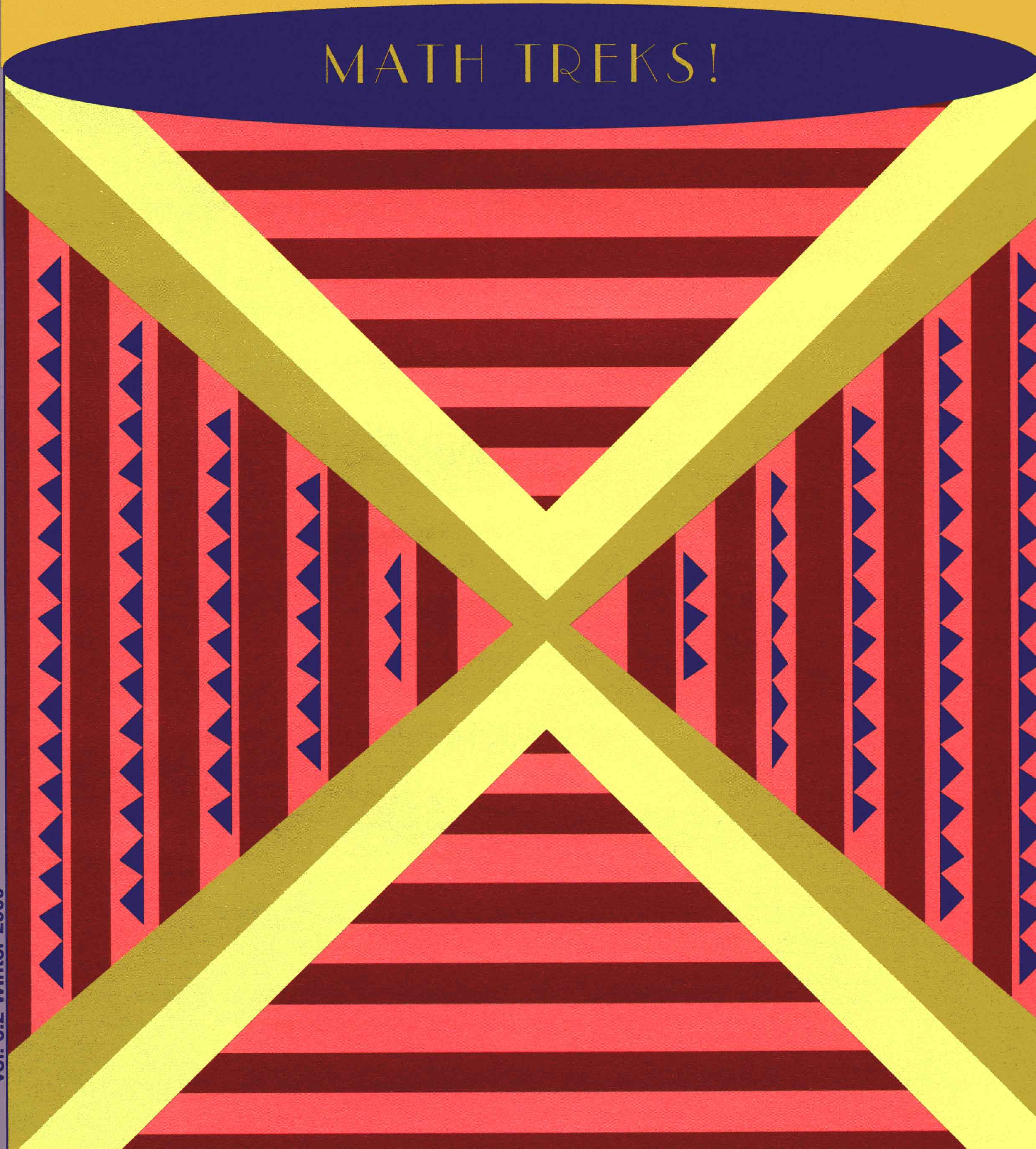
Math Reader



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Math Reader

Contents

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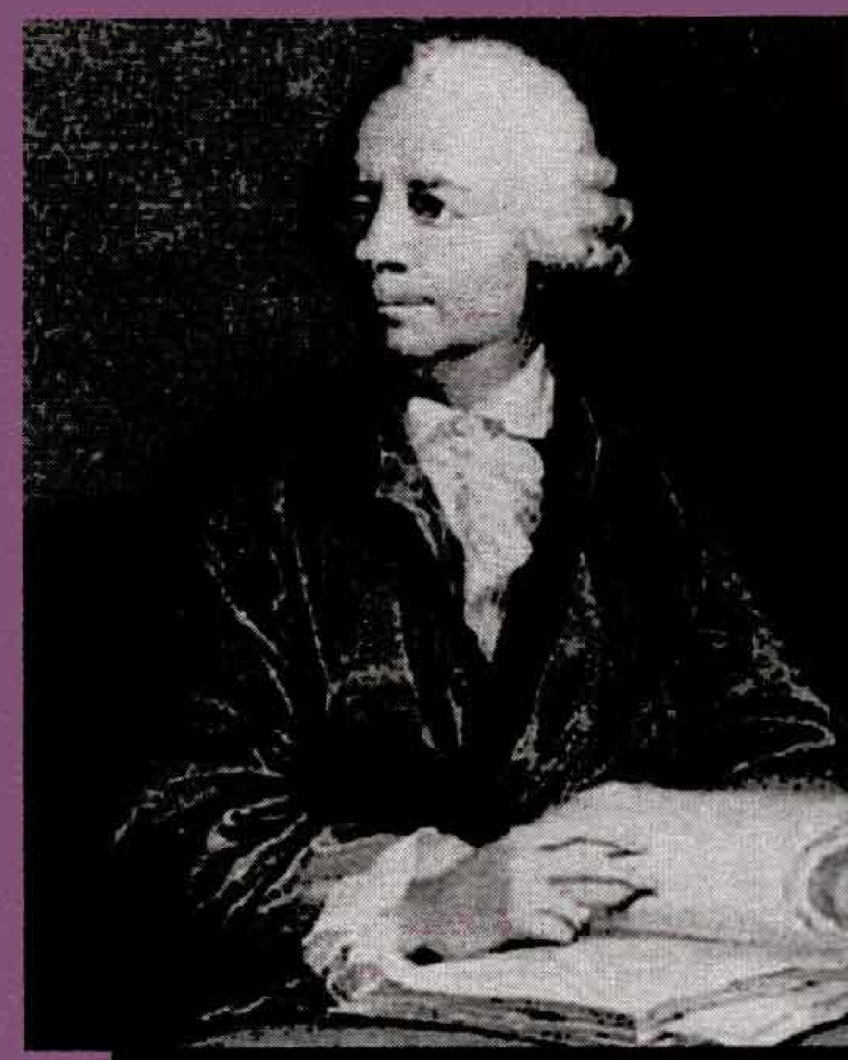
SWT

Leonhard Euler

By Hiroko K. Warshauer



Mathematicians communicate the results of their work in articles, which they publish and share with other mathematicians. One of the most prolific writers the world of mathematics has ever known is Leonhard Euler. Euler was born in Basel, Switzerland in April 1707. Switzerland was also home to another mathematical family, the famous Bernoulli's, with whom Leonhard's father was acquainted. It was Johann Bernoulli who discovered Leonhard's interest and potential in mathematics at an early age, and he took time to discuss mathematical ideas with him often. Euler completed his studies at the