2016 Mathworks Math Contest

Mathworks Math Contest
For Middle School Students
November 10, 2016

PROCTORING TEACHER COVER SHEET

- Please complete the following fields and return this cover sheet with ALL student exams
- Only one Teacher Cover Sheet is required
- Each student must fill out the student cover sheet

Proctoring Teacher
First Name: _____

Last Name:_____

E-mail Address: _____

Name of School:_____

Your students' scores will be sent to the e-mail address you provide above.

Please return all student exams so that we receive it by November 17th. Please mail to:

Mathworks - MMC
601 University Dr., ASBS #110
Texas State University
San Marcos, TX 78666

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Mathworks Math Contest (MMC)
For Middle School Students
November 10th, 2016

SCORE
(for Mathworks use)

STUDENT COVER SHEET

Please write in all information neatly and clearly to ensure proper grading. Thank you!

Student First Name:_____ Last Name:_____

Current Grade in School: _____

Home Address:_____

City:_____ State: _____ Zip: _____

Home Phone: (_____) _____E-mail Address: _____

School Name: _____

Check Math Courses Taken Or Currently Taking:
☐Pre-Algebra ☐Algebra 1 ☐Algebra 2 ☐Geometry

Student Birth Date (MM/DD/YYYY): _____ / _____/ _____

Gender: ☐Male ☐Female

Are you a U.S. Citizen or Permanent Resident? ☐Yes ☐No

Return Completed Test by November 17th to:
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601 University Dr., ASBS #110
Texas State University
San Marcos, TX 78666

Contest Directions

- Please write as neatly as possible
- We award points only if we can read your work!
- 15 problems in 120 minutes (2 hours)
- No calculators allowed. Use additional paper as needed
- Show all your work and how you obtained each answer
- **Please BOX your final answers**

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1. A turtle starts at the point (0, 0) in the coordinate plane and makes three kinds of moves: steps one unit to the right, steps one unit upward, and steps in a diagonal direction that moves him one unit to the right and one unit upward. The turtle makes a total of 23 moves and ends up at the point (12, 18). How many diagonal moves did the turtle make?

2. Find the largest eight-digit integer with two 1's, two 2's, two 3's and two 4's such that there are exactly four digits between the two 4's, exactly three digits between the two 3's, exactly two digits between the two 2's and exactly one digit between the two 1's.

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3. A concave pentagon ABCDE has its vertices at the points A = (3, 3), B =(3, -3), C = (-3, -3), D = (-3, 3), and E = (t, u), for some numbers t and u, where $-3 < t < 3$. If the area of pentagon ABCDE is 22 square units, what is the value of u?

4. If $(x^2 + y^2)/(x + y) = 2017$, find the value of the expression: $y - (2y^2/(x + y)) - x$.

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5. Marissa cuts 2016 regular pentagons, each with sides of length 2 inches, out of sheets of paper. She then cuts 2017 equilateral triangles, each with sides of length 2 inches, out of paper. She then creates a very large concave polygon by attaching all of the triangles and pentagons together in a chain, starting with a triangle, then attaching a pentagon to a side of the triangle, then attaching another triangle to a different side of the pentagon, then attaching another pentagon to a different side of the triangle, and so on, making sure that no two of the triangles and pentagons overlap. The triangles and pentagons alternate until the chain ends with the last triangle. What is the perimeter, in inches, of the large polygon Marissa creates?

6. In triangle ABC, M is the midpoint of AB.
MA=MC=4, BC=6.
Find AC.

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7. Erika runs laps around a circular track at a constant speed while wearing a digital watch that gives the time in hours, minutes, and seconds. She passes a trash can at 6:28:50, and then passes the same trash can on her next lap at 6:30:30. At this point, Erika's friend Frank starts walking around the track at a constant speed. Erika passes Frank for the first time at 6:31:10, and for the second time at 6:33:50. What is the ratio of Frank's walking speed to Erika's running speed?

8. An integer n is chosen at random from the range $1000 \leqslant n \leqslant 9{,}999$.
What is the probability that the product of its digits is even?

9. Five students take a test with ten multiple-choice questions. When the test is over, the five students have 9, 8, 7, 6, and 5 correct answers, respectively. If there were q questions on the test that a majority of the five students answered correctly, what is the least possible value of q?

10. Find the remainder when 2^{2000} is divided by 13.

11. Three different integers are selected from the set of integers 1 to 12 inclusive.
What is the probability that the sum is divisible by 3?

12. We say that a positive integer N is divisorous if the ones digits of the positive divisors of N include all of the base-ten digits from 0 to 9. What is the least positive integer that is divisorous?

13. A triangle has altitudes with lengths of 24, 24, and 20 units. What is the measure of the radius of its inscribed circle?

14. Gil is 17 years older than Sheila. If his age were written after hers, the result would be a four-digit perfect square. The same statement could be made 13 years from now. What is Sheila's present age?

15. Fernando has a 5x5 array of light bulbs, with all the bulbs initially turned off. Each light bulb has a switch; flipping the switch turns the light bulb on if it is off, and turns the light bulb off if it is on. Fernando chooses two rows of the array, and flips the switches of all of the light bulbs in the two rows. He then chooses two columns of the array, and flips the switches of all of the light bulbs in the two columns (including the ones for light bulbs that he turned on in the previous step). Finally, he chooses one of the two main diagonals of the square array, and flips the switches of all of the light bulbs on the diagonal. He then counts the number of light bulbs that are on. If L is the least possible number of light bulbs that are on and M is the greatest possible number of light bulbs that are on, what is the value of the product $L \cdot M$?