

In reference to problem 7.4a and 7.4b of the MC review:

Consider an Amperian loop which is a circle in the space between the plates of a capacitor. The capacitor is charging, so a current is flowing through the left wire towards the positive plate and a current is flowing through the right wire away from the negative plate. We can do Ampere's law on this Amperian loop. Note that we would need to use Ampere's law and Maxwell's correction, i.e.

$$\nabla \times \vec{B} = \mu_o \vec{J} + \mu_o \epsilon_o \frac{\partial \vec{E}}{\partial t}$$

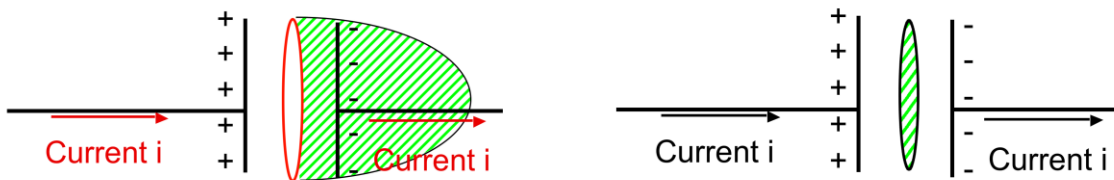
Let us first convert this differential law to an integral form by taking the surface integral on both sides:

$$\iint \nabla \times \vec{B} \cdot d\vec{a} = \iint \mu_o \vec{J} \cdot d\vec{a} + \mu_o \epsilon_o \iint \frac{\partial \vec{E}}{\partial t} \cdot d\vec{a}$$

Apply Stokes law on the left side gives:

$$\oint \vec{B} \cdot d\vec{l} = \mu_o I_{enclosed} + \mu_o \epsilon_o \frac{\partial}{\partial t} \iint \vec{E} \cdot d\vec{a} \quad [3]$$

Note that the very left integral is the electric flux through the surface bounded by the Amperian loop. We can choose this surface arbitrary. So the green surface in the picture on the left is as good as the green surface in the picture on the right. As long as we assume that the Amperian loop is the same for both cases.



For the picture on the right I do not have any electric flux going through the green surface. So equation [3] becomes:

$$\oint \vec{B} \cdot d\vec{l} = \mu_o I_{enclosed} = \mu_o i \quad [4]$$

For the picture on the left I do not have any enclosed current so equation [3] becomes:

$$\oint \vec{B} \cdot d\vec{l} = \mu_o \epsilon_o \frac{\partial}{\partial t} \iint \vec{E} \cdot d\vec{a} \quad [5]$$

As the left part does not change both expression should be the same. It is possible to convert the right side of [5] into the right side of [4] with a little effort. The E-field between the plates of a capacitor is

equal to V/s where s is the distance between the plates. The voltage drop across a capacitor is proportional to the charge on the plates, i.e. $Q=CV$. We can take the derivative of both sides of this equation and we get:

$$C \frac{dV}{dt} = \frac{dQ}{dt} = i$$

Substituting $E=V/s$ gives:

$$i = C \frac{d(sE)}{dt} = Cs \frac{dE}{dt}$$

Now plugging this expression into equation [5] gives us:

$$\oint \vec{B} \cdot d\vec{l} = \mu_o \epsilon_o \iint \frac{\partial E}{\partial t} da = \mu_o \epsilon_o \iint \frac{i}{Cs} da = \mu_o \epsilon_o \iint \frac{i}{\frac{\epsilon_o A}{s}} da = \mu_o \iint \frac{i}{A} da$$

Note that the integral on the far right is equal to i as long as the Amperian loop has a surface area equal to A , so it includes all the electric flux. So now we get indeed:

$$\oint \vec{B} \cdot d\vec{l} = \mu_o \iint \frac{i}{A} da = \mu_o i$$

QED.