1. The figure below consists of five congruent rectangles that together form one large rectangle. If the area of the large rectangle is 60 square centimeters, what is the perimeter of the large rectangle?



2. In the following list, each positive integer appears exactly seven times, and all integers occur in nondecreasing order. There are seven consecutive numbers in the list that add to 365. What is the middle number in this set of seven consecutive numbers?

1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 5, 5, . . .

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3. The integers a, b and c are all positive.

Given that a + b + c = 14 and 156a + 13 b + c = 873, find 100a + 10b + c.

4. Suppose each cell of the 3 × 3 grid shown below is filled with one of the digits 2, 3, 5, 6, or 7 so that the following conditions are met:

1. No two adjacent cells (horizontally or vertically) contain the same digit.

2. The three-digit number in the bottom row is equal to the sum of the three-digit number in the top row and the three-digit number in the middle row,

3. The three-digit number in the bottom row is as large as possible. What is the sum of all 9 digits entered in the cells?

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5. Positive integers x and y satisfy the equation x^3 - y^3 = 218. Find x + y.
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6. A six-digit positive integer is *sixish* if it has six different **non-zero** digits and the product of each pair of adjacent digits is divisible by 6. What is the greatest sixish six-digit positive integer?

- 7. What two-digit positive integer satisfies **exactly** three of the following criteria?
 - 1. The number is a perfect square.
 - 2. The number is the product of two distinct prime numbers.
 - 3. The number has two even digits.
 - 4. The number is less than 30.
 - 5. The number's tens digit and ones digit are the same.
 - 6. The number is divisible by 9.

8. ABCD is a square with area 36 square meters. E lies on AB and F lies on CD. AE:EB = CF:FD =1:2. G is the midpoint of BC. H is the intersection of AG with EF. Find the area of the quadrilateral CFHG.

9. The hexagon shown below has sides of length 40, 30, 30, 40, 30, and 30, and six interior angles each measuring 120°. If a *triangle unit* is the area of an equilateral triangle with sides of length 1, how many triangle units are in the area of the hexagon shown?



10. In a sequence of 1000 positive integers, the first number is 42, and each subsequent number is double the previous one. How many of the 1000 numbers in the sequence have odd tens digits?

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11. A point P lies between two parallel lines l and m. The perpendicular distances from P to l and m are 2 units and 3 units, respectively. Points Q and R lie on lines l and m, respectively, so that QR = 13, the area of Δ PQR is 27 square units, and the distance from P to R is greater than the distance from P to Q. What is the distance PR?

12. An integer is *repetitive* if it uses the same digit at least twice in a row in base ten. Thus 100, 110, 998 and 999 are the first two and last two repetitive three-digit integers. Find the sum of all the three digit repetitive integers. 13. The numbers 1, 2, ..., 8 are reordered into a sequence $x_1, x_2, ..., x_8$. How many possibilities are there for this sequence if the first two terms differ by at least 2, and the last two terms also differ by at least 2?

14. Trey and Quatro play a game where they repeatedly flip a fair coin until it either comes up heads three times in a row or tails four times in a row. Trey wins if the coin comes up heads three times in a row, and Quatro wins if it comes up tails four times in a row. What is the probability Trey wins the game?

15. A circuit is a path which has the same starting and ending point, but never visits any other point twice. How many circuits are there on the grid which start at A and follow a path to B along edges moving right or up only at each stage, and then returning to A moving along edges going left or down at each stage? [The direction in which a circuit is traversed matters, so that reversing a circuit gives a different circuit.]

