are interested. When I created a slide show of art from Bridges, a math-art conference, I tweeted a link to an applet created by David Chappell, one of the featured artists. Using Twitter this way gave me the chance to give more content to those who were interested without overwhelming casual readers. I am still learning how to use social media effectively for math communication, but I think it has the potential to be a valuable resource.

I loved the variety of my assignments and the challenge of learning new science quickly, but I missed the process of discovery itself. In some ways, I feel like two different people. One of me wants to take any possible opportunity to educate people about all kinds of math, while the other one wants to discover new theorems. During the 2012–2013 academic year, since I do not have an academic position, I am continuing to write for *Scientific American* as a freelancer while also attending math conferences and working on my own research. In August 2013, when I begin my postdoc at the University of Utah, I plan to continue science and math communication for a general audience through blogging and freelance writing. Writing has joined research and teaching as an important and rewarding part of my career.

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— ***doceamus*** . . . let us teach

"A Modest Proposal" Research Base

Max Warshauer

Jonathan Swift's original "A Modest Proposal" [1] was a thought-provoking satire designed to help deal with a food famine and its root causes of overpopulation. Schoenfeld's "Modest Proposal" [2] is designed to deal with a different kind of famine, namely, an intellectual famine brought about by not incorporating real sense making into the teaching of mathematics.

We extend Schoenfeld's proposal by suggesting that all students can do mathematics at a high level if properly challenged and engaged, and provide a collection of research questions to examine this proposition. Evidence abounds that students are having great difficulties in learning algebra [6]. We realize algebra is not a synonym for "sense making", and indeed algebra can be taught in a procedural way that does not encourage sense making at all. The reason that we focus on algebra as a proxy for sense making is that, when properly taught, algebraic reasoning provides students a powerful tool for explaining ideas rigorously and precisely.

Over the past six years a group of mathematics faculty has developed a curriculum, Math Explorations [13], designed to prepare all students for algebra by grade 8 or earlier, weaving in the type of sense making that Schoenfeld so elegantly describes. Students are engaged in exploring problems deeply and learn to explain simple ideas, such as adding and subtracting integers, with visual models. Variables and algebra are integrated throughout. Here we shall describe research questions that have arisen and propose a framework for further study.

Consider Suzuki's work on talent development in music [3], [4]. Suzuki observed that "any child is able to display highly superior abilities if only the correct methods are used in training." A method for teaching mathematics with the Suzuki method was described by the author previously in [5]. One of Suzuki's fundamental tenets is that, if one begins learning music when older, this may be a harder task than if one learns music as a child,

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since much time must be spent "unlearning" misconceptions. Suzuki observed [3],

> If they have learned the wrong fa by hearing it five thousand times, one must make them listen to the right fa six thousand or seven thousand times.

Reasons for suggesting that all students can succeed in algebra when younger if given the opportunity include:

1. The National Math Panel [6] observed:

> Teachers and developers of instructional materials sometimes assume that students need to be a certain age to learn certain mathematical ideas. However, a major research finding is that what is developmentally appropriate is largely contingent on prior opportunities to learn. Claims based on theories that children of particular ages cannot learn certain content because they are "too young", "not in the appropriate stage", or "not ready" have consistently been shown to be wrong.