University of Basel and was awarded an academic position at the St. Petersburg Academy of Science in Russia at the age of 19. While at his post, he was surrounded by other mathematicians and made many mathematical discoveries.

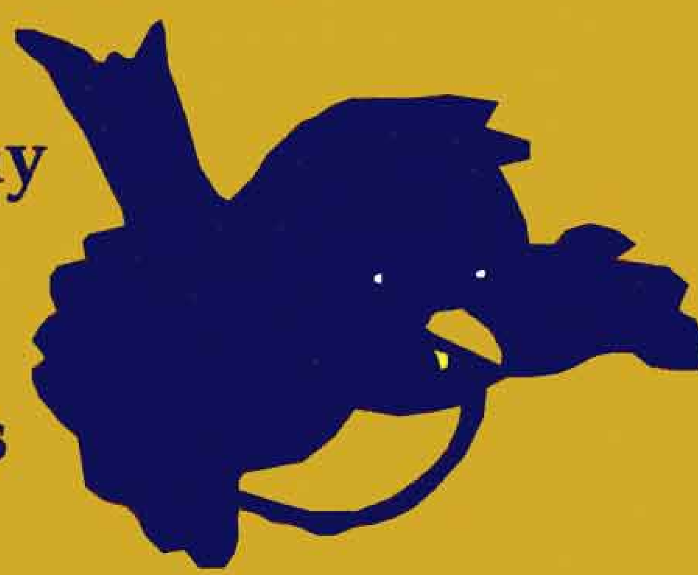
In 1741 Euler left Russia, which at that time was in political turmoil, and accepted a position in Berlin, Germany where he would remain for 25 years. Euler returned to St. Petersburg and by 1771 the gradually deteriorating eyesight of many years left him totally blind. Despite his lack of eyesight, Euler continued to work on mathematics and other areas of scientific interest. With remarkable memory and help from his sons and other colleagues at the Academy, Euler produced almost half of his total works after his blindness struck. Euler lived to the age of 76 and he died in Russia in 1783.

