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13th Primary Mathematics World Contest
Team Contest 2010

All the figures are not drawn to scale.)

Question 1:

There are two boxes of balls. The number of balls in box A is between 133 and 200.

Step 1	take the same number of balls as there are in box B from box A and put them into box B
Step 2	take the same number of balls as there are currently in box A from box B and put them into box A
Step 3	take the same number of balls as there are currently in box B from box A and put them into box B
Step 4	take the same number of balls as there are currently in box A from box B and put them into box A
Step 5	take the same number of balls as there are currently in box B from box A and put them into box B

After these five steps, there will be an equal number of balls in both boxes. How many balls were there in box A initially?

Question 2:

A line segment with a length of 2010 units is divided into three new line segments. The length of each segment is an integer. The three new line segments can form a triangle. What is the maximum difference between the longest side and the shortest side of any triangle formed?

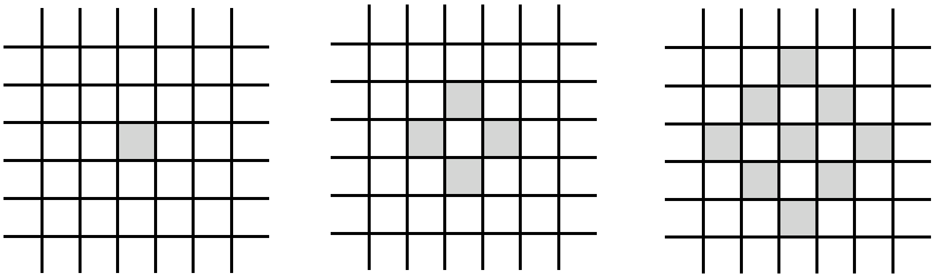
Question 3:

Only four teams A, B, C and D participated in 4 sports events. They were awarded 5, 3, 2 and 1 point for the 1st, 2nd, 3rd and 4th positions respectively. The total number of points for each team is the sum of the points for the four sports events. Given that the total number of points for the four teams are all different and team A is first for three of the sports events, find the largest possible total points for the team that was ranked last.

Question 4:

A worm lives in a rectangular nest with square cells. It starts in one cell and each new day it must move to a neighboring cell (a cell which shares a common side). It may move back to a cell it occupied on the previous day.

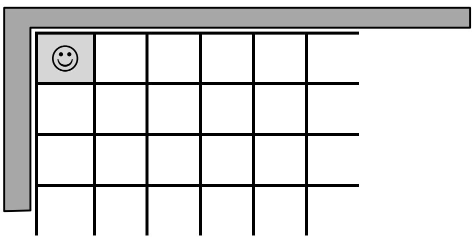
For example:



1st day 2nd day 3rd day

Note: The shaded cells are the possible cells where the worm may move to. On the second day, the worm could be in one of four possible cells. On the third day, the worm could be in one of nine possible cells.

If the worm starts in a corner cell on the first day as shown below, in how many different cells could the worm be on the 2010th day?



Question 5:

A machine changes the order of the cards that were put into it, in the same way every time. If 13 cards are fed into the machine in the order A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K , then the cards come out in the order 6, A, 7, 4, 2, 9, K, J, 10, Q, 8, 5, 3.

Original order

A	2	3	4	5	6	7	8	9	10	J	Q	K

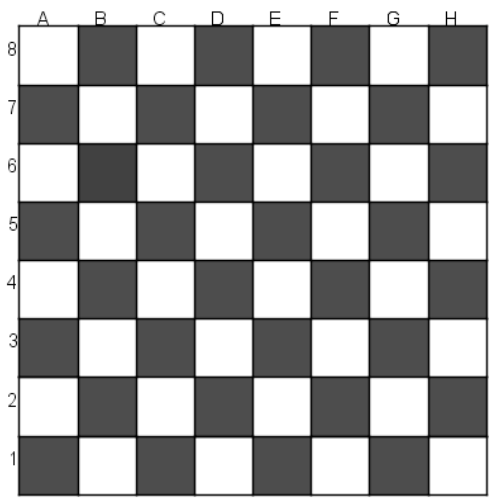
After one change

6	A	7	4	2	9	K	J	10	Q	8	5	3

How many times must the cards be fed into the machine so that they come out in their original order?

Question 6:

In how many ways can two identical coins be placed on the black squares of an eight by eight board if they cannot be arranged in the same row, same column or same diagonal?



Question 7:

Two numbers are written using the digits 1, 2, 3, 4, 5, 6, 7 and 8 so that each digit is used exactly once. One of the two numbers is a perfect square and the other number is a perfect cube of the same positive integer. Find these two numbers.

Question 8:

An animal shelter has twelve cages in two rows and six columns. Four dogs, four cats and four rats are in the shelter, one in each cage.

i. A cat is noisy if and only if it is in the same column as a rat.

ii. A rat is noisy if and only if it is immediately between two cats on the same row.

Iii. A dog is noisy if and only if it is immediately between a cat and a rat on the same row.

One night, there are six noisy cages, as shown by the dark circles in the diagram below.

		•	•		•
•	•	•			

What animal is in each of the twelve cages?

(Please fill in the table using **D** for Dog, **C** for Cat and **R** for Rat)

Question 9:

A "PLK" number is a counting number which has the following features. When 1 is added to it, the sum is a perfect square. When 1 is added to its half, the sum is another perfect square. For example, 48 is a "PLK" number since $48 + 1 = 49$ and $\frac{48}{2} + 1 = 25$, and both 49 and 25 are perfect squares. Find the next "PLK" number which is greater than 48.

Question 10:

Put the numbers from 2000 to 2010 into the boxes in the figure below. Each box consists of one number and each number must be used exactly once. The sum of the three numbers along each of the ten segments must be the same.