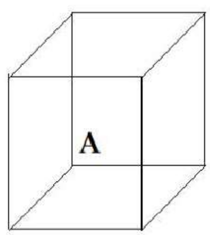
MCC Problems 2017:

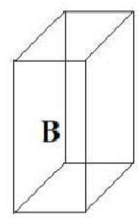
Problem 1. The following equation is false, but we can change it into a true equation by changing some of the addition signs to subtraction signs. What is the smallest number of addition signs that must be changed into subtraction signs in order for the equation to become true?

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 = 0$

Problem 2. The two containers shown below are right rectangular prisms. Container A has a base measuring 4 inches by 6 inches and is 6 inches tall. Container B has a 3-inch square base and is 6 inches tall. Eva fills container A with water to a height of 5 inches, and fills container B to a height of 2 inches. She then pumps water from container A into container B until the two containers are filled to exactly the same height. In inches, what is the height of the water in both containers when this process is completed?

Express your answer as a mixed number.





Problem 3. Mark draws a number line, marks the number 0, and puts his pencil there. He performs the following four steps in some order (not necessarily the one given):

He moves his pencil 2 units to the right, stops, and marks the number there.

He moves his pencil 3 units to the right, stops, and marks the number there.

He moves his pencil 4 units to the right, stops, and marks the number there.

He moves his pencil to the point halfway between the last two numbers he has marked, and marks the number there.

At the end of this process, Mark has marked four nonzero numbers. What is the greatest possible value of the product of these four numbers?

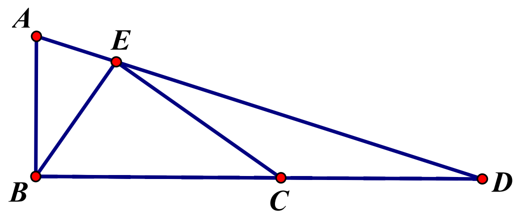
Problem 4. Maria creates a sequence of integers where the first and second terms are both 1. At each step she adds the last two terms she has obtained. If the sum is less than 9, then she enters that sum as her new term. Otherwise she subtracts 8 from the sum and records that.

Her sequence therefore starts 1, 1, 2, 3, 5, 8, 5, 5, . . .

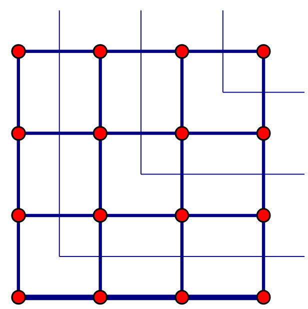
Find the 2020th term in the sequence.

Problem 5. Two spheres with radii of 9 units and 4 units lie on a table so that they touch each other. What is the distance between the two points where they touch the table?

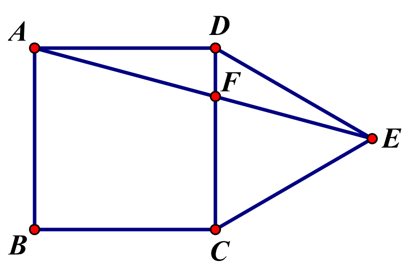
Problem 6. In the triangle ABD, E lies on AD and C lies on BD. Angles ABD and BEC are right angles. The segments AB and BE both have length 3. The length of BC is 5. Find the length of AD.



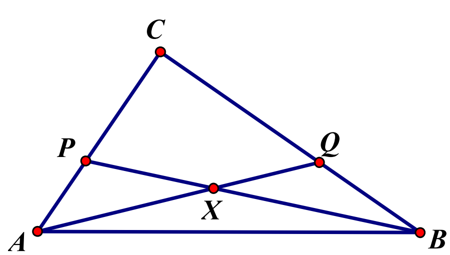
Problem 7. The figure below shows a 4 x 4 square array of dots. The L-shapes in the picture divide the array into four sets of dots, containing 1, 3, 5, and 7 dots, respectively. How many ways are there to choose exactly one dot from each of these sets so that no three dots chosen are collinear?



Problem 8. Figure ABCD is a square and figure CDE is an equilateral triangle. Segments AE and CD intersect at F. What is the degree measure of angle CFE?



Problem 9. In the diagram shown below, AC = 15, BC = 20, and angle ACB is a right angle. Points P and Q lie on sides AC and BC, respectively, so that AP = BQ = 7. Segments AQ and BP intersect at X. What is the positive difference between the areas of ΔAPX and ΔBQX?



Problem 10. Positive integers a, b and c satisfy the equations

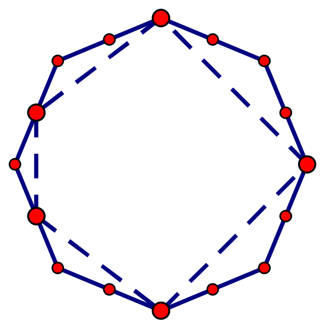
$$29^2 - a^2 = 28^2 - b^2 = 27^2 - c^2.$$

If a<20, what is the value of a+b+c?

Problem 11. A circle passes through one vertex of an equilateral triangle and is tangent to the opposite side at its midpoint. What is the ratio of the segments into which the circle divides one of the other sides if the ratio is less than 1?

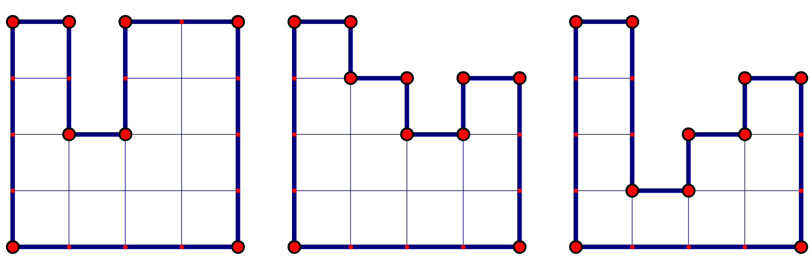
Problem 12. Jar A contains 50 balls, 15 are green and 35 are red. Jar B also contains 50 balls, 20 are green and 30 are red. Three balls are chosen at random from jar A and placed in jar B. Jar B is then turned and shaken to mix up the balls. Then a ball is chosen at random from jar B. What is the probability that it is red?

Problem 13. A regular octagon is shown in the picture below, with the vertices marked and midpoints of the sides marked, and a dashed inner polygon drawn. An *inner polygon* is a polygon formed by traversing the octagon in a clockwise manner, selecting some of the marked points as you go, ensuring that each side of the octagon contains exactly one selected point. Then each chosen vertex is connected to the next with a line segment, and the last is connected to the first to complete the inner polygon.
How many inner polygons does the regular octagon have?



Problem 14. A line through (4, 3) is tangent to the circle $x^2 + y^2 = 5$. Let (a, b) be the point of tangency so that the slope of the tangent is a minimum. What are the coordinates of (a, b)?

Problem 15. Three views of a stack of unit cubes are shown. What is the maximum number of cubes in the stack?



Top View, with the front side at the bottom of diagram. Front View. Right Side View.

In[1]:=
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answers = {4, "4 2/11", 2016, 3, 12, 3 √10, 60, "75 degrees",
   "35/2 or 17.5", 22, "1/3 or 1:3", 321 / 530, 47, "(-2/5, 11/5)", 30};
TableForm[Table[{i, answers[[i]]}, {i, 15}]]
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Out[1]//TableForm=

1	4
2	4 2/11
3	2016
4	3
5	12
6	$3\sqrt{10}$
7	60
8	75 degrees
9	35/2 or 17.5
10	22
11	1/3 or 1:3
12	$\frac{321}{530}$
13	47
14	(-2/5, 11/5)
15	30