithm that requires adding from right to left. Thus, working with the abacus can increase students' comprehension of the nature of the calculations and deepen their understanding of the standard algorithm procedure. Similarly, students can practice subtracting on the abacus.

See **figure 3**'s example of adding 23 cats + 49 dogs (answer to which will be 72 pets). A video demonstration of this addition can be found at https://youtu.be/bW-8HfXdnpQ.

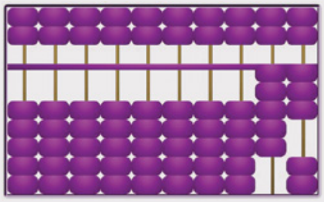
Fig. 3 This example illustrates adding 23 + 49.

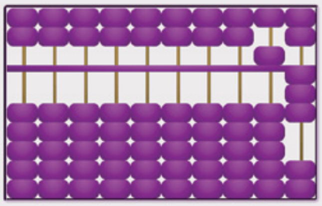
Example: Add 23 + 49

Step 1: Start with 23 on the abacus.

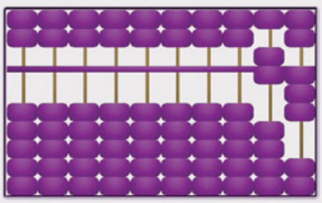


Step 2: We want to add 4 beads in the ten's column. Add 3 beads and then substitute the 5 beads in the lower deck for 1 bead in the upper deck. The two pictures are equivalent.



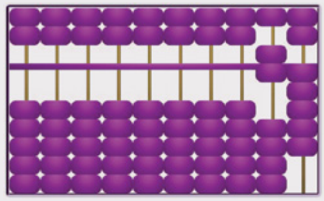


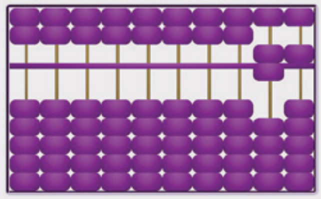
Step 3: Add 1 more bead in the ten's column.



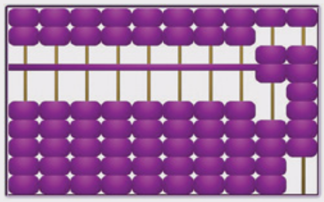
Step 4: We now want to add 9 beads in the unit's column. Add 2 beads and then substitute the 5 beads in the lower deck for 1 bead in the upper deck. The two pictures are equivalent.

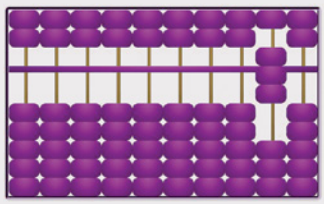
Step 4 continued:





Step 5: Add 5 more beads in the unit's column and then substitute the 5 lower beads and the 1 upper bead for 1 lower bead in the ten's column. The two pictures are equivalent.





Step 6: We then add the last 2 beads in the unit's column to get the answer of 72.

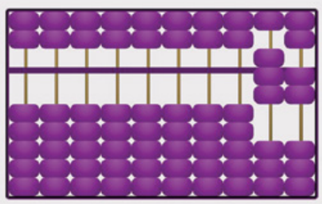
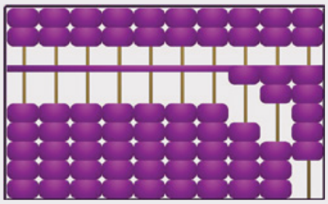


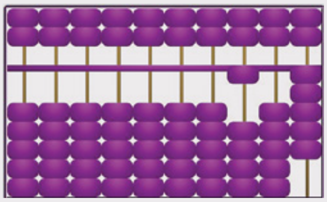
Fig. 4 This example illustrates subtracting 48 from 125.

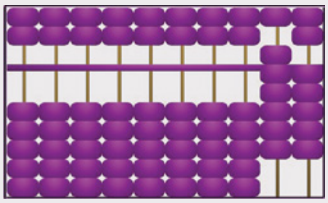
Example: Subtract 125 – 48

Step 1: Start with 125 on the abacus.

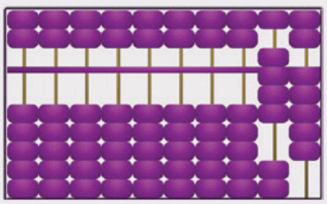


Step 2: We want to take away 4 beads in the ten's column. Take away 2 beads and then substitute 1 bead in the hundred's column for 1 upper bead and 5 lower beads in the ten's column. The two pictures are equivalent.

