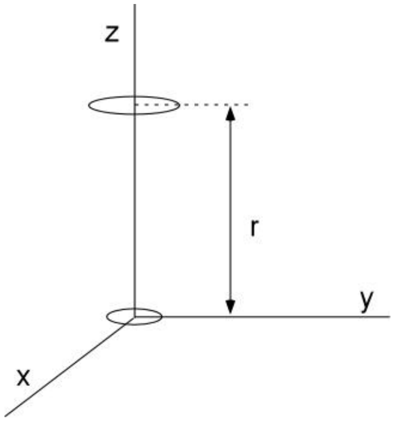
Semester exam chapter 7.

PHYS4315

Work at home and turn in by 11:00 AM on Friday morning. I will take the weighted average of the test you worked in class (2/3) and the take home test (1/3) for your 2^{nd} semester exam grade.

1. Two small circular loops of wire (of radii a and b) lie at two different points on the z-axis. Assume that both loops have their normal parallel to the z-axis. What is the mutual inductance between the loops if the distance r is sufficiently large that the dipole approximation may be used?



2. In a perfect conductor, the conductivity is infinite, so E=0 and any net charge resides on the surface.
   a. Show that the magnetic field is constant ($\partial B/\partial t = 0$), inside a perfect conductor.
   b. Show that the magnetic flux through a perfectly conducting loop is constant.

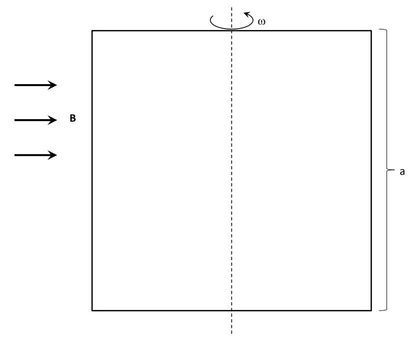
   A superconductor is a perfect conductor with the additional property that the (constant) B inside is in fact zero. (This “flux exclusion” is known as the Meissner effect.

   c. Show that the current in a superconductor is confined to the surface.
   d. Superconductivity is lost above a certain critical temperature (T_C), which varies from one material to another. Suppose you had a sphere (radius a) above its critical temperature, and you held it in a uniform magnetic field $B_o\hat{z}$ while cooling it below T_C. Find the induced surface current density **K**, as a function of the polar angle θ.

3. A square loop of side axa is rotated at angular velocity ω around a vertical shaft as shown below. Assume that the magnetic field is uniform but time dependent:

   $$B = 5t$$

   Find the time-dependent emf for this situation. Show your work.



4. Assume a coaxial cable of 1 meter length with an inner wire radius of a and an outer radius of b. The space between the electrodes is filled with a weakly conducting material whose conductivity is inhomogeneous and depends on the distance of the material to its center axis of the coaxial system, i.e.

$$\sigma = 3s$$

   a. Assume that a current of I flows from the inner electrode, to the outer electrode. Find an expression for the current density as a function of s, i.e. the radial parameter (a<s<b).
   b. Use your answer of (a) to determine the electric field as a function of the position in the conducting material (i.e. determine E from J, so not from Gauss' law).
   c. Use your answer of (b) to determine the Electric potential between the inner and outer electrodes.
   d. Determine an expression for the resistance of the device.