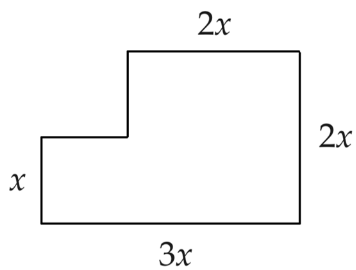
1. In the polygon shown below, every angle is a right angle. Four consecutive sides of the polygon have length x, 3x, 2x, and 2x as shown. If the area of the polygon is 180, what is the value of x?



2. One gallon is equal to four quarts, one quart is equal to two pints, and one pint is equal to two cups. If we have an endless number of gallon containers, quart containers, and pint containers, what is the smallest total number of containers needed to hold exactly 90 cups of water, assuming that all containers used must be completely filled?

3. A tortoise walks around a track that is exactly $\frac{1}{4}$ mile long at a constant speed of $\frac{1}{5}$ mile per hour. If the tortoise starts walking at noon on Monday and continues walking until noon the next day, then at noon the next day it is in the middle of its Nth lap around the track. What is the value of N?

4. A seventh-grade class goes on a field trip to an amusement park. Some of the students bring their own lunches, but most of the students buy their lunch from the concession stand at the park. The concession stand offers only two food items: a large slice of cheese pizza for $4.99, and a large slice of pepperoni pizza for $6.99. Both prices include tax. If the class spends a total of $130.77 on pizza slices, how many slices of pepperoni pizza did the class order?

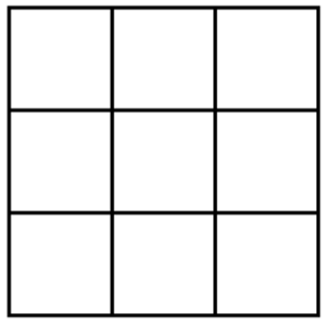
5. If the numbers 2.75 and $2.\overline{3} = 2.333\ldots$ are multiplied, the product is p. If p is expressed in decimal form, what is the sum of the first 20 digits to the right of the decimal point of p?

6. If k is an integer such that $10^{19} < 5^{k} < 10^{20}$, what is the value of k?

7. In right triangle ΔABC with $AC = 7$, $BC = 24$, and $AB = 25$, a circle of radius r is drawn centered at C, and a circle of radius s is drawn centered at a point D on hypotenuse AB. The two circles are externally tangent to each other, and the circle of radius s is also tangent to side BC.
If $r = \frac{2}{3}s$, what is the value of r? Express your answer as a fraction in simplest form.

8. The four-digit positive integer ABCD is a perfect square. The sum of the four digits is 18, and when we separate ABCD into two two-digit numbers AB and CD, both two-digit numbers are divisible by 7. What is the four-digit positive integer ABCD?

9. Each cell of the 3 x 3 array below is filled at random with one of the numbers 0 or 1. What is the probability that the product of the four numbers in every 2 x2 square is 0? (There are four such 2 x 2 squares.) Express your answer as a fraction in simplest form.



10. It is possible to write the number 1813 as the sum of a four-digit positive integer, a three-digit positive integer, and a two-digit positive integer so that the following conditions are satisfied:

• The four-digit number is divisible by the three-digit number, which is divisible by the two-digit number.

• The four-digit number, three-digit number, and two-digit number use a total of nine different digits.

What is the three-digit number?

11. In isosceles trapezoid ABCD with parallel bases AB and DC, side AD is perpendicular to diagonal DB. [The trapezoid is isosceles means that the perpendicular bisector of AB is a line of symmetry for the figure.] The perpendicular from vertex C to diagonal DB intersects the diagonal at a point E such that CE = EB = 6. What is the area of trapezoid ABCD? Express your answer in simplest radical form.

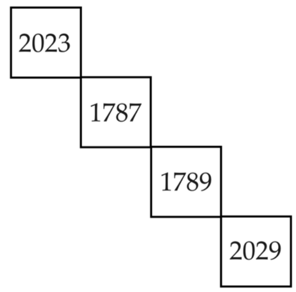
12. The letters A and B represent different digits.
The 5 digit integer AB5AB = x^3-3 x^2 for some two digit integer x.
Find x.

13. All ten distinct digits from 0 to 9 are arranged to form two five-digit positive integers. The difference between the two five-digit numbers is 32017. What is the greatest possible value of the larger five-digit number?

14. A jar contains 30 marbles, identical apart from color. 10 are red, 10 are blue and 10 are green. Judy puts her hand in the jar and takes out two marbles at random, placing them in her pocket without looking at their colors. An hour later she takes one marble out of her pocket and sees it is green, and puts it back in her pocket with the other marble. An hour later she again takes a marble from her pocket (it could be the same marble as the first time or the other marble), looks at it and sees it is green. What is the probability that both of the marbles she took from the bag are green?

15. Max builds an infinite array of numbers by placing a 0 in one cell, then moving to the right and placing a 1, then moving up and placing a 2, and continuing in this manner in a spiral pattern as shown, counting up by 1 each time.

64	63	62	61	60	59	58	57	56
65	36	35	34	33	32	31	30	55
66	37	16	15	14	13	12	29	54
67	38	17	4	3	2	11	28	53
68	39	18	5	0	1	10	27	52
69	40	19	6	7	8	9	26	51
70	41	20	21	22	23	24	25	50
71	42	43	44	45	46	47	48	49
72	73	74	75	76	77	78	79	80



That night, while Max is asleep, a gremlin changes the array by picking an integer k and adding that integer to every number in the array. The next morning, Max looks along a diagonal of the array and sees the numbers 2023, 1787, 1789, and 2029, as shown on the right above. What was the value of k?