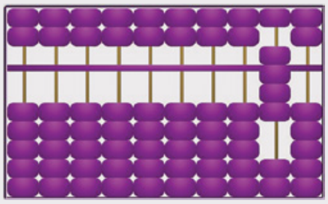


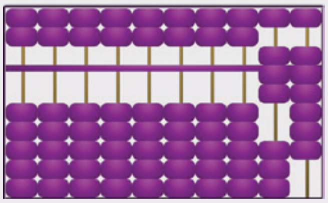


Step 3: Take away 2 more beads in the ten's column.

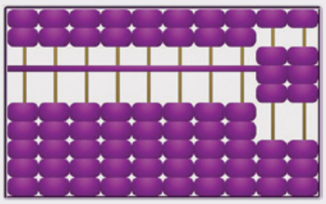


Step 4: We now want to take away 8 beads in the unit's column. Take away 5 beads and then substitute 1 lower bead in the ten's column for 1 upper bead and 5 lower beads in the unit's column. The two pictures are equivalent.





Step 5: Take away 3 more beads in the unit's column. We get an answer of 77.



An example of subtracting 125 – 48 appears in **figure 4**.

IMPLEMENTING THE ACTIVITY

For this activity, students will be creating their own abacuses and performing the calculations on the provided activity sheet. However, before students construct their own abacuses and receive their activity sheets, we recommend that the teacher either project an online abacus on the board (see the above-mentioned Web resource) or create one from magnets to use on a whiteboard to provide students with examples of representing numbers, adding, and subtracting. When the authors implemented this activity, they led examples at the front of the class and then had student volunteers come up and do the motions to complete the calculations as well. This allowed students to understand the steps used in calculating on an abacus and showed them the usefulness of this tool.

Next, have students construct their own abacuses. Follow the steps on the Abacus Construction Instructions sheet provided. Examples of completed abacuses are included with the instruction sheet to assist in the process.

After each student has an abacus, distribute the activity sheet. We recommend that you implement the activity sheet first as individual student work, and then have students discuss their work in small groups. Place the emphasis on the actions required to perform the operations on the abacus, not just the final answers. Then have students present their step-by-step processes in front of the class. This activity can be completed in two 90-minute class periods. Two of the authors implemented this activity at a summer math camp with a diverse set of students ranging from grades 5–7.

The first question on the **activity sheet** should act as a review of place value that the students should have learned in a previous lesson. Questions 2–4 allow students to visualize place value and get familiar with representing numbers on their abacus to prepare for the calculations in questions 5 and 6. Question 7 should suggest to the student that the very top bead and the very bottom bead are not necessary for the calculations. At this point, the teacher can bring up the Japanese version of the abacus, which has four lower beads and only one upper bead. Question 8 calls students' attention to the connections between how they would perform arithmetic operations on paper versus on the abacus. This will allow students to make more sense of the mechanics behind standard algorithms.

The goal of the extension is to introduce the base 5 numeration system without explicitly using the powers of 5 as the place values. Students should be able to follow similar steps to those used on their base 10 abacuses to represent numbers in base 5. We leave it to the teacher to extend the activity further by having students add or subtract numbers in base 5, convert numbers between base 10 and base 5, or introduce other bases.

CONCLUSION

In this activity, students have an opportunity to explore the use of the Chinese abacus, an ancient calculation tool. There is value in creating and learning about a historical tool that is still commonly used in China, Japan, and other Asian countries. This activity allows students to develop a deeper understanding of place value by repeatedly decomposing numbers into expanded form, recording them on their abacus, and translating numbers off the abacus. Instead of learning by rote, when students manipulate an abacus, they can visualize and make sense of the operations so that the meaning behind the algorithm becomes clear.

ACKNOWLEDGMENTS

This activity was implemented by two of the authors at Mathworks Junior Summer Math Camp 2016 in San Marcos, Texas, with students ranging from grades 5–7. The authors would like to thank Mathworks for providing a setting for the authors to test the problems in a classroom and to Max Warshauer, of the mathematics department at Texas State University, for offering insightful suggestions, proofreading the manuscript, and for his overall guidance in orchestrating this project.

