Mock Semester Exam EMT2, Spring 2015.

Evidently this exam is way too long, but I just copied and pasted what I found in my folder of last year. Come and see me if you need help.

Additional information:

The magnetic field on the z-axis at a point P(0,0,z) caused by a current carrying circular wire that is situated in the xy-plane (radius r, current I) is given by:

$$B(z) = \frac{\mu_o I}{2} \frac{r^2}{\left(r^2 + z^2\right)^{3/2}}$$

Other relevant equations:

$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi} \iiint \frac{\vec{J}(\vec{r})}{r_s} d\tau'$$
$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi} \oint \frac{I}{r_s} d\vec{l}'$$
$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi} \iint \frac{\vec{K}(\vec{r})}{r_s} da'$$

The following integrals might be useful to solve some of the problems:

$$\int \frac{x^3}{\left(x^2 + b^2\right)^{3/2}} dx = \frac{2b^2 + x^2}{\sqrt{x^2 + b^2}}$$

- 1. J.J. Thomson "discovered" the electron by measuring the charge-to-mass ratio of "cathode rays" (actually, streams of electrons, with charge q and mass m) as follows:
- a. First he passed the beam through uniform crossed electric and magnetic fields E and B (mutually perpendicular, and both of them perpendicular to the beam), and adjusted the electric field until he got zero deflection. What, then was the speed of the particles (in terms of E and B)?
- b. Then he turned off the electric field, and measured the radius of curvature, R, of the beam as deflected by the magnetic field alone. In terms of E, B, and R, what is the charge-to-mass ratio (q/m) of the particles?
- Assume that we have a wire along the positive x-axis and a wire along the positive y-axis and that they are connected at the origin. Furthermore assumes that a current I flows through the wire. Positive charge is transported from x=+infinity along the x-axis to the origin and then through the wire segment along the y-axis to y=+infinity.

- a. Use the right hand rules to determine the forces acting on different parts of the wire: indicate the direction of the forces on two places of each wire segment (make sure that the direction of the force and the relative magnitude are correct when drawing your arrows).
- 3. Consider two infinite plates parallel to each other at a distance d. The top plate carries a surface current density K C/(s.m) in the positive x-direction. The bottom plate carries a surface current denisty K C/(s.m) in the negative x-direction. Use superposition concepts.
- a. Describe the magnetic field between the plates (magnitude¹ and direction). Explain. If you have forgotten the magnetic fields around a plane that carries a homogeneous surface current, you might want to derive it using ampere's law.
- b. Describe the magnetic vector potential (magnitude¹ and direction) between the plates. If you have forgotten the magnetic vector potential around a plane that carries a homogeneous surface current, you might want to derive it from B using the definition of curl, or from the A-K relation provided above.

Footnote problem 3: ¹) Is the magnitude constant or does it vary through space, is the direction constant or varies through space?

- 4. Assume that we have a rubbed CD (radius R) that has a non zero surface charge density σ C/m². The disk is situated in the xy-plane so that its center coincides with the origin. It is spinning around an axis perpendicular to the disk along the z-axis. Its angular velocity is ω radians per second.
- a. Make a sketch of the problem. Indicate the coordinate system and the provide parameters.
- b. Determine an expression for the surface current density of the disk from the provided information.
- c. What is the magnetic vector potential for a point P on the z-axis that is z meters above the disk?
- d. What is the direction of the magnetic field at point P (assume σ is positive and ω is pointing in the positive z-direction).
- e. Determine an integral expression for the magnetic field in point P from Biot-Savart's law or superposition. Show your work which means, first determine the magnetic field caused by a small segment of the surface, identify the surface, determine the total current of the surface segment, use symmetry arguments for the magnetic field direction in P, and then add up all the components. If you want you can use the result on a current carrying circular wire (radius r, current I) provided above.
- f. Evaluate the integral you found in (e).
- 5. Derive an expression for the magnetic moment m of the spinning CD of problem 4. Show your work.
- 6. (i). A thin wire carries a uniform current I. This current produces a magnetic field, **B**. You have always been told that magnetic fields loop around a current-carrying wire (Figure a., below), but how do you know there are no other components to the magnetic field? Perhaps the magnetic field has a z-component (Figure b.) or a radial component (Figure c.).



Think of convincing arguments for why the magnetic field should have no z- or s-components. You may want to consider symmetry, Maxwell's equations, and any laws that have recently been covered in class.

- 7. Permanent magnets, magnetic recording materials, and novel magnetic field sensors are made from magnetic material.
 - a. The materials in the devices mentioned above are they mainly ferromagnetic, antiferromagnetic, paramagnetic, or diamagnetic?
 - b. What property on an atomistic level distinguishes ferromagnetic materials from other magnetic materials?
 - c. Several effects contribute to the energy of a magnetized object. Name three magnetic related energy contribution of a somewhat magnetized pair of scissors made of iron.
 - d. If I demagnetize a pair of scissors made of iron by placing it in a alternating magnetic field whose amplitude decreases, does it make the magnetization zero everywhere in the material? Explain.
- 8. The static magnetic field in a region inside some material is given in by an the following expression

 $\vec{B} = B_o r \left(\cos(\phi) \hat{r} + 5k \sin(\theta) \sin(\phi) \hat{\phi} \right)$

Where B_o and k are constants with appropriate units.

- a. What is the value for k?
- b. Find the total current density within the material.
- c. Use the answer of (b) to find $\frac{\partial \rho}{\partial t}$

9. Consider an infinite slab of a conducting material with magnetic susceptibility χ_M carrying a certain current distribution. The slab is parallel to the xy plane, between z = -a and z = a. It carries a free volume current density given by

$$\boldsymbol{J}(\boldsymbol{z}) = \left(\frac{J_0 \boldsymbol{z}}{\boldsymbol{a}}\right) \boldsymbol{\hat{\iota}}$$

Outside the slab is vacuum.

- a. Make a sketch of the problem. Indicate **J** and **B** in the figure.
- b. The magnetic field outside the slab is equal to zero. Explain!
- c. Use symmetry argument to determine where the magnetic H-field is maximum.
- d. Determine the H-field inside the slab.
- e. How do we determine the B-field and M-Field from the H-field.
- 10. A current flows up a long straight wire of radius 1. If the wire is made of copper (linear material) with susceptibility χ_m , and the free current is distributed non-uniformly according to:

$$\vec{J} = \left(5s - 5s^3\right)\hat{z} \qquad s < 1$$

Where s is the distance from the center of the wire.

- (a) What is the magnetic B-field a distance s from the axis inside the wire? In the wire so assume s<1. Note that B is a function of the distance s from the center of the wire.
- (b) What is the magnetic B-field a distance s from the axis outside the wire? Assume s>1. Note that B is a function of the distance s from the center of the wire.
- (c) Find all the bound current densities, i.e. J and K. Indicate in a figure the direction of J and K and where they are unequal to zero.
- (d) What is the net bound current flowing through the wire?