MATHEMATICS EDUCATION THROUGH THE MIDDLE GRADES

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AT
ARITHMETIC TEACHER

VOLUME 37
NUMBER 3
NOVEMBER 1989

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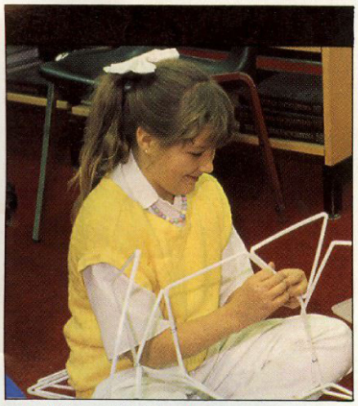
CONTENTS



STRAW CONSTRUCTIONS
SEE PAGE 4

OLYMPIC MEDALS
SEE PAGE 47

On the cover: Students develop their spatial reasoning using flexible straws as described in the article by Gerard Prentice. Readers are encouraged to submit nonreturnable color slides of children involved in elementary school mathematics for possible use on the cover.

ISSN 0004-136X

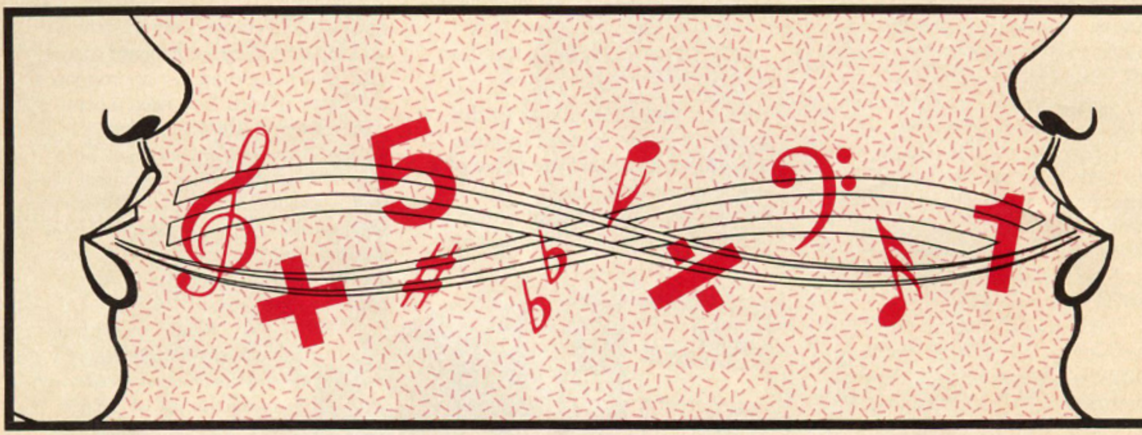
The *Arithmetic Teacher* is an official publication of the National Council of Teachers of Mathematics, Inc., 1906 Association Drive, Reston, VA 22091. See masthead for subscription information.

FEATURES

DEPARTMENTS

SUZUKI MEETS PÓLYA
TEACHING MATHEMATICS TO YOUNG PUPI

Donald G. Hazlewood, Sandy Stouffer, and Max Warshauer



The Suzuki method (Suzuki 1969, 1981, 1984) of teaching young pupils to play the violin has been widely used with remarkable results. Suzuki credits his approach to a simple observation, namely that all Japanese children learn to speak their native language and, thus, must have high ability for learning. Suzuki reasoned that this ability, if properly developed, would enable children to learn to play the violin. Since his basic premise was that all children are capable of high achievement, the problem for any child who could not learn music must be in the way music was taught. In an attempt to solve this problem, Suzuki developed his "mother tongue" approach to music instruction based on his observations of children learning to speak, stressing the following:

1. All children have high ability.
2. Repetition is the key.
3. Don't rush, but at the same time, don't rest.
4. Always praise first, since constant criticism dulls the brain.

However, as Suzuki himself writes (Suzuki 1981), his approach to learning could and should be applied in teaching other subjects. In this article, we describe how Suzuki's methods can be combined with Pólya's ideas on problem solving (Pólya 1957) to teach mathematics to elementary school pupils.

Donald Hazlewood and Max Warshauer teach in the mathematics department at Southwest Texas State University, San Marcos, TX 78666. Sandy Stouffer is director of the ACE Program at Bowie Elementary School of San Marcos IDS, a program for gifted-talented students. They are interested in several areas related to mathematics and computer science and work with gifted-talented students to encourage their interests in those areas.