Abacus Construction Instructions

Materials Needed

- Foam board or cardboard cut into 8 1/2 in. × 11 in. rectangles
- Copies of abacus template
- Pipe cleaners (13 per student)
- Beads (84 per student)
- Tape
- Scissors

Steps of Construction

1. Mount the abacus template on the foam board with tape.
2. Cut the pipe cleaners to the desired length for the vertical rods. Some students may wrap the pipe cleaners around the end of the board if desired.
3. Put seven beads on each rod and then tape the rod onto the template at the ends.
4. Mount a center beam across the vertical rods using tape on the ends.
5. Add extra tape for support if necessary.

Helpful Tips

- Having vertical rods all the way across the template is unnecessary, but students should have a minimum of five rods.
- Some students found that putting small pieces of tape on the center beam between the vertical rods helped keep the center beam pushed against the template.
- Different colors of beads may be useful in distinguishing between upper and lower beads and beads of different place values.
- Twisting two pipe cleaners together also strengthened the center beam. Examples of a student at work and a final product are shown in **figures 5** and **6**.

Fig. 5 A work in progress

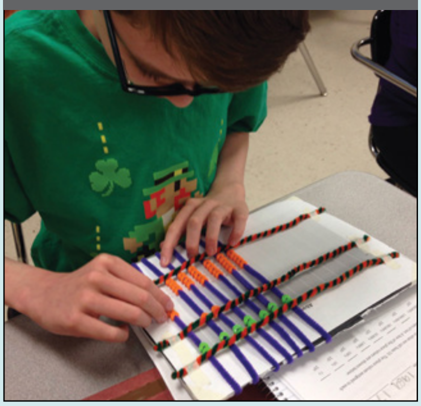
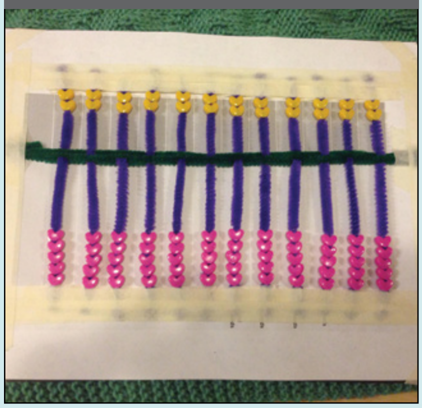


Fig. 6 A completed abacus



REFERENCES

Common Core State Standards Initiative (CCSSI). 2010. Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf

Heffelfinger, Totten, and Gary Flom. 2010. "Abacus: Mystery of the Bead." http://webhome.idirect.com/~totton/abacus/pages.htm

National Council of Teachers of Mathematics (NCTM). 2010. *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

Socher, Katja. 2016. "A Short Explanation on How to Use the Chinese Abacus (Suanpan)." http://www.tuxgraphics.org/giniandkarlsworld/mathblog/abacusexplained.shtml

Texas Educational Agency (TEA). 2015. Texas Essential Knowledge and Skills for Mathematics. http://ritter.tea.state.tx.us/rules/tac/chapter111/index.html

Yoke, Ho Peng. (2000). *Li, Qi and Shu: An Introduction to Science and Civilization in China*. Mineola, NY: Courier Dover Publications.






Ellen Robinson, ebr21@txstate.edu, is a lecturer in the mathematics department at Texas State University. Her research interests are in the areas of graph theory and combinatorial matrix theory. **Xiaowen Cui**, x_c5@txstate.edu, is a doctoral candidate in mathematics education at Texas State University. She is interested in instructional coherence in the classroom on the topic of introduction of fractions. **Nama Namakshi**, namakshi@uark.edu, is a clinical assistant professor in the mathematics department at the University of Arkansas. Her research interests include math camps and the effect they have on women's participation in STEM fields, development of teacher noticing skills among preservice teachers, and curriculum development at elementary/middle-school levels. **Hiroko K. Warshauer**, hw02@txstate.edu, is an assistant professor of mathematics at Texas State University. Her research interests include areas of teaching and learning that foster productive struggle and investigation of teacher noticing of student thinking at preservice and in-service levels. **Sonalee Bhattacharyya**, sb1212@txstate.edu, is a graduate student in mathematics education at Texas State University. Her research interests include how teachers can be supported in their profession, professional teacher noticing, and how rich mathematical tasks promote interest in mathematics. **Christina Koehne**, crz7@txstate.edu, holds a master's degree in mathematics from Texas State University, a bachelor of science degree in mathematics with a Teaching Certification from Texas A&M University Kingsville, and is currently a doctoral student at Texas State University studying mathematics education.