The world of mathematics owes to Euler many concepts and symbols that are still used today. Some of them even bear his name: the **Euler's**

(continued on p.7)

1. Each letter stands for one of the digits 0,1,2,…9.
What do X, Y and Z stand for?

$$\begin{array}{r} X\ Y \\ +\ \ \ X \\ \hline Y\ Z\ Z \end{array}$$



2. If five artists can paint five paintings in five weeks, how many paintings can ten artists paint in ten weeks?

3. How many different four-digit numbers have 3 as their thousands digit, 4 as their units digit and can be evenly divided by 3?

4. What is the smallest whole number that is divisible by at least half of the numbers in the following list?
5, 6, 7, 8, 9, 10

5. Mr. Finch puts birdseed out for wild birds. He left a barrel with 100 pounds of bird seed out without a lid. Blue jays came and ate 10% of the birdseed. A little while later, wrens came and ate 10% of what was left. How much bird seed was left?

6. Which fraction is greater: 13/19 or 7/10?

7. A drama class is planning to perform a play. There are 6 roles to cast and only 6 students (3 boys and 3 girls) who have volunteered to be in the play. There are 2 female roles and 2 male roles and 2 roles where gender does not matter. How many different ways can the play be cast?

8. How many 3-digit numbers are there that are not divisible by 7? How many 3-digit odd numbers are not divisible by 7?

9. We could join four 1 foot x 1 foot tiles along edges to cover a connected area with perimeter 8 or perimeter 10. Do you see how? What perimeters could you obtain for connected areas covered by 5 of these tiles? What about 6 tiles? Can you guess what perimeters are possible for 20 tiles?

Send us your solutions! Every month, we will publish the best solutions on our website: **www.mathexplorer.com**. If we print your solutions, we will send you and your teacher free **Math Reader** pens!

Ingenuity: If four lines are drawn on a sheet of paper, so that each line intersects each of the other lines and no three lines intersect at the same point, then 4 triangles are formed. Do this and find the 4 triangles. How many triangles are formed if you use 5 lines? 6 lines? Can you find and explain any pattern to the answers?

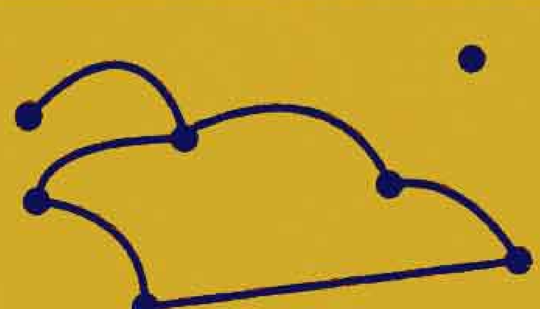
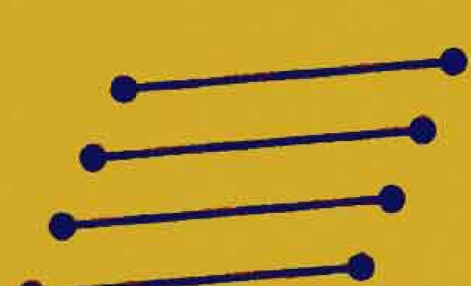
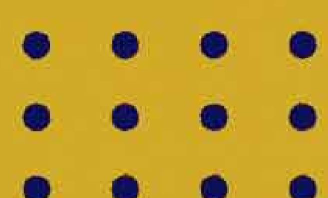
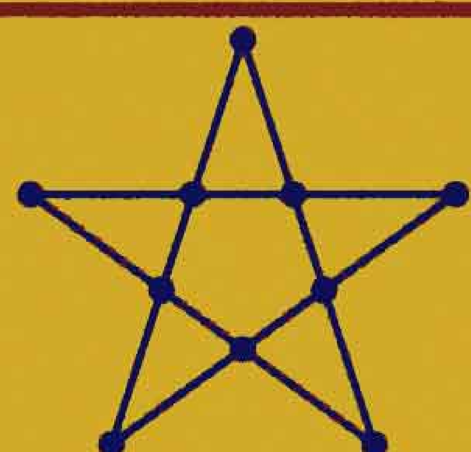
Doodling Around With Euler

by Kevin S. Jones

When most people hear the word "math" the first thing they think of is working with numbers. That's true, but it turns out that working with numbers is only a tiny bit of what math is. It's like when you hear the word "grocery store" and you think of bread and peanut butter: yes it's there, yes it's important, but there is so much more!

Perhaps the best way to describe mathematics is by looking at different kinds of patterns. Let's go down another aisle in the mathematical supermarket and look at some of the "groceries" on the shelf.
Mathematicians and Math Readers can find patterns anywhere, even when doodling. For example, put down some points with your pencil on a clean sheet of paper. Connect the points however you like with sides (sometimes called **edges**), and the condition that two sides have to meet at a point. Count the number of **points**, the number of **sides**, the number of **areas** created and the number of **groups** of points that are connected to each other. Below are some examples.

