Key HW7_2

1. Read the following paper:
   http://physics.niser.ac.in/labmanuals/sem6/four_point_resi_ajp2k3.pdf
   a. Explain why one would use the four point probe technique rather than a 2 point probe technique to determine the resistivity of a silicon wafer.

We use the 4pp technique so the contact resistance between the sample and electrode does not show up in the measurement results. Note that contact between electrode and material depends on the properties of both the electrode and the sample. Not only intrinsic (work-function) but also extrinsic properties are important, such as defect concentration. Bottom line is that the resistance of a 2pp setup can be mainly caused by the contact resistance of the electrodes with the sample. If we apply a 4pp setup though and we inject the current via the outer electrodes but measure the voltage over the inner electrodes we will not measure the voltage over the contacts and thus get a resistance that is only caused by the sample material. Note that no current flows through a good voltmeter, so there is no voltage drop across the electrode-sample contacts that are connected to the voltmeter. The voltage drop across the electrode-sample contacts that are connected to the current source are not influencing the measurement.

   b. Explain why the current density has no component that is perpendicular to the water tray near the edge of the water tray.

The transport of charge happens in the water. Under steady state conditions we can assume that the derivative of the local charge density is zero, so no charge will pile up on the edge of the water tray. So the component of J perpendicular to the edge of the tray is zero at the edge of the tray, and J has only a component parallel to the edge of the tray.

   c. Explain in your own words why the measured resistance will be twice as large if we would place the electrodes at the edge of the water tray.

When we put the electrodes in the center of the tray the current can go through the left half plane and the right half plane. If we put the electrodes near the edge of the tray, the current can only go through one side. This will lead to a current density that is twice as large, and thus to a voltage that is two times larger, resulting in a two times larger resistance.

   d. Derive equation (1) of the paper from the continuity equation and the E-V relation we introduced in chapter 2.

The continuity equations gives:

$$\nabla \bullet \vec{J} = -\frac{\partial \rho}{\partial t} \qquad [1]$$

Where r is the charge density. For the stationary case the derivative of the charge density is constant, so its derivative is zero.

$$\nabla \bullet \vec{J} = 0 \qquad [2]$$

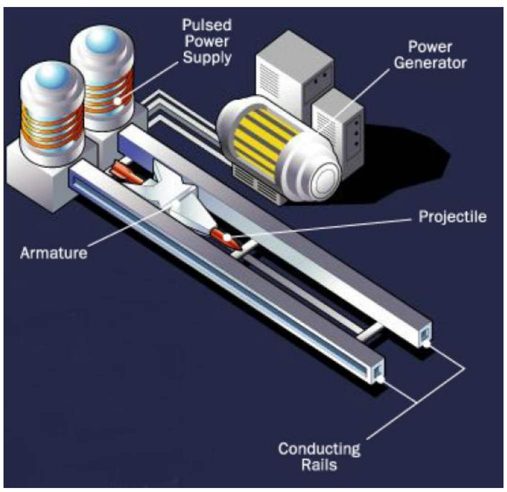
Furthermore I can use the relation between E and J:

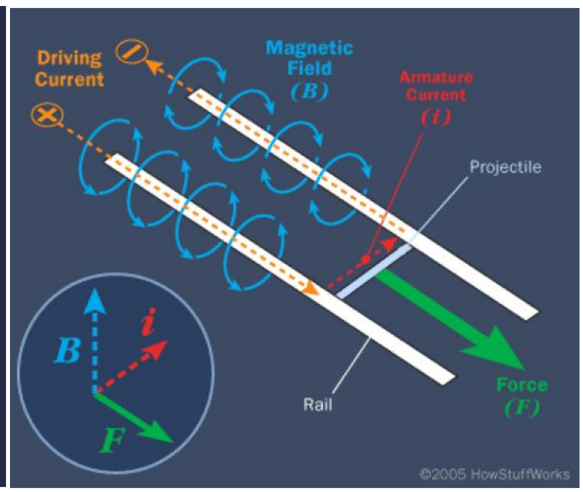
$$\vec{J} = \sigma\vec{E} \Leftrightarrow \vec{J} = \frac{\vec{E}}{\rho} \qquad [3]$$

And of course the relation between E and V:

$$\vec{E} = \nabla V \qquad [4]$$

Plugging [4] into [3] and plugging the result into [2] gives equation (1) of the paper.





2. A. We are testing the rail gun sketched in the left figure above. A projectile is positioned on top of two conducting rails and can slide over it. A current source that can provide a large current pulse is connected to the end of the rails. The housing of the projectile completes the circuit, i.e. current source, rail 1, projectile, rail 2, current source. When the current source is switched on a large force is exerted on the projectile parallel to the rails which propels the projectile along the rails away from the current source. Use the figure on the top right to explain the force that propel s the projectile along the rails.

