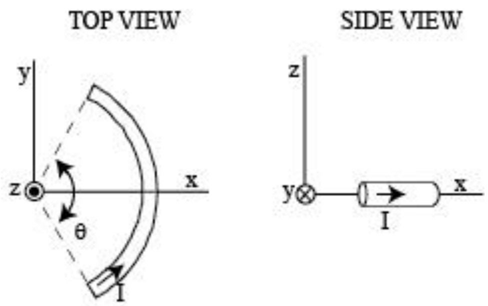
Homework 5.2: (grading rubric: P1: 2, P2: 2, PR:3, P4: 2, P5:3, P6:3, P7: 3 (only participation credit for problem 7))

1. Consider a current carrying circular arc that covers an angle of θ degrees and that is situated in the xy-plane. Furthermore assume that its center of curvature coincides with the origin (see the figure below).



   Consider an arbitrary field point on the z-axis at a distant z from the origin. Determine the component of the magnetic field parallel to the z-axis from Biot-Savart's law. Start of with making a drawing that shows the definition of all the parameters. Then determine expressions for r_script and dl', and finally evaluate the integral.

   See example 5.6 on page 218. Note that we now have to integrate θ not from 0 to 2π but from –$\theta/2$ to $\theta/2$. Note that the xy-components of the magnetic field no longer cancel out, but because we are only interested in the z-component of B we are still ok. I found the following expression:

$$B = \frac{\mu_o I}{4\pi}\theta\frac{R^2}{\left(R^2+z^2\right)^{3/2}}$$

2. Work problem 5.8 a.

   (a) Use Eq. 5.35, with $z = R, \theta_2 = -\theta_1 = 45°$, and four sides: $B = \boxed{\frac{\sqrt{2}\mu_0 I}{\pi R}}$.

3. Work problem 5.9.

   (a) The straight segments produce no field at P. The two quarter-circles give $B = \boxed{\frac{\mu_0 I}{8}\left(\frac{1}{a}-\frac{1}{b}\right)}$ (out).

   (b) The two half-lines are the same as one infinite line: $\frac{\mu_0 I}{2\pi R}$; the half-circle contributes $\frac{\mu_0 I}{4R}$.

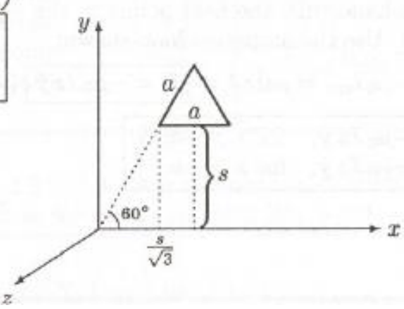
   So $B = \boxed{\frac{\mu_0 I}{4R}\left(1+\frac{2}{\pi}\right)}$ (into the page).

4. Work problem 5.10.

Problem 5.10

(a) The forces on the two sides cancel. At the bottom, $B = \frac{\mu_0 I}{2\pi s} \Rightarrow F = \left(\frac{\mu_0 I}{2\pi s}\right) Ia = \frac{\mu_0 I^2 a}{2\pi s}$ (up). At the top, $B = \frac{\mu_0 I}{2\pi(s+a)} \Rightarrow F = \frac{\mu_0 I^2 a}{2\pi(s+a)}$ (down). The net force is $\boxed{\frac{\mu_0 I^2 a^2}{2\pi s(s+a)}}$ (up).