The first example has seven points, six sides, one area that is surrounded by sides and two groups of points: six points that are connected make one

Graph	Points (P)	Sides (S)	Areas (A)	Group (G)
	7	6	1	2
	6	5	4	5
	8	4	0	4
	12	0	0	12
	10	15	6	1

group, and the single point makes another group by itself.

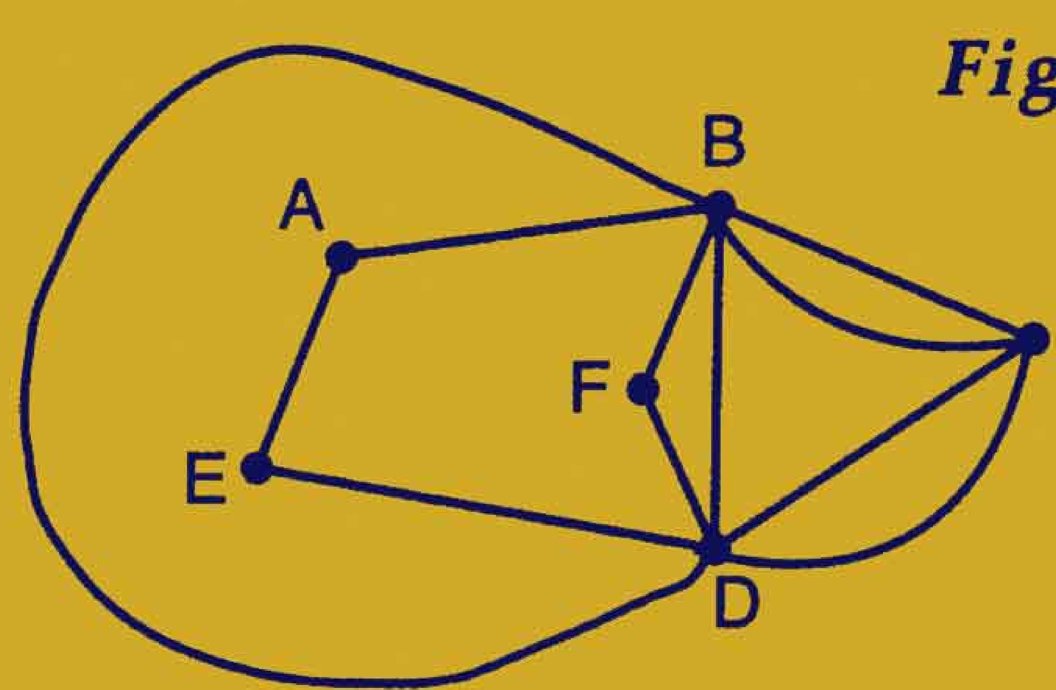
It may seem hard to believe, but it turns out there is a pattern to the number of points, sides, areas and groups, regardless of how many points and how you connect those points with sides. This pattern was discovered by the Swiss mathematician Leonard Euler (pronounced "oiler") over 200 years ago: the number of points plus the number of areas is always equal to the number of sides plus the number of groups.

$$P + A = S + G.$$

In the first example notice that 7 + 1 = 6 + 2. This pattern works every single time! Check out the other examples. Try doing two (or more if you like) for practice below.

There is another pattern in points and sides that is surprising: suppose you put down some points and connect them with one or more sides, making one group of points. For example:

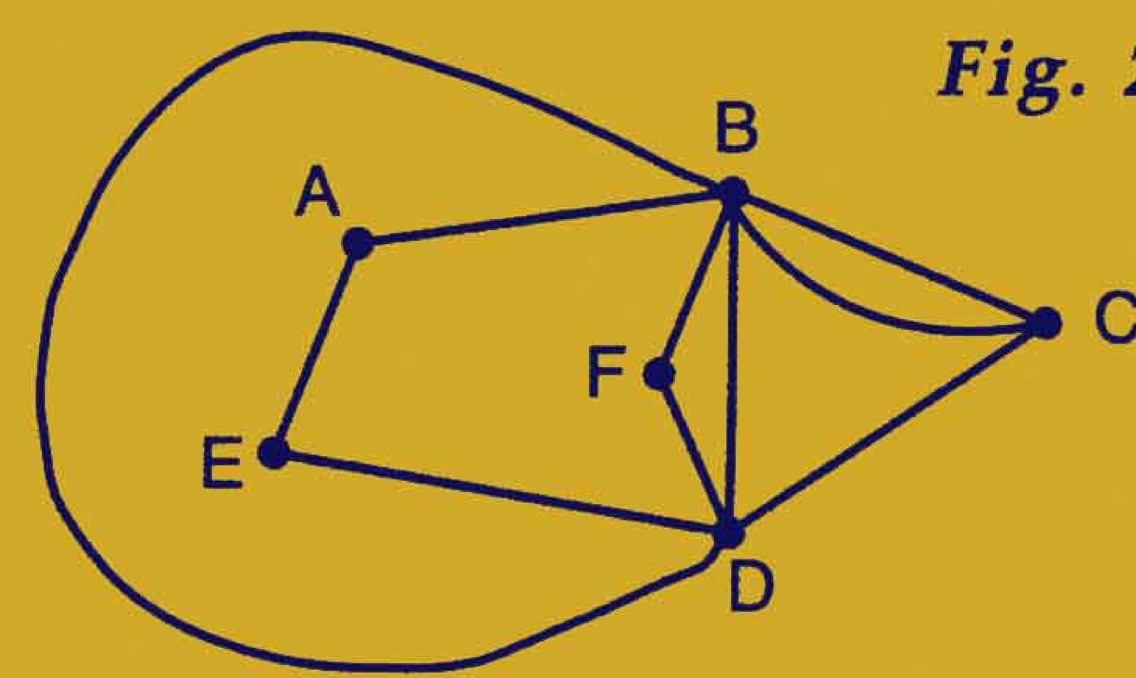
Fig. 1



A question you might ask is: can you start at a point (say, point A) and be able to trace over the graph, covering every side, without lifting your pencil or backtracking? It's OK to visit a particular point more than once, but a side must be "walked down" exactly once. If you can, it is called an **Euler Circuit**, named (once again) after Leonhard Euler. Try it on the graph in Figure 1, starting at A. You should be able to figure it out pretty quickly.

Now try the same thing on the graph in Figure 2, which is the same as the graph in Figure 1 except that there is a single side deleted:

Fig. 2



No matter what you try, you won't be able to do it you'll either miss a side, or you'll have to go over at least one side more than once. In fact, it is impossible!

The reason is simple: to go over each side exactly once, you have to pass through the points. Every time you go into a point on a side you have to leave the point along another side; therefore, a graph which is one connected group of points has an Euler circuit only if every point has an even number of sides connected to it. If you count the sides which go into each point of Figure 1, you'll see that every point has an even number of sides connected to it. If you do the same for Figure 2, you'll find that points D and C have an odd number of sides and so there cannot be an Euler Circuit.

Euler circuits have real-life applications. For example, suppose you were a postal worker delivering mail or the driver of a garbage truck picking up garbage down a number of streets. You would want to find the most efficient way to cover all the streets on your job with as little backtracking as possible. You could use the doodles that a famous mathematician figured out many years ago to help solve your problem.

Graph	P	S	A	G

Kevin Jones teaches mathematics at Southwest Texas State University.

Puzzle Page

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Word Search

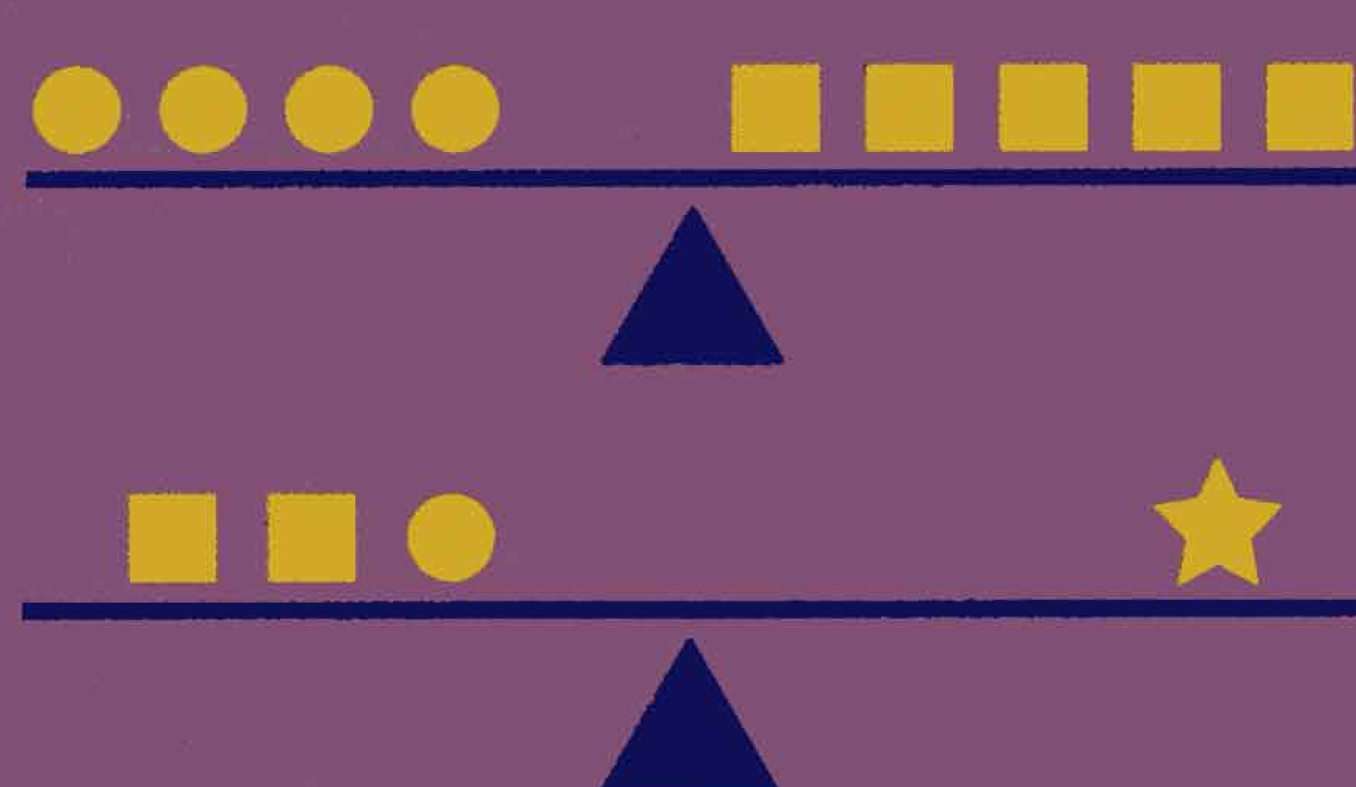
Forwards or backwards, up, slanted, or down.
Where can the words in this puzzle be found?

