Key HW 7_5

1. Consider a toroidal coil with a rectangular cross section (inner radius a, outer radius b, and height h), which carries a total of N turns.

a. Calculate the energy stored in toroidal coil from the magnetic B-field.

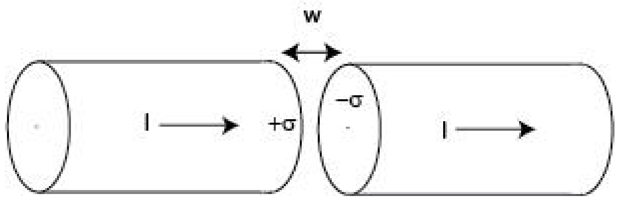
b. Calculate the energy stored in the toroidal coil from $0.5LI^2$.

Problem 7.27

$$B=\frac{\mu_0 n I}{2\pi s};\ W=\frac{1}{2\mu_0}\int B^2\,d\tau=\frac{1}{2\mu_0}\frac{\mu_0^2 n^2 I^2}{4\pi^2}\int\frac{1}{s^2}hr\,d\phi\,ds=\frac{\mu_0 n^2 I^2}{8\pi^2}h2\pi\ln\left(\frac{b}{a}\right)=\boxed{\frac{1}{4\pi}\mu_0 n^2 I^2 h\ln(b/a).}$$

$\boxed{L=\frac{\mu_0}{2\pi}n^2 h\ln(b/a)}$ (same as Eq. 7.27).

2. A fat wire, radius a, carries a constant current I, uniformly distributed over its cross section. A narrow gap in the wire, of width w<<a, forms a parallel-plate capacitor, as shown in the figure below. Find the magnetic field in the gap, at a distance s<a from the axis.



Problem 7.31

The displacement current density (Sect. 7.3.2) is $\mathbf{J}_d=\epsilon_0\frac{\partial\mathbf{E}}{\partial t}=\frac{I}{A}=\frac{I}{\pi a^2}\,\hat{\mathbf{z}}$. Drawing an "amperian loop" at radius s,

$$\oint\mathbf{B}\cdot d\mathbf{l}=B\cdot 2\pi s=\mu_0 I_{d_{enc}}=\mu_0\frac{I}{\pi a^2}\cdot\pi s^2=\mu_0 I\frac{s^2}{a^2}\Rightarrow B=\frac{\mu_0 I s^2}{2\pi s a^2};\quad\boxed{\mathbf{B}=\frac{\mu_0 I s}{2\pi a^2}\,\hat{\boldsymbol{\phi}}.}$$

3. See water at frequency $n=4\times10^8$ Hz has permittivity $\varepsilon=81\varepsilon_0$, permeability $\mu=\mu_0$ and resistivity $r=0.23\ \Omega$.m. What is the ratio of the conduction current to displacement current?

Problem 7.37

$$E=\frac{V}{d}\Rightarrow J_c=\sigma E=\frac{1}{\rho}E=\frac{V}{\rho d}.\ J_d=\frac{\partial D}{\partial t}=\frac{\partial}{\partial t}(\epsilon E)=\epsilon\frac{\partial}{\partial t}\left[\frac{V_0\cos(2\pi\nu t)}{d}\right]=\frac{\epsilon V_0}{d}\left[-2\pi\nu\sin(2\pi\nu t)\right].$$

The ratio of the amplitudes is therefore:

$$\frac{J_c}{J_d}=\frac{V_0}{\rho d}\frac{d}{2\pi\nu\epsilon V_0}=\frac{1}{2\pi\nu\epsilon\rho}=\left[2\pi(4\times10^8)(81)(8.85\times10^{-12})(0.23)\right]^{-1}=\boxed{2.41.}$$