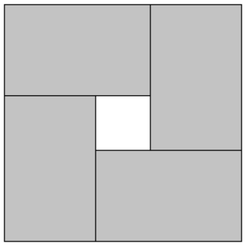
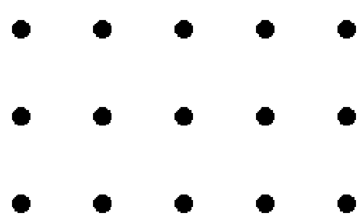
1. The diagram shown consists of a white square surrounded by four congruent shaded rectangles. The white square has area 36; each shaded rectangle has area 216. What is the perimeter of one of the shaded rectangles?



2. There are two single-digit positive integers a and b such that $a^b - b^a = a + b$. What is the value of $a \cdot b$?

3. Consider the set of all fractions that can be written using four distinct digits, with each digit used exactly once. Which of these fractions is closest to 1?

4. In the 3×5 array of points shown below, adjacent points are 1 unit apart. If we want to color each point in the array so that no three points on the same line have the same color, what is the minimum number of different colors needed to do this?



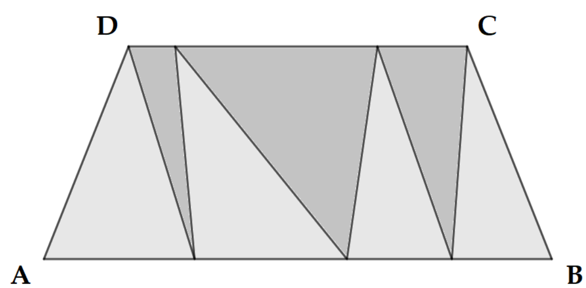
5. A 12-hour digital clock uses up to six digits to report the time: two for seconds, two for minutes, and one or two for hours. A *binary time* is a time on the clock that uses no digits other than 0 and 1; for example, 1:11:11 and 10:01:10 are both binary times. How many different binary times occur between 12:00 AM and 12:00 PM on the same day?

6. The numbers 7, 9, 11, and 13 are placed in the blanks in the expression below, using each number exactly once, so that the value of the expression is as large as possible. If the value of the expression is written as a fraction in simplest form, what is the numerator?

$$\left(\frac{1}{\square}+\frac{1}{\square}\right)\left(\frac{2}{\square}+\frac{2}{\square}\right)$$

7. There are two three-digit positive integers, both with tens digit 0, whose product is 328,042. What is the sum of these two three-digit numbers?

8. In trapezoid ABCD, AB $= 12$, CD $= 8$, and sides $\overline{AB}$ and $\overline{DC}$ are on parallel lines. The trapezoid is divided by some line segments into four light-gray triangles and three dark-gray triangles. If the area of trapezoid ABCD is 50 square units, what is the result (in square units) when the total area of the dark-gray triangles is subtracted from the total area of the light-gray triangles?



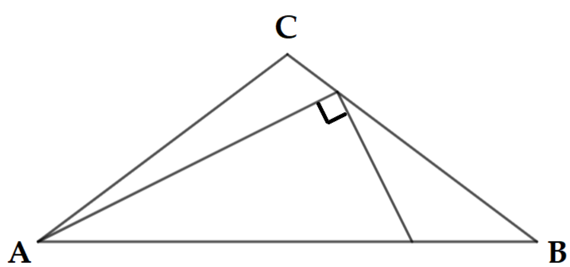
9. A positive integer is *supercool* if it has an odd number of even positive divisors. Find the sum of all supercool integers between 100 and 300.

10. An Olympic triathlon consists of a 1.5-kilometer swim, a 40-kilometer bike ride, and a 10-kilometer run. Cori bikes 3 times as fast as she runs, and runs 3 times as fast as she swims. Danielle bikes 2 times as fast as she runs, and runs 4 times as fast as she swims. Amazingly, Cori and Danielle finish an Olympic triathlon in exactly the same amount of time! What is the ratio of Cori's biking speed to Danielle's biking speed? Express your answer as a fraction in simplest form.

11. Assume that the ten variables in the expression below represent the ten integers from 1 to 10, in some order. What is the greatest possible **odd** value of the expression?

$$(ab + cd)(pqr + stu)$$

12. In triangle ΔABC, $AB = 16$ and $AC = BC = 10$. A right triangle is inscribed in ΔABC so that the longer leg of this right triangle is twice as long as the shorter leg, the hypotenuse lies on $\overline{AB}$ with one vertex at A, and the right-angled vertex lies on $\overline{BC}$. What is the length of the hypotenuse of this right triangle?



13. In the new American Jackpot Lottery, each state begins with a jackpot of 100 million dollars. At the end of each week there is a drawing for one winner in the United States. Immediately after the drawing, the winner wins their state's jackpot, that state's jackpot starts over at 100 million dollars, and every other state's jackpot goes up by 1 million dollars. After the 100^{th} drawing, the average of all 50 states' jackpots is 144.88 million dollars. In total, many dollars have been awarded in the first 100 drawings?

14. Find the number of possibilities for the sequence of numbers (a, b, c, d, e) if a, b, c, d and e are distinct positive single digit integers such that $a + b + c$ is a multiple of 3 and $c + d + e$ is a multiple of 3.

15. The regular octahedron shown below has six vertices, including one at the top and one at the bottom. How many different ways are there to travel along a sequence of seven edges, starting at the top vertex and ending at the bottom vertex, assuming that there are no restrictions on repeating edges or vertices?