

## Homework 6.2.

1. A coaxial cable consists of two very long cylindrical tubes, separated by linear insulating material of magnetic susceptibility  $\chi_m$ . A current  $I$  flows down the inner conductor and returns along the outer one; in each case the current distributes itself uniformly over the surface of each conductor.
  - a. Find the magnetic H-field in the region between the tubes.
  - b. Use the answer of (a) to calculate the magnetic B-field.
  - c. Use the answer of (a) to calculate the magnetization of the material.
  - d. Calculate from the answer of (c) the bound current densities.
  - e. Now use the standard Ampere's law in terms of B to check the answer to (b).
2. A current  $I$  flows down a long straight wire of radius  $a$ . If the wire is made of copper (linear material) with susceptibility  $\chi_m$ , and the current is distributed uniformly.
  - (a) What is the magnetic B-field a distance  $s$  from the axis?
  - (b) Find all the bound currents.
  - (c) What is the net bound current flowing down the wire?
3. If  $\nabla \times \mathbf{H} = 0$  everywhere, the curl of  $\mathbf{H}$  vanishes and we can express  $\mathbf{H}$  as the gradient of a scalar potential  $W$ , i.e.

$$\vec{H} = -\nabla W$$

According to equation 6.23 then:

$$\nabla^2 W = (\nabla \cdot \vec{M})$$

So  $W$  obeys Poisson's equation, with  $\nabla \cdot \vec{M}$  as the source. This opens up all the machinery of Chapter 3. Find the field inside a uniformly magnetized sphere (i.e. example 6.1) by separation of variables (Hint:  $\nabla \cdot \vec{M} = 0$  everywhere except at the surface ( $r=R$ ), so  $W$  satisfies Laplace's equation in the regions  $r < R$  and  $r > R$ ; use equation 3.65, and from Eq 6.24 figure out the appropriate boundary condition on  $W$ ).

4. A sphere of linear magnetic material is placed in an otherwise uniform magnetic field  $\mathbf{B}_0$ . Find the new field inside the sphere. Use the same method as in problem 3.
  - (a) Determine the B for large  $r$ .
  - (b) Determine the H for large  $r$ .
  - (c) Determine the W for large  $r$ .
  - (d) How does  $W$  look inside the sphere and outside the sphere.
  - (e) Apply the boundary conditions on  $W$  and  $dW/dr$  at the surface of the sphere. Solve for  $W_{in}(r, \theta)$ .
  - (f) Determine  $H_{in}$  from  $W_{in}$ .
  - (g) Determine  $B_{in}$  from  $H_{in}$ .