Mathematics Is a Language

J. Willard Gibbs observed that "mathematics is a language" (Wheeler 1952). Gibbs, a nineteenth-century mathematical physicist, made this statement while sitting in the lounge at Yale University where his colleagues were debating language requirements for the baccalaureate degree. His contention was that if language is a tematic means for communic ideas, then mathematics is equiv to Greek, Latin, or French. In mathematics has developed int universal language of science.

Assuming Gibbs's observatio premise, we propose that math ics can be taught using Suzuki's er-tongue approach in which mathematical concept is intro by examining it in the context of problems. A word problem c thought of as a "story" with p pants in the story acting out v roles. The story is then tran from the verbal problem into a ematical one as an equation. this method, pupils will gain flue mathematics just as they gain fl in reading.

The Four-Step Method

Mathematics is a medium fo cisely expressing ideas. To young pupils how to use this me a systematic framework is nece Exactly such a framework is su by the four-step approach of Pólya (Pólya 1957):

1. Understand the problem.
2. Make a plan.
3. Carry out the plan.
4. Check the answer.

Translated for second graders, these four steps mean that one must first determine the list of "characters" in the story and which of them need to be represented by variables; second, use the characters together with their roles in the story to write an equation that tells the story; third, solve the equation; and finally, verify that the solution actually makes sense.

Fun Is the Key!

Of course, it is one thing to teach problem solving and another to make it enjoyable for young pupils. The key to the approach is that it should be fun! As one develops the statements in step 1, care should be taken to talk about the characters in the story together with their individual roles and relationships that eventually lead to the equation in step 2.

This approach to problem solving requires an understanding of the concept of variables and equations. But, just as a pupil in the primary grades learns that letters are part of words and that these words express ideas, so also should a pupil learn that variables can represent numbers of objects in stories. Where stories describe actions in words, mathematics has operations for actions such as addition and multiplication. Finally, where a story expresses relationships with sentences, mathematics uses equations.

Variables and Equations

This observation brings us to our second premise: to attempt to teach mathematics without using variables and equations is like teaching reading without using the alphabet or sentences.

Our second premise is not universally accepted. Prevost (1985) attempts to draw a distinction between algebra and mathematics and concludes that eighth graders are not ready to learn algebra. We contend, however, that even second and third graders should begin learning algebra because algebra is really the language of mathematics. Prevost does suggest that what is needed is a "challenging and engaging alternative." We suggest that Suzuki's mother-tongue approach is just such an alternative but should be used with a much younger group than eighth or ninth graders.

Mathematics can be taught using Suzuki's approach.

Practice and Repetition

Suzuki's method can be summed up in an equation:

repetition + desire = results

Suzuki maintains that one learns only by continual practice of basic concepts. Repetition is the key to learning to speak fluently, whether the language be English, Japanese, music, or mathematics. We conclude that one needs a program in which practice is strongly encouraged. Before practice, however, pupils need to know basic number facts, such as those from addition and multiplication tables. This knowledge is equivalent to learning how to hold the bow correctly before playing the violin. Then practice is needed. However, a basic warning is in order: monotony is our worst enemy. Therefore, it is crucial to propose word problems that are interesting and challenging, just as Suzuki uses interesting and challenging music. As the pupil solves problems using variables and equations, fluency is developed in reading and writing the language of mathematics. When mathematics is taught in an exciting and interesting way, the pupil develops the desire to practice and comes to appreciate that practice is useful in solving the problems. Only through practice in using variables and equations to model real problems will fluency in the language of mathematics be developed.

Mathematics is not taught as a disjoint collection of facts and drill but rather as a vehicle for understanding problems. By being introduced to a systematic method for analyzing problems, Pólya's four-step method, the pupils themselves develop confidence in their own ability to solve difficult problems. As a result, they view mathematics not as an abstract subject dealing with numbers but as the art of refined human thinking that can be employed to solve a host of interesting problems.

Some Examples

This process is exemplified by the following problems:

> Jason has some marbles. If Jason could get three more marbles, the number of marbles that he has would be the same as twice the number of marbles that Claire has. Claire has five marbles. How many marbles does Jason really have?

