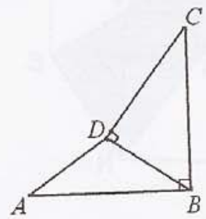
Po Leung Kuk
14^{th} Primary Mathematics World Contest
Individual Contest 2011

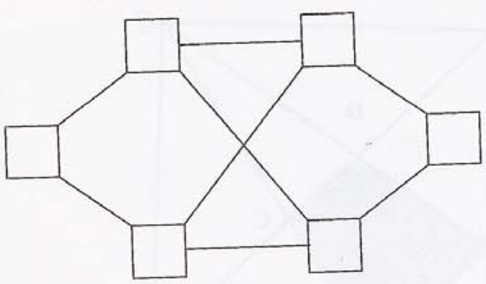
final

1. In the diagram below, $AB \perp BC$, $AB=BC$, $\angle BDC=90°$, BD=3cm and CD=5cm. Find the area of triangle $\triangle ABD$, in cm^2.



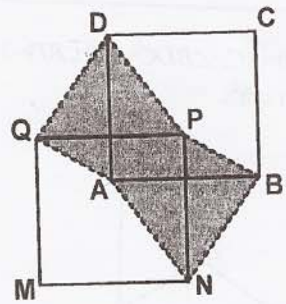
2. Five boys went fishing and caught 31 fish. The boy who caught the most number of fish has 3 times as many fish as the boy who caught the least number of fish. How many fish has the boy who caught the second largest number of fish if all of them caught a different number of fish?

3. In the diagram below, six boxes are joined by eight line segments. The numbers 1, 2, 3, 4, 5 and 6 are to be placed into the boxes without repetition and each box can only contain one number. At most how many line segments are there connecting two boxes with non-consecutive numbers?



4. Find the value of
$100 \times 99 - 99 \times 98 + 98 \times 97 - 97 \times 96 + \ldots + 4 \times 3 - 3 \times 2 + 2 \times 1$.

5. In the diagram below, squares $ABCD$ and $MNPQ$ have segments AB and PQ which are parallel and equal. Find the ratio of the area of the shaded part to the area of square $ABCD$.



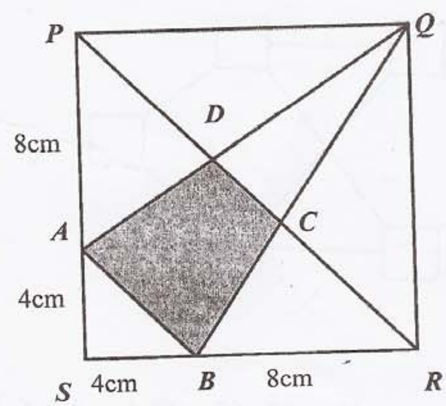
6. Three numbers 70, 98 and 143 are divided by a positive integer. If the sum of the three remainders is 29, find this positive integer.

7. The sum of 888 consecutive positive integers

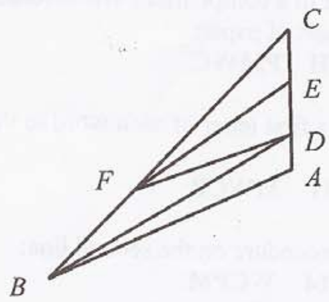
$$n + (n+1) + (n+2) + (n+3) + \ldots + (n+886) + (n+887)$$

is a perfect square. Find the smallest possible value of n.

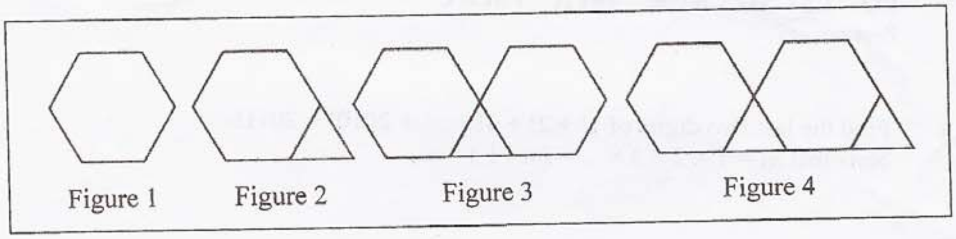
8. In the diagram below, square $PQRS$ has sides of 12 cm. If $AS = BS = 4$cm, $PA = BR = 8$cm, find the area of trapezoid $ABCD$, in cm^2.



9. ΔABC is divided by segments BD, DF and FE into four small triangles as shown in the diagram below. Those four small triangles have equal areas. If $BF = 2DE$, find the ratio of $AC : BC$.



10. The regular hexagons and the equilateral triangles, with each side 1 cm, are placed together as a polygon as shown in Figures 1, 2, 3, 4, … Find the outer perimeter of the polygon in Figure 2011, in cm.



11. What is the last digit of the sum of 7777^7 and 77^{777}?

12. There are three boxes of marbles. Each box contains a different number of marbles. From the first box, I remove $\frac{1}{3}$ of the number of marbles, from the second box, I remove $\frac{1}{4}$ of the number of marbles and from the third box, I remove $\frac{1}{5}$ of the number of marbles. Finally, there is an equal number of marbles remaining in all the three boxes. What is the smallest possible number of marbles which I may have removed in total?

13. Maria is preparing to take part in a competition. While relaxing, she wrote the following text on the blank sheet of paper:
PO LEUNG KUK 14TH PMWC

On the first line she moved the first letter of each word to the end of the word as follows:
OP EUNGL UKK 4TH1 MWCP

Then she repeated the same procedure on the second line:
PO UNGLE KKU TH14 WCPM

etc.

In what line will
PO LEUNG KUK 14TH PMWC
first occur?

14. Find the last two digits of $1! + 2! + 3! + \ldots + 2010! + 2011!$.
Note that $n! = 1 \times 2 \times 3 \times \ldots \times (n-1) \times n$.

15. Suppose $1 \times 2 \times 3 \times \ldots \times 2010 \times 2011 = 14^n \times A$, where n and A are both positive integers. What is the maximum value of n?