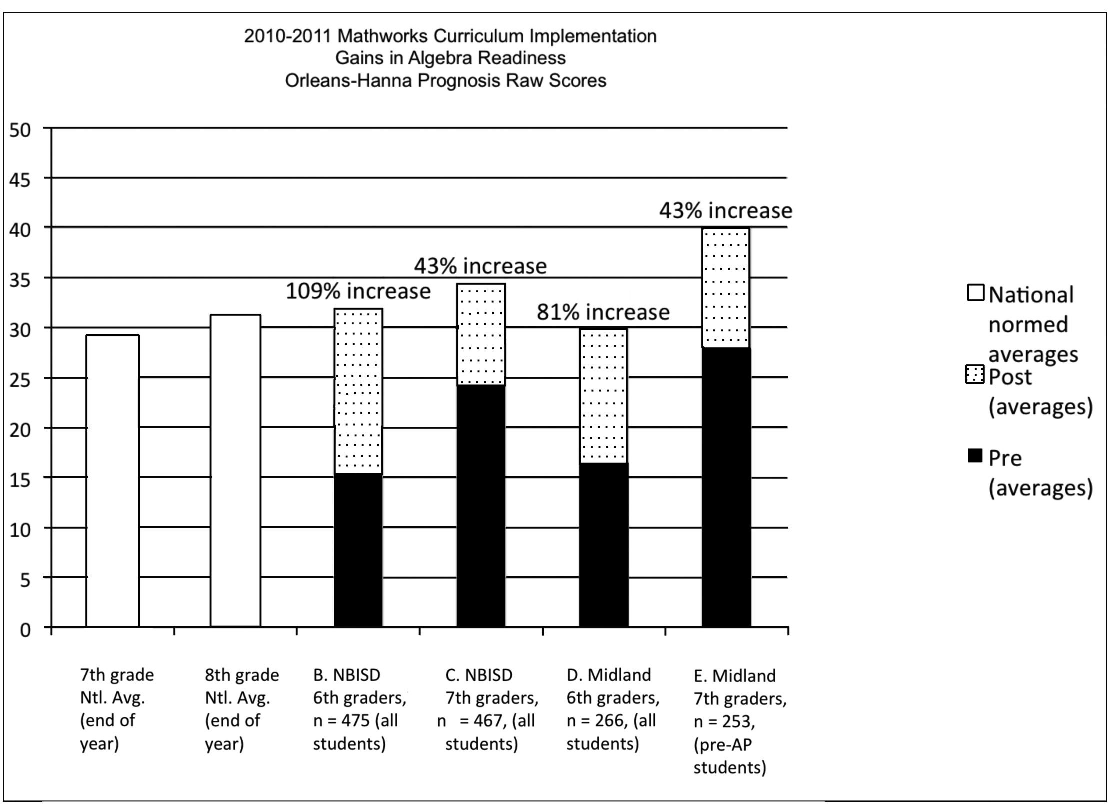
2. If waiting to learn music till later makes the task significantly harder, would not the same logic suggest that waiting to learn "algebra" till later might also be a much more challenging task?

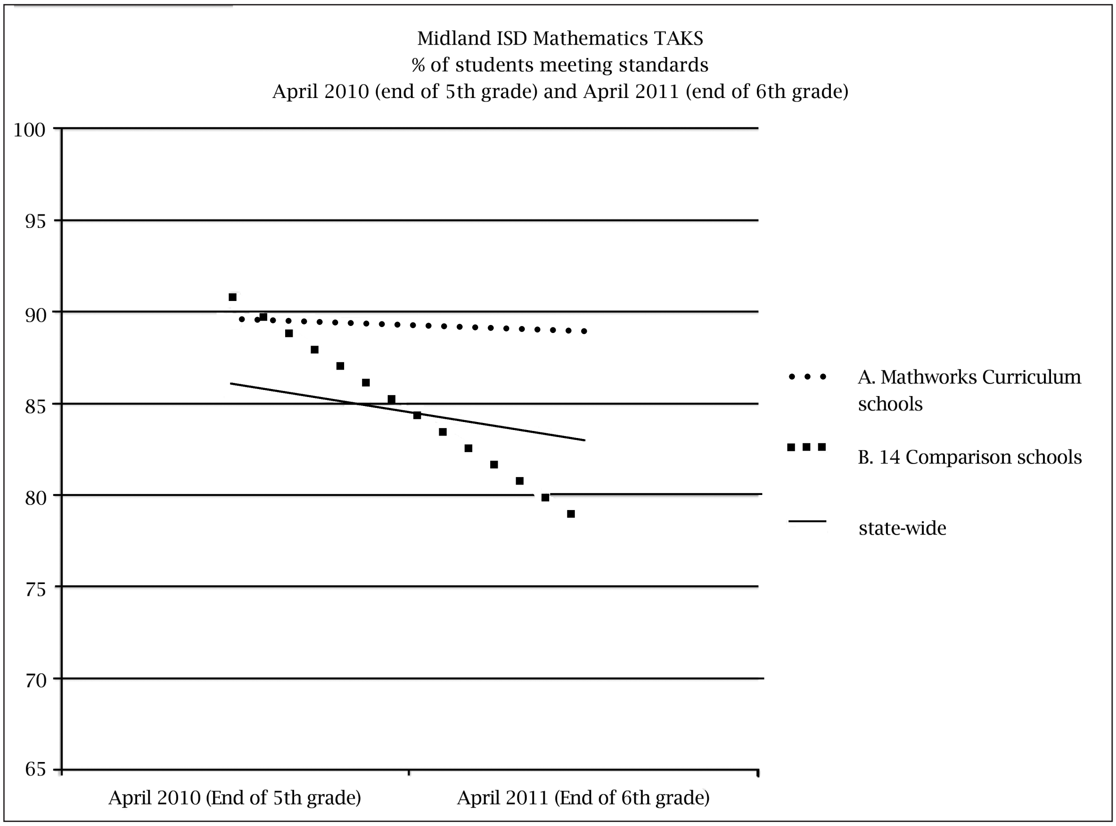
3. Is waiting till ninth grade to teach algebra part of the problem? The NCTM *Standards* [12] advocates weaving in algebraic ideas throughout elementary and middle school.