The projectile or its carrier completes the circuit, i.e. power supply right rail, carrier, left rail, power supply. Once the power supply is switched on a large current will flow through the circuit. The current through the rails will create a large magnetic field in between the rails including at the position of the carrier. The current carrying carrier is thus situated in a large magnetic field and will experience a large Lorentz force propelling it along the rails.

B. Derive an equation for the magnetic field in the middle between the rails at the position of the projectile. Use the field of an infinite current carrying conductor and superposition. Of course the magnetic field is not constant in between the rails. You could use two times the magnetic field in the center as a reasonable approximation of the average magnetic field at the position of the projectile.

The magnetic field caused by an infinite wire that carries a current I is given by:

$$B = \mu_o \frac{I}{2\pi\, r}$$

Note that the rails are only half an infinite wire, so I expect the magnetic field to be half of this. There is two rails though so I need double this half. Furthermore I realize that the magnetic field is not constant along the carrier but depends on the distance to the rails. I will assume that on average the magnetic field along the current carrier will be twice the magnetic field in the center between the rails, so

$$B_{average} = 2\mu_o \frac{I}{2\pi\, s/2} = 2\mu_o \frac{I}{\pi\, s}$$

C. The advantage of a rail gun is that it can make muzzle velocities twice larger than what is currently possible with explosive. This results in a larger firing range. Assume that the projectile has a mass of 20 kg. How much current is required to accelerate the projectile to 2000 m/s assuming the total length of the rail gun is 10 meters, and the rails are 1 meter apart?

The work should be equal to the change in kinetic energy:

$$\frac{1}{2}mv^2 = F * length \Leftrightarrow F = \frac{1}{2}\frac{mv^2}{length} = \frac{1}{2}\frac{20 * 2000^2}{10} = 4E6 Newton$$

The force acting on the carrier can now be calculated from the length of the carrier, the current and the magnetic field, i.e.

$$F = IBs = 2\mu_o \frac{I^2}{\pi} \Leftrightarrow I = \sqrt{\frac{F\pi}{2\mu_o}} = 2.2E6 Ampere$$

D. How much time will it take for the projectile to accelerate from 0 to 2000 m/s?

Assuming a constant force, approximately

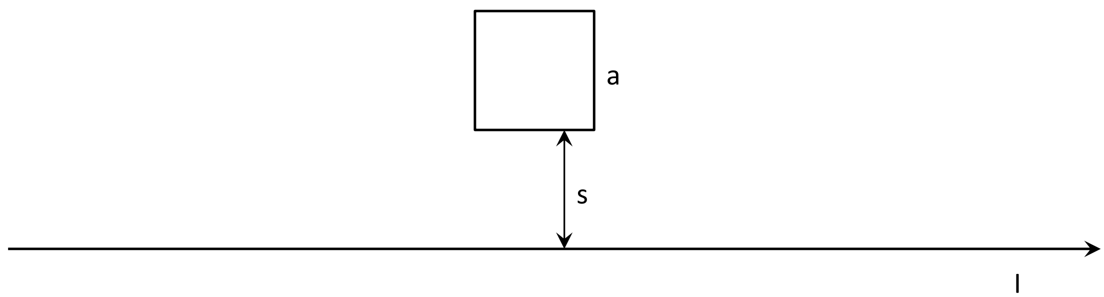
$$2000 = v = at = 2E5 * t \Leftrightarrow t = \frac{2000}{2E5} = 0.01\sec$$

3. A square loop of wire (side a) lies on a table, a distance s from a very long straight wire which carries a current I, as shown.

a) Find the flux of B through the loop.

b) If someone pulls the loop directly away from the wire at speed v, what emf is generated? In what direction does current flow?

c) What if the loop is pulled to the right instead?



(a) The field of long wire is $\mathbf{B} = \frac{\mu_0 I}{2\pi s}\hat{\phi}$, so $\Phi = \int \mathbf{B} \cdot d\mathbf{a} = \frac{\mu_0 I}{2\pi} \int_s^{s+a} \frac{1}{s}(a\,ds) = \boxed{\frac{\mu_0 I a}{2\pi} \ln\left(\frac{s+a}{s}\right)}.$

(b) $\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{\mu_0 I a}{2\pi}\frac{d}{dt}\ln\left(\frac{s+a}{s}\right)$, and $\frac{ds}{dt} = v$, so $-\frac{\mu_0 I a}{2\pi}\left(\frac{1}{s+a}\frac{ds}{dt} - \frac{1}{s}\frac{ds}{dt}\right) = \boxed{\frac{\mu_0 I a^2 v}{2\pi s(s+a)}}.$

The field points *out* of the page, so the force on a charge in the nearby side of the square is *to the right.* In the far side it's also to the right, but here the field is weaker, so the current flows $\boxed{\text{counterclockwise.}}$

(c) This time the flux is *constant*, so $\boxed{\mathcal{E} = 0.}$