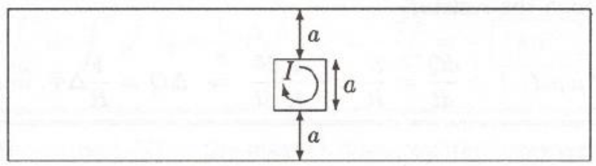
Key HW 7_4

1. Work problem 7.22
   We worked this already for HW7_3

2. Work problem 7.23

Problem 7.21

$$\mathcal{E} = -\frac{d\Phi}{dt} = -M\frac{dI}{dt} = -Mk.$$



It's hard to calculate M using a current in the little loop, so, exploiting the equality of the mutual inductances, I'll find the flux through the *little* loop when a current I flows in the *big* loop: $\Phi = MI$. The field of *one* long wire is $B = \frac{\mu_0 I}{2\pi s} \Rightarrow \Phi_1 = \frac{\mu_0 I}{2\pi}\int_a^{2a} \frac{1}{s} a\, ds = \frac{\mu_0 I a}{2\pi}\ln 2$, so the *total* flux is

$$\Phi = 2\Phi_1 = \frac{\mu_0 I a \ln 2}{\pi} \Rightarrow M = \frac{\mu_0 a \ln 2}{\pi} \Rightarrow \boxed{\mathcal{E} = \frac{\mu_0 k a \ln 2}{\pi},} \text{ in magnitude.}$$

Direction: The net flux (through the big loop), due to I in the little loop, is *into the page*. (Why? Field lines point *in*, for the inside of the little loop, and *out* everywhere outside the little loop. The big loop encloses *all* of the former, and only *part* of the latter, so *net* flux is *inward*.) This flux is *increasing*, so the induced current in the big loop is such that *its* field points *out* of the page: it flows $\boxed{\text{counterclockwise.}}$

3. Determine the Neumann formula for the self-induction of a circular coil with radius a. Use the same approach as on page 322, so first determine the flux in terms of B, then in terms of A and then substitute A with 5.66.

   Use the same approach as on page 322. The flux through the coil is per definition:

$$\Phi = \iint \vec{B} \bullet d\vec{a} = \iint \nabla \times \vec{A} \bullet d\vec{a}$$

   Now use Stokes theorem, i.e.

$$\Phi = \oint \vec{A} \bullet d\vec{l}$$

   Realize that A is the magnetic vector potential of the same coil. Now use expression 5.66 on page 245, to determine A from the current through the loop:

$$\Phi = \oint \frac{\mu_o}{4\pi}\oint \frac{I}{r_s} d\vec{l}\,' \bullet d\vec{l} = \frac{\mu_o I}{4\pi}\oint\oint \frac{d\vec{l}\,' \bullet d\vec{l}}{r_s}$$

   L can now be found by dividing by I, so

$$L = \frac{\mu_o}{4\pi}\oint\oint \frac{d\vec{l}\,' \bullet d\vec{l}}{r_s}$$

   Which is exactly identical to the Neumann equation for mutual inductance.