Making Mobius Magic

One of the most interesting "creations" to come from topology is the Mobius Strip. Despite its simple appearance and construction, this shape has a lot of complex things going on with it. It is described as a surface having only one side and one edge. The Mobius strip was "discovered" by two different mathematicians in the same year, independently. Both August Ferdinand Mobius, and Johann Listing are credited with the discovery in 1858, although there are indications that similar ideas have been around for several centuries: the Ouroboros, a mythical creature from Greek and Egyptian stories may have been a form of Mobius strip.

Since then, artists and mathematicians both have been fascinated with this simple, yet complex object, and incorporated it into their work. M.C. Escher has several examples of Mobius-type creations in his paintings.

Today, you will explore some of the more interesting properties of the Mobius strip.

You will need:
- several strips of paper
- a pair of scissors
- some tape
- lots of imagination ;)

I. Lets first start by creating a Mobius strip. Take a strip of paper, put a ½ twist into it, at attach the ends together with a piece of tape.
That's it. The Mobius Strip.

Topologists often talk about the properties of the spaces they work with. List some of the things you notice initially about the Mobius strip.

II. For comparison purposes, lets create a simple hoop or ring: without twisting the paper strip in any way, tape the two ends of the paper together. What properties do you notice about this space? Compare it with the Mobius Strip.

Locally, (in topology that means "on a small scale") these two objects are essentially identical, but when looked at as a whole, their differences stand out.

III. One property of the Mobius strip is that is has only one edge, or boundary. To see this, mark a point near one edge of the Mobius strip, then with your finger, trace along the outside edge of the paper. What happens?

IV. What happens when you try to do the same thing with the hoop? How many edges or boundaries does it have?

V. The Mobius strip also has only one side. Again, take your pencil, and place it on a spot in the center of the mobius strip. Without picking up your pencil, draw a line down the center of the strip, moving the mobius strip along like a conveyor belt. (You may need assistance from a friend to do this.) What happens? Try the same thing with the simple hoop and compare the results.

Now, this is all good and interesting, but the real fun of the Mobius strip comes from what happens when we start cutting it.

VI. If we were to cut the simple hoop down the center line you drew, just about anyone could predict what will happen – we will get two objects, similar in shape to the original, both a little bit thinner, however. The Mobius strip, on the other hand, is a different beast entirely. What do you think will happen if you cut the Mobius strip down the center line you drew? After making your prediction, go ahead and try it. What do you get as a result?

VII. Look carefully at the object you made. Is it a Mobius strip? If you are not sure, try tracing a line down the center and seeing what happens. What did you get, cutting the mobius strip? What do you think will happen if you cut this new hoop down the middle? After guessing, try it. Describe the result.

VIII. There is something special about the number 2 (or the number ½) and the Mobius strip. What do you think will happen if instead of cutting it in half, we try cutting it in *thirds*? Make a new mobius strip, and this time, instead of cutting it down the middle, pick one "edge" and try to stay about a third of the distance across the strip from that edge as you cut. What happens? Did you get the same thing, or something different?

XI. Now, we know about hoops with no twists, and have explored those with both one twist, and two twists (what you got from cutting the Mobius strip in half had two twists). What about three twists? Four? What do you think happens with cutting those shapes? Even if you do not try any others physically yourself, try the one with three twists (also called the 1 ½ twist loop). Cutting this down the center yields a very interesting shape – a trefoil knot, having 8 twists.

This shape is actually topologically equivalent to the torus but requires a fourth dimension in order to wrap/unwrap it.