more4U

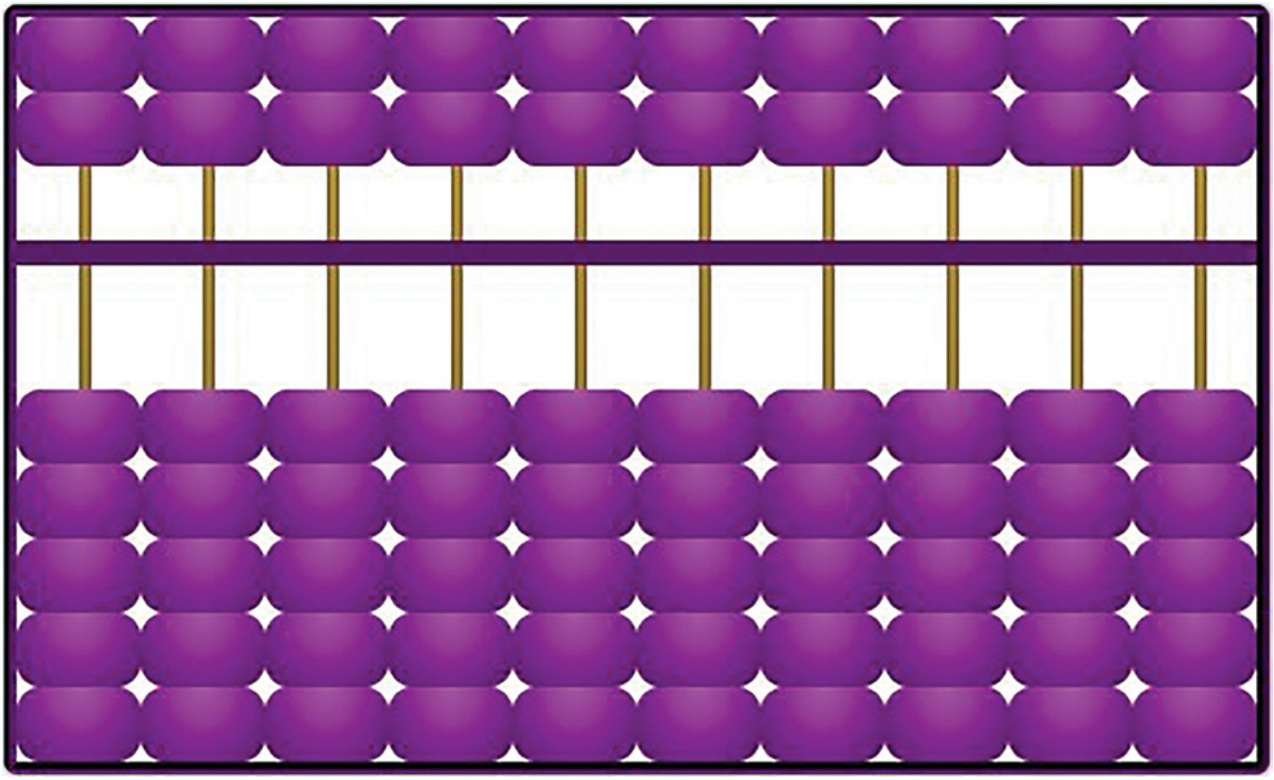
The solutions to Mathematical Explorations, online at **http://www.nctm.org/mtms,** are a members-only benefit.


activity sheet 1

Name ______________________________

ABACUS INTERACTIVE ACTIVITY

1. Write the place values below the beams of the abacus. The units place has been done for you.



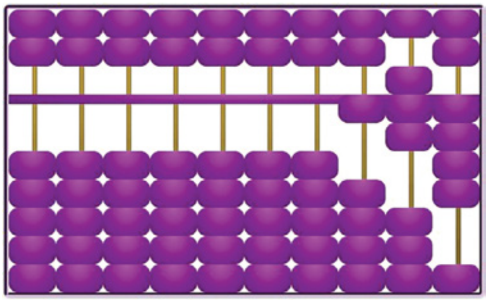
									Units
									1

2. Count up to 20 on your abacus.

from the March 2018 issue of mathematics teaching in the MIDDLE SCHOOL

Name ______________________________

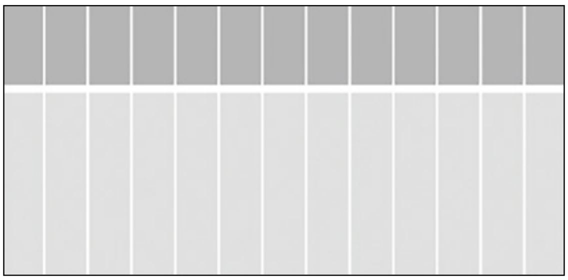
3. What number is shown on the abacus?