The mathematics curriculum presently used in many middle schools, as Schoenfeld observes, is having the exact effect that Suzuki describes—it is dulling the students' minds. They are being taught that mathematics is not about making sense, that math is a mysterious, incomprehensible topic. We hypothesize that not teaching sensemaking throughout middle school may be training students with attitudes about mathematics that are difficult to overcome. As Suzuki points out, once misconceptions are ingrained in young students, it can be very difficult to "undo" this poor training.

One argument that schools often make for not allowing all young students to begin algebra is that some students are not up to the challenge. Evidence from the Charles A. Dana Center suggests the opposite. In a study [8] of 378 schools and 26,363 students who took the Algebra I EOC in 1997 and 26,334 who took the Algebra I EOC in 1998, the following was a key finding:

> Teachers at schools with improving scores on the Algebra I EOC exam stated repeatedly that they believed that all their students could perform in algebra and successfully pass the exam—this critical issue was identified as an "Algebra for All" vision. In





contrast, the schools with declining scores reported that their students worked very hard but suggested that there should be mathematics classes less difficult than Algebra I available for some students.

By waiting to introduce some students to algebra, is it possible that we are sustaining and perpetuating an educational path that is destined to failure?

One measure of student ability to do algebra is the Orleans-Hanna Algebra Prognosis test. Each summer we give the Orleans-Hanna algebra prognosis test as a pre- and posttest in an intensive two-week Junior Summer Math Camp for rising fourth- to eighth-grade students. Students completing level 3 of our summer camp score on average above eighth-grade level. The results from pilots of the Math Explorations are even more convincing. After using this curriculum for one year, seventh-graders scored on average over eighth-grade level in their preparation for algebra, as measured by the Orleans-Hanna algebra prognosis test, and over 65 percent of the sixth-graders scored over eighth-grade average; see the chart below.

The group of students using Math Explorations also scored significantly higher than comparison groups on the state math assessment (TAKS). We compared four schools using Math Explorations to fourteen similar schools in the same district. All student groups entered the school year with approximately the same passing percentages on TAKS. By the end of their sixth-grade school year, students in the fourteen comparison schools experienced a sharp drop in passing percentages, while students using Math Explorations experienced no such decrease.

This brings us to the central question raised by this article: What are the keys to teaching mathematics with real sense making? We suggest a collection of research questions to address this fundamental question:

1. Is sense making as described by Schoenfeld correlated to providing students with algebra and algebraic models as a tool? Is there evidence that teaching sense making without algebra is more or less effective than teaching the same concepts with algebra?

2. Can all students succeed in algebra when young (grade 8 or earlier)? Several studies [9], [10], [11] develop a research base for early algebra and suggest that much more can be done.

3. Is the data from multiple pilots of Math Explorations the result of the curriculum, teaching, teacher training that includes observing math camps, school environment, faculty support, or a combination of these or other factors?

4. If students wait until the ninth grade to begin algebra, does this yield significantly worse results than beginning algebra in eighth grade (or younger)?

5. Is there a correlation between students who are placed into early algebra and the factors of ethnicity, race, or gender? Is "early algebra" for "some" one of the root causes of the gaps that we are seeking to close?

6. If we begin teaching students algebra in grades 6–8, will these students complete all of their math requirements earlier in high school and stop taking further mathematics? Do students who take algebra when younger do worse in subsequent courses and college than students who began taking algebra later? Longitudinal studies are needed.

These are testable questions. More work needs to be done. The key is to find schools that are willing to challenge all of their students. To convince a school to offer Algebra I for all students in grade 8 or earlier may require evidence of the kind we are proposing to gather, which is like a Catch-22.

However, there are encouraging developments. Programs such as the TEX Prep Program, begun by Manuel Berriosabal at University of Texas-San Antonio, have opened the door of opportunity for many Hispanic students. That program may be a fertile ground for research about young students succeeding in early algebra. The Math Explorations curriculum described above weaves in algebra for all students in grades 6–8 and is currently being tested at several sites. These are samples of what can be done. The challenge now will be to gather more evidence and to continue our efforts to engage all students in real "sense making", as suggested by Schoenfeld, while giving a rigorous definition of what this means in terms of introducing all students to algebra and higher-level mathematics.

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