To solve this problem with second graders, first we should read the story carefully with the students and make up a list of all of the characters. The characters are Jason, Jason's marbles, the number 3, the number 2 related to twice, Claire, Claire's marbles, and the number 5. From these characters, we need to decide what our variables must be. Examining the question at the end of the paragraph, we decide that the basic unknown is the number of marbles that Jason has, and so we write

J = the number of marbles Jason really has.

For step 2, that is, to write an equation that tells the story, we must consider the characters and the roles they play in the story. The characters in this story are J, 3, 2, and 5. What else? Well, our mathematical sentence requires a relational sign. Specifically, an = sign comes from the phrase "would be the same as." Rereading the story and underlining the phrase "If Jason could get three more marbles," we see that the operation of "getting more" is adding, so we will also need the character + in our story. Anything else? The phrase "twice the number" means to multiply by 2, so we also need the character × for multiplication.

Thus, we translate our story into the mathematical story

If Jason could get three more,	$+ 3$
the number of marbles that he has	$J + 3$
would be the same as	$=$
twice the number	$\times 2$
number of marbles that Claire has	5

to get the equation

$$J + 3 = 2 \times 5.$$

Solving the equation is done intuitively, namely

$$J + 3 = 10;$$
$$J = 7.$$

One could, for example, ask what number added to 3 equals 10. The important thing, at this stage of development, is not to get bogged down in Pólya's steps 3 and 4.

Finally, we check the solution:

$$7 + 3 = 2 \times 5$$
$$10 = 10$$

Thus, we see how we can translate a story into a mathematical story. This activity is fun and requires thinking. Similarly, we can give the pupils an equation, for example, $J + 3 = 2 \times 5$, and ask that they write a story that corresponds to the mathematical equation.

The first attempts may deal only with numbers.

> Jill is thinking of a number. If her number was three larger it would be twice as large as five. What number is Jill thinking of?

But remember, J can stand for the number of anything. Suppose Jill has J pencils. What does it mean for Sara to give her three more pencils? What does multiplication mean? Is twice as many as five the same as five times as many as two? By asking a series of questions, the following story might arise:

> Jill had some pencils. Her friend Sara gave her three more. Now Jill has twice as many pencils as Christy. Christy has five pencils. How many pencils does Jill have now?

We have also found that money problems are interesting to pupils. Here is an example:

> Nathan had a pocket filled with change consisting only of dimes and quarters. When he counted his money he found that he had seven coins and \$1.15 total. How many dimes and how many quarters did he have?

Monotony is our worst enemy.

Again, the first step is to decide what the characters are. In this problem, we have dimes and quarters, the number 7, and 115—the total amount of change in cents. From these characters we must decide what the variables are. The question at the end furnishes the clue, and we let

D = number of dimes;

Q = number of quarters.

For step 2, what are the relationships among D, Q, and the numbers? This time we get two equations:

$$D + Q = 7$$
$$10 \times D + 25 \times Q = 115$$

This second equation is not at all easy to create. One may have to ask several leading questions, for example, if you have two dimes, how many cents do you have? If you have four dimes, how many cents do you have? If you have D dimes, how many cents do you have?

A long silence may ensue. But with patience, the pupils will begin to see that to find the number of cents, given the number of dimes, one just multiplies by ten. A series of simpler problems will lead the pupils to this realization.

Solving these equations is again done intuitively, by querying the pupils for possible answers that satisfy both equations. For example, $D = 0$ and $Q = 7$ satisfies the first equation. Do these values for D and Q also satisfy the second equation? Pupils quickly recognize the process and can offer a variety of choices. Care should be taken to allow the pupils to determine which possibility is correct and to explain why.

Epilogue

We stress that the language of mathematics can be used to solve real problems; indeed, this is the reason it was invented. By using algebra, we can get a beautiful translation of stories into mathematical equations that would not be possible otherwise. As the pupils grow mathematically, more specificity is required in finding all solutions. A pupil who is well versed in constructing the models will find the rote algebraic manipulation, commonly considered boring by most students today, also interesting since she or he knows why such solutions are useful.

Mathematics is a rich and powerful language for precisely expressing ideas. The language of mathematics is a language of algebra. This language can and should be taught to elementary school pupils. Only by learning and using this language can they truly understand mathematics, not as an abstract subject but as a practical language for expressing and solving everyday problems.

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