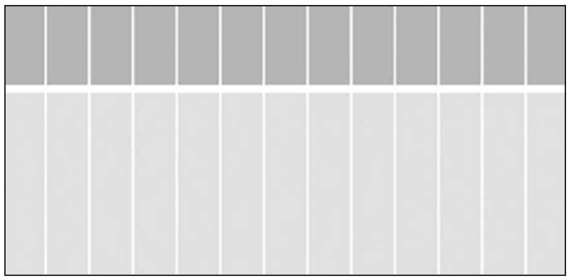
4. Represent the following numbers on your abacus and record them in the spaces provided.

a. 85 yards of fencing
b. 396 hummingbirds
c. 1,174 Dalmatian puppies
d. 600 high school graduates
e. Pick a number to represent on your abacus.

a.

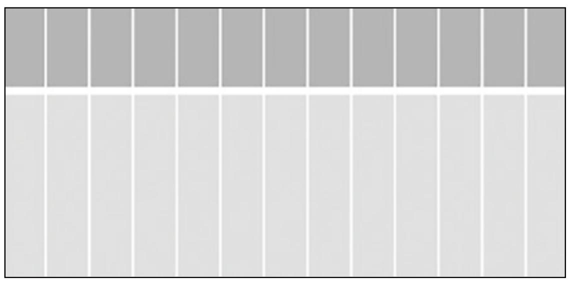


b.

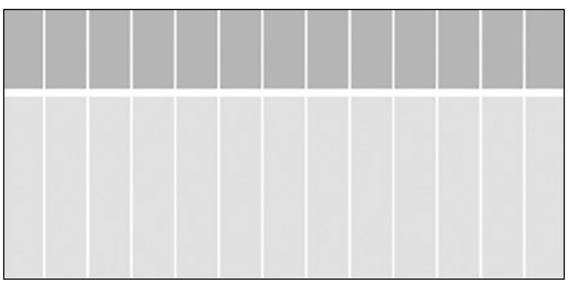


c.

d.



e.



activity sheet 1 (continued)

Name ______________________________

5. Solve the following problems on your abacus and record the final answer in the spaces provided. Represent the final answer for each on the abacus.

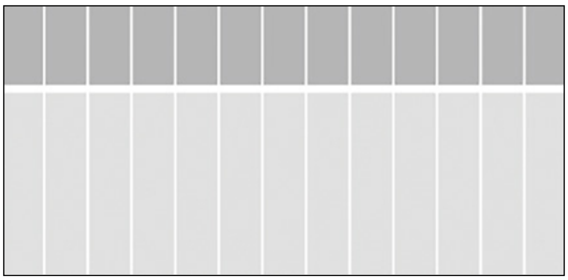
a. After shell collecting at the beach, Abby counts 38 shells in her collection and August counts 51 in his. How many shells do the two shell collectors have altogether? Answer: ___________

b. Oliver and Hugo set up chairs at a wedding. Oliver sets up 126 chairs, and Hugo sets up 34 chairs. How many chairs do the two helpers set up in total? Answer: ___________

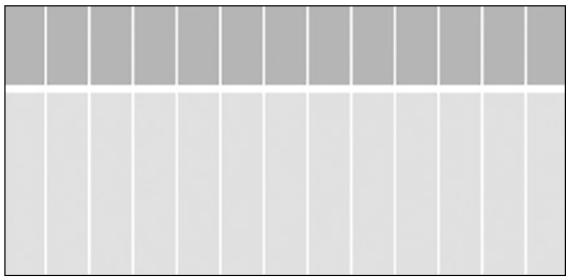
c. Penelope and Victoria are driving home from two different destinations. Penelope will drive 571 miles, and Victoria will drive 629 miles. How many total miles will the two girls have driven by the time they get home? Answer: ___________

d. Write your own addition problem:

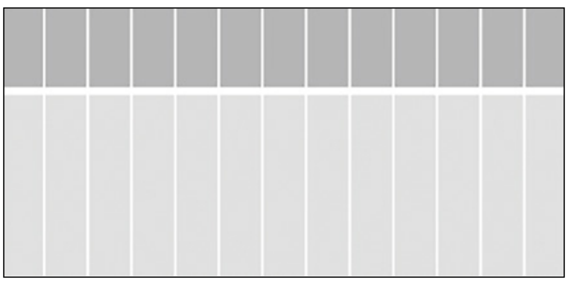
a.