Euler
Graph
Edges
Circuit
Function
Bernoulli
Vertices
Pentagram
Diagonal
Sides

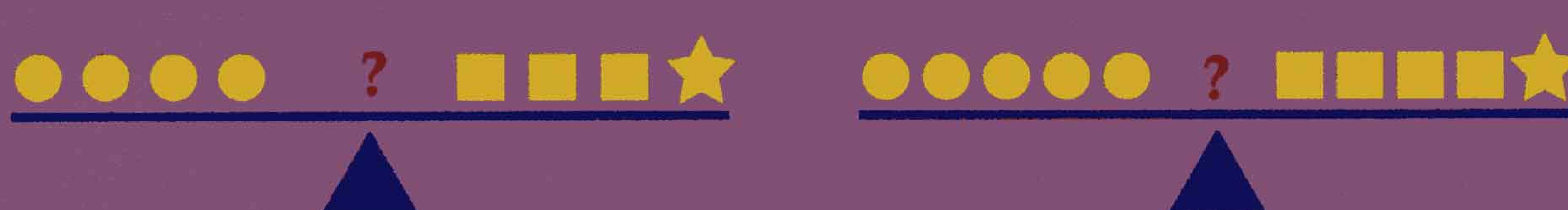
I	A	U	A	E	N	O	I	T	C	N	U	F	B
P	S	B	M	W	N	Q	L	P	H	U	P	C	B
E	B	C	X	D	O	T	L	S	E	D	P	U	B
N	U	T	O	M	T	Y	A	P	P	S	D	D	E
T	M	I	C	B	G	M	V	G	S	V	E	U	R
A	B	U	E	U	L	E	R	X	R	L	E	Y	N
G	U	C	O	O	R	G	D	Z	F	A	K	E	O
R	D	R	H	T	A	K	T	C	T	N	M	D	U
A	M	I	I	Y	C	V	K	K	M	O	J	T	L
M	O	C	O	T	S	T	D	S	E	G	D	E	L
U	E	Y	H	U	E	P	Q	T	J	A	B	T	I
S	U	U	G	S	I	D	E	S	A	I	S	U	F
U	H	D	T	D	V	H	E	U	H	D	Y	X	M
E	D	E	H	P	A	R	G	Z	S	Z	S	U	C

Each letter below represents a different digit. What numbers make this problem true?

$$\begin{array}{r} \text{TWO} \\ +\text{TWO} \\ \hline \text{FIVE} \end{array}$$



If the scales to the left are balanced, which side is heavier in the scales below? Why?



