2024 Mathworks Math Contest

1. The seventh grade class at Katherine Johnson Middle School has exactly 400 students. All 400 students were polled and asked how many pets they have. Results are shown in the table; the percentages shown are exact.

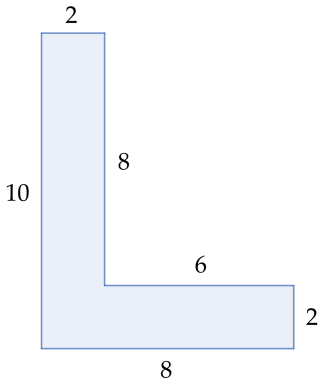
Number of pets	**Percent of students**
0	35%
1	40%
2	20%
3	5%

Assuming no two seventh graders have any pets in common, how many pets in total does the seventh grade class have?

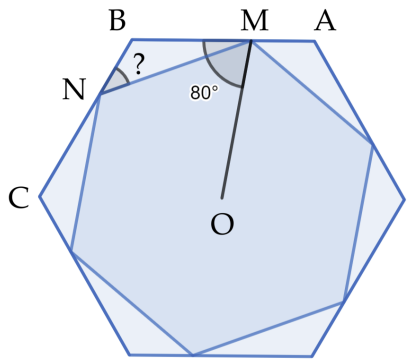
2. Max fills in each of the empty boxes in the figure with a number. When he is done, every box except for the two leftmost boxes contains the sum of the numbers in the two boxes immediately to its left. What number does Max place in the box immediately to the right of the 3?

3			19		49

3. In the L-shaped polygon shown, each consecutive pair of sides meets at a right angle, and the lengths of the sides in centimeters are 10, 8, 2, 6, 8, and 2, in that order. What is the area in square centimeters in the L-shaped polygon?



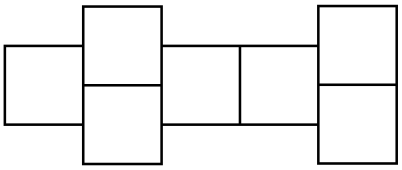
4. Shown are two regular hexagons, with all vertices of the inner hexagon lying on the sides of the outer hexagon. Let A, B, and C be consecutive vertices of the outer hexagon, and let M be the vertex of the inner hexagon that lies on $\overline{AB}$. The segment $\overline{OM}$ from the center O of both hexagons intersects side $\overline{AB}$ so that $m\angle OMB = 80^\circ$. If N is the vertex of the inner hexagon that lies on $\overline{BC}$, what is the degree measure of $\angle BNM$?

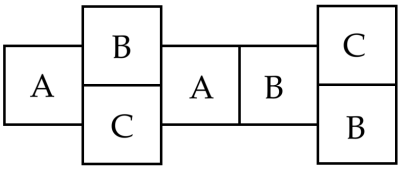


5. Find the largest possible value for the sum $b + br + br^2 + br^3 + br^4$, given that b and r are positive integers, and br^3 is a two-digit number while br^4 is a three-digit number.

6. The reflection of the point $(2, 10)$ across the line $y = x + 6$ in the coordinate plane is the point Q, and the reflection of the point Q across the line $y = x + k$ (for some number k) is the point $(15, -3)$. What is the value of k?

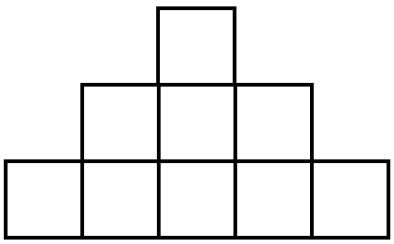
7. Each of the seven boxes in the figure is to be filled with one of the letters A, B, or C. A box is *gregarious* if the letter it contains, together with the letters in the neighboring boxes, include all three of the letters A, B, and C. For example, in the example on the right, the box in the middle containing an A is gregarious, since it contains an A and its neighbors contain at least one each of B and C, but the box on the lower right corner is not gregarious, since it neither contains nor neighbors an A. How many ways are there to fill the boxes so that all seven boxes are gregarious?





8. If a, b, c, and d are positive integers such that $a + b + c + d = 24$, $ab + cd = 77$, $abcd = 360$, and $ab > cd$, then what is the value of $ab - cd$?

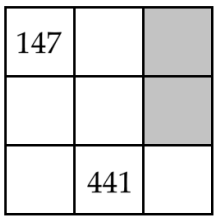
9. Trey wants to number the squares in the figure below with the integers 1 through 9, putting one number in each square and using each number exactly once, so that the number in the top square is divisible by 3, the sum of the three numbers in the middle row is divisible by 3, and the sum of the five numbers in the bottom row is divisible by 3. How many ways are there for Trey to do this?



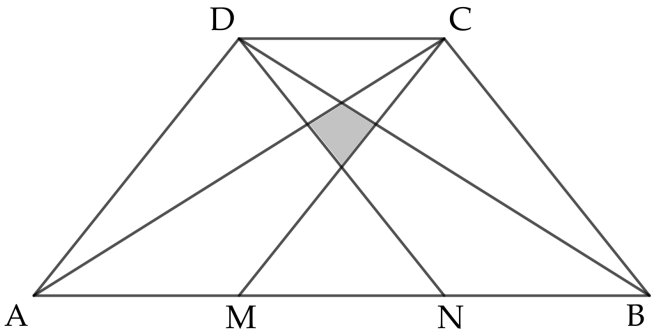
10. The product of a positive integer with exactly eight positive divisors and a positive integer with exactly twelve positive divisors is a perfect square. What is the greatest possible number of positive divisors this perfect square has?

11. Praise plays a game in which she begins by writing down the number 1. In each round of the game, she tosses a coin and rolls a standard die with faces labeled 1 through 6. If the coin comes up heads, she adds the number on the die to the number previously written down, and writes down the result. If the coin comes up tails, she multiplies the number on the die by the number previously written down, and writes down the result. What is the probability that after four rounds of the game (that is, four tosses each of the coin and the die), the last number Praise writes down is even? Express your answer as a fraction in lowest terms.

12. Consider a three-by-three square array in which two cells are already filled with the numbers 147 and 441 as shown. Suppose we fill each of the remaining cells with a positive integer so that the product of the three numbers on each row, on each column, and on each main diagonal is the same. If we do this so that the sum of the nine numbers in the array is as small as possible, what is the sum of the numbers in the two shaded cells on the right?



13. In trapezoid ABCD, sides $\overline{AB}$ and $\overline{DC}$ are parallel, and points M and N lie on side $\overline{AB}$ so that $AM = MN = NB = DC$. Segments are drawn from vertex C to points A and M, and from vertex D to points B and N as shown. These four segments contain the sides of a kite that lies in the interior of the trapezoid; this kite is shaded in the figure. What fraction of the area of trapezoid ABCD is the area of the shaded kite? Express your answer as a fraction in lowest terms.



14. The letters A, P, Q, R, and Z represent five distinct base-ten digits. If the product of the three-digit numbers APZ and AQZ is the five-digit number 3ARZ4, what is the value of $A \cdot (P + Q + R) \cdot Z$?

15. In right triangle ΔABC, $AC = 4$, $BC = 3$, and $AB = 5$. Three circles of equal radius r are inscribed in the triangle so that one circle is tangent to sides $\overline{AB}$ and $\overline{AC}$, one circle is tangent to sides $\overline{AB}$ and $\overline{BC}$, and one circle is tangent to side $\overline{AC}$ and to the other two circles. What is the value of r? Express your answer as a fraction in lowest terms.