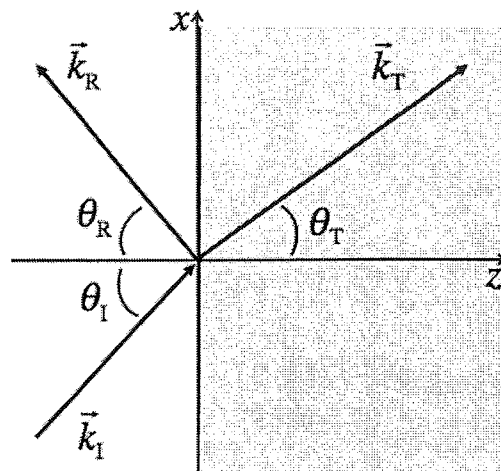


A. An electromagnetic plane wave traveling through vacuum in the x - z plane (i.e., $\vec{k}_I = k_{IX}\hat{x} + k_{IZ}\hat{z}$ & no propagation in the y -direction) is incident upon a material at an angle θ_I relative to the z -axis.



With the incident wave linearly polarized in the y -direction (out of the page), the electric fields of the incident, reflected and transmitted waves can each be represented by the following complex exponentials:

$$\vec{E}_I(\vec{r}, t) = E_I \exp[i(\vec{k}_I \cdot \vec{r} - \omega t)] \hat{y}$$

$$\vec{E}_R(\vec{r}, t) = E_R \exp[i(\vec{k}_R \cdot \vec{r} - \omega t + \delta_R)] \hat{y}$$

$$\vec{E}_T(\vec{r}, t) = E_T \exp[i(\vec{k}_T \cdot \vec{r} - \omega t)] \hat{y}$$

According to Faraday's Law, the parallel component of the total electric field on either side of the boundary must be the same at all times:

$$\vec{E}_I^{\parallel}(z=0, t) = \vec{E}_R^{\parallel}(z=0, t) + \vec{E}_T^{\parallel}(z=0, t) \quad \rightarrow \quad \vec{E}_I^{\parallel}(z=0, t) + \vec{E}_R^{\parallel}(z=0, t) = \vec{E}_T^{\parallel}(z=0, t)$$

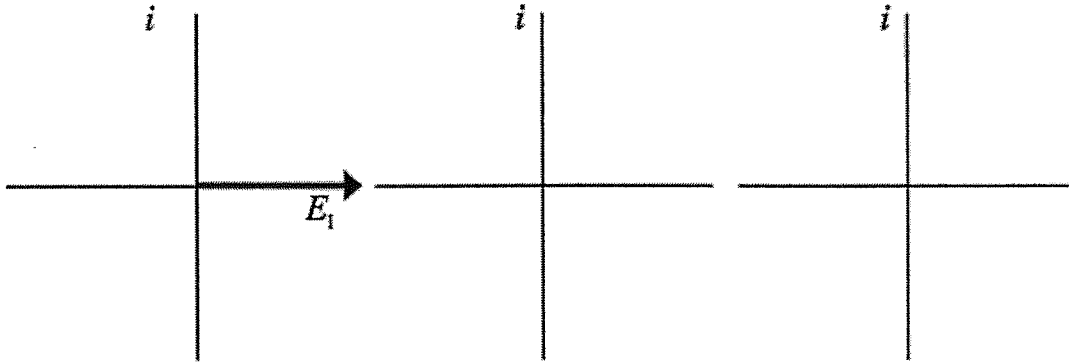
Use the information given to re-write this boundary condition for $t = 0$, using the complex exponential notation from above.

Now, write out this boundary condition for $t = 0$ & $x = 0$.



You may continue, but be sure to check your answers with an instructor.

B. With the *incident* E-field at $t = 0$ & $x = 0$ shown below at left, use the other axes to draw arrows representing the E-fields of the *reflected* and *transmitted* waves for the case $E_T < E_I$. Be sure to label your arrows.



What is the phase shift δ_R of the *reflected* wave in this case?

Use the diagram above to draw the *incident* E-field at a distance $k_{IX}x = \pi/4$ from the origin ($x = z = 0$). Be sure to think carefully about the direction $E_I \exp[i(k_{IX}x - \omega t)]$ rotates in the complex plane as x increases and t is held constant.

Boundary conditions must be satisfied everywhere along the boundary (not just at $x = 0$). We can solve for E_I in the time-independent equation from the first page:

$$E_I = E_R \exp[i(k_{RX} - k_{IX})x] + E_T \exp[i(k_{TX} - k_{IX})x]$$

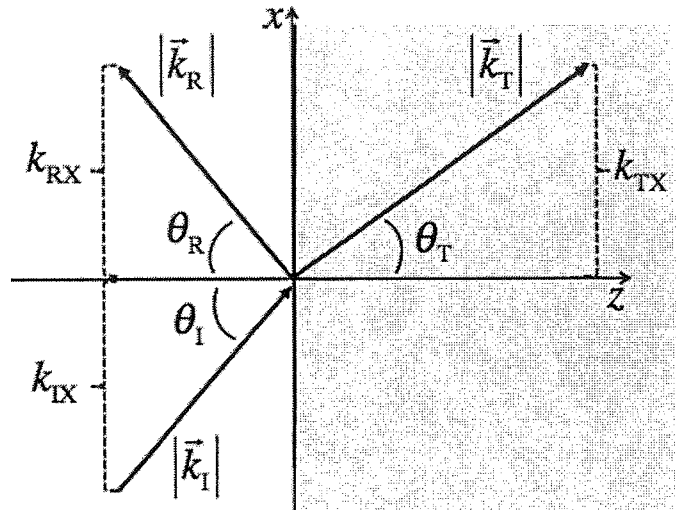
For this