Bulletin Board

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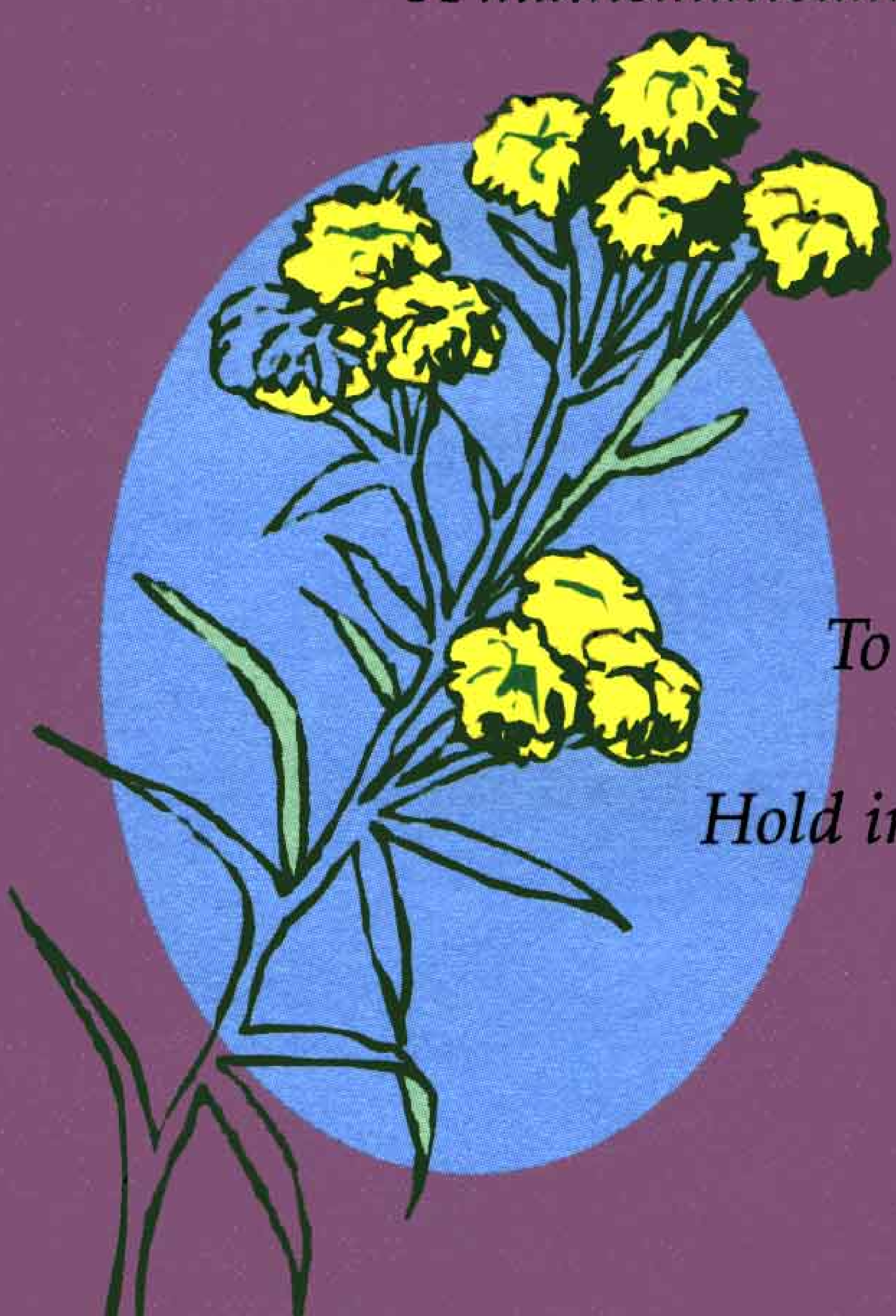
Mathcounts

Mathcounts is a national math coaching and competition program that promotes middle school mathematics achievement. For more info look up *www.mathcounts.org*

Words of Wisdom

"A mathematician, like a painter or a poet, is a maker of patterns."

-- G. H. Hardy



Imagine Infinity

To see the world in a grain of sand.
And heaven in a wildflower:
Hold infinity in the palm of your hand,
And eternity in an hour.

-- William Blake

Euler (cont'd)

formula we refer to in this issue's main article, **Euler's phi function**, $\phi(x)$, which occurs in number theory and a number, γ , called **Euler's constant**. Another constant, *e*, does not use his whole name, only the first letter of Euler. In addition to mathematics, Euler made substantial contributions to the areas of physics, cartography, navigation, the motion of the moon, and even music.

Reference: http://www-groups.dcs.st-andrews.ac.uk/~history/Mathematicians/Euler.html

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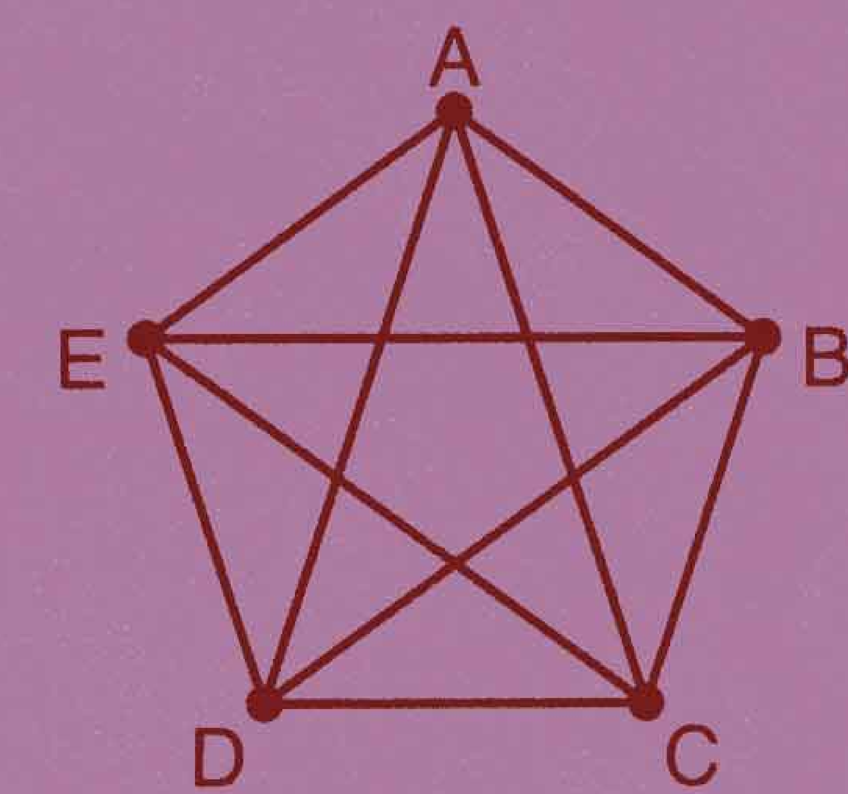
MATH ODYSSEY

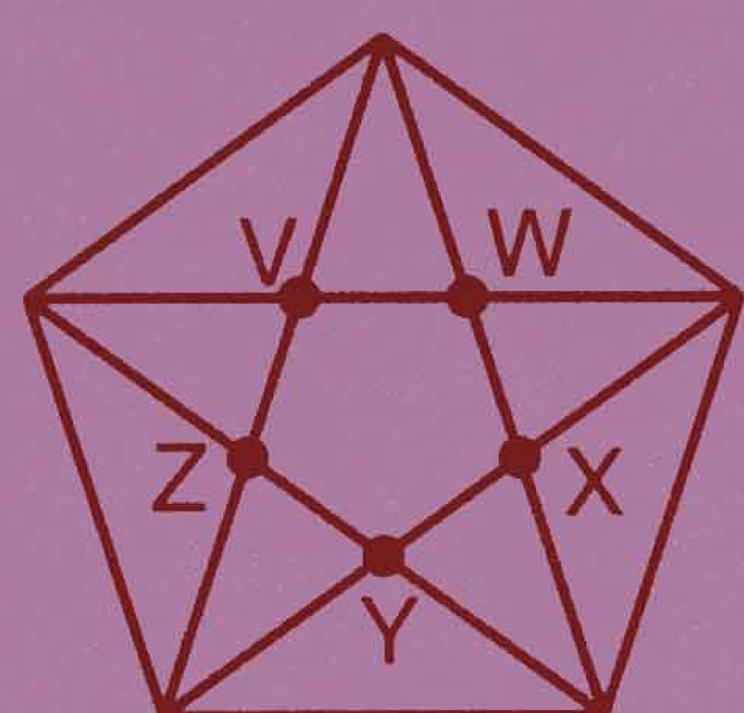
by Hiroko Warshauer

Triangles and Pentagons

The figure to the right is a pentagon ABCDE. Notice the five vertices and the five sides, with all the diagonals drawn in. The diagonals connect vertices that are not next to each other. AC is one such diagonal. How many different diagonals are in a pentagon?



Notice that the diagonals form a five pointed star. If we label the intersection of the diagonals VWXYZ, then there is a inner pentagon as well.



Pythagoras of Samos (c 580-500 BC), the famous Greek mathematician, used this shape, called the **star pentagon** or **pentagram,** as a symbol for the school of mathematics and philosophy that he founded. Take a close look at the pentagram and see if you can answer some of these questions.

1. If Euler starts at point E, is it possible for him to draw the shape without lifting up his pencil? Try it and see.

2. How many triangles can you find? (warning: There are a lot!)

3. What if you draw in the diagonals to pentagon VWXYZ? Do you see the tiny inner pentagon? Do you notice a pattern? If we continue this process, how many pentagons can we construct? (note, we may not be able to see them all!)

Math explorations take us on many different paths. In our winter issue of Math Reader, we take a closer look at how points on flat and curved surfaces connect. Graph theory and topology are some of the most fascinating areas of mathematics. We hope this is just the beginning of your exploration.

Happy Holidays and Good Wishes for the year 29 x 23 x 3.

Sincerely,

Hiroko K. Warshauer

Hiroko K. Warshauer, editor