(b) The force on the bottom is the same as before, $\mu_0 I^2/2\pi$ (up). On the left side, $\mathbf{B} = \frac{\mu_0 I}{2\pi y}\,\hat{\mathbf{z}}$; $d\mathbf{F} = I(d\mathbf{l} \times \mathbf{B}) = I(dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}) \times \left(\frac{\mu_0 I}{2\pi y}\,\hat{\mathbf{z}}\right) = \frac{\mu_0 I^2}{2\pi y}(-dx\,\hat{\mathbf{y}} + dy\,\hat{\mathbf{x}})$. But the x component cancels the corresponding term from the right side, and $F_y = -\frac{\mu_0 I^2}{2\pi}\int_{s/\sqrt{3}}^{(s/\sqrt{3}+a/2)} \frac{1}{y}\,dx$. Here $y = \sqrt{3}x$, so $F_y = -\frac{\mu_0 I^2}{2\sqrt{3}\pi}\ln\left(\frac{s/\sqrt{3}+a/2}{s/\sqrt{3}}\right) = -\frac{\mu_0 I^2}{2\sqrt{3}\pi}\ln\left(1+\frac{\sqrt{3}a}{2s}\right)$. The force on the right side is the same, so the net force on the triangle is $\boxed{\frac{\mu_0 I^2}{2\pi}\left[1 - \frac{2}{\sqrt{3}}\ln\left(1+\frac{\sqrt{3}a}{2s}\right)\right]}$.

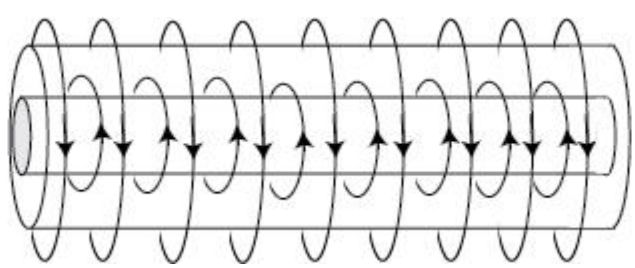


Note that force on the bottom wire segment of the triangle should read:

$$\frac{a\mu_o I^2}{2\pi s}$$

Also the last answer of 5.10 b should read a/s instead of 1.

5. Two long coaxial solenoids each carry current I, but in opposite directions, as shown in the figure below. The inner solenoid (radius a) has n_1 turns per unit length, and the outer one (radius b) has n_2. Find B is each of the three regions: (1) inside the inner solenoid, (ii) between them, and (iii) and outside both.



Problem 5.15

The field inside a solenoid is $\mu_0 nI$, and outside it is zero. The outer solenoid's field points to the left $(-\hat{\mathbf{z}})$, whereas the inner one points to the right $(+\hat{\mathbf{z}})$. So: (i) $\boxed{\mathbf{B} = \mu_0 I(n_1 - n_2)\,\hat{\mathbf{z}},}$ (ii) $\boxed{\mathbf{B} = -\mu_0 I n_2\,\hat{\mathbf{z}},}$ (iii) $\boxed{\mathbf{B} = 0.}$

Problem 5.16

6. A thick slab extending from z=-a to z=a (a>0) carries a uniform volume current

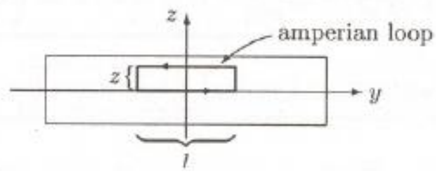
$$\vec{J} = J\hat{x}$$

Find the magnetic field, as a function of z, both inside and outside the slab.

By the right-hand-rule, the field points in the $-\hat{\mathbf{y}}$ direction for $z > 0$, and in the $+\hat{\mathbf{y}}$ direction for $z < 0$. At $z = 0, B = 0$. Use the amperian loop shown:

$\oint \mathbf{B}\cdot d\mathbf{l} = Bl = \mu_0 I_{\text{enc}} = \mu_0 l z J \Rightarrow \boxed{\mathbf{B} = -\mu_0 J z\,\hat{\mathbf{y}}}$ $(-a < z < a)$. If $z > a$, $I_{\text{enc}} = \mu_0 l a J$,

so $\boxed{\mathbf{B} = \begin{cases} -\mu_0 J a\,\hat{\mathbf{y}}, & \text{for } z > +a; \\ +\mu_0 J a\,\hat{\mathbf{y}}, & \text{for } z > -a. \end{cases}}$



7. A large parallel-plate capacitor with uniform surface charge σ on the upper plate and –σ on the lower is moving with a constant speed v, as shown in the figure below.
   a. Find the magnetic field between the plates and also above and below them.
   b. Find the magnetic force per unit area on the upper plate, including its direction.
   c. At what speed v would the magnetic force balance the electric force.