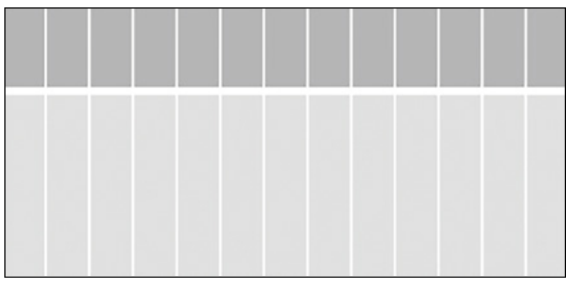
b.



c.



d.



Name ______________________________

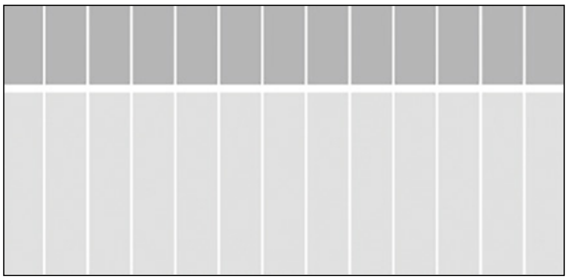
6. Solve the following problems on your abacus and record the final answer in the spaces provided. Represent the final answer for each on the respective abacus.

a. Madeline wants to save $28 to make a purchase. So far she has $16 saved up. How much more money does Madeline need to save to get to her goal? Answer: ___________

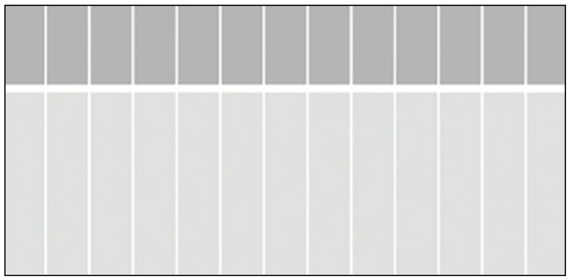
b. Fineas ran 125 miles in the month of October; Jasper ran 48. How many more miles did Fineas run than Jasper in the month of October? Answer: ___________

c. Kensey is reading a book that is 977 pages long. So far, she has read 384 pages. How many pages does Kensey have left to read? Answer: ___________

a.



b.



c.

7. Will your abacus still work with fewer beads in each column? If so, explain why.

8. What similarities and differences do you notice between doing arithmetic on the abacus and doing arithmetic by hand on your paper?

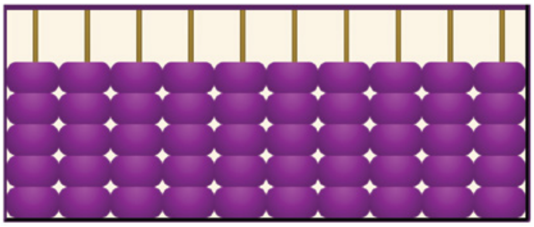
activity sheet 1 (continued)

Name ______________________________

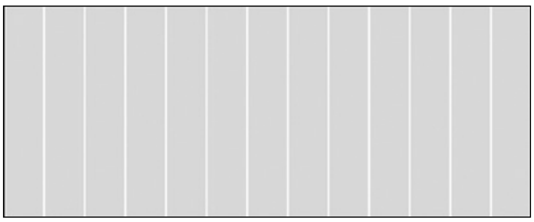
EXTENSION

Another planet has a race called the Pentanese, who represent their numbers differently from those of us on Earth. The abacus they use is represented in the figure below. Notice that their abacus is very similar to the Chinese abacus but uses only the lower beads.

Each bead has a value of 1, but the Pentanese do substitutions differently. Five beads in the rightmost column substitute for one bead in the second column from the right. These aliens have a different place-value system than we do!



1. Count up to 17 on their abacus and represent the final position below.



2. How would the Pentanese represent a value of 25 on their abacus?

3. Create an abacus that has the same place value as the Pentanese while using fewer beads. Can you think of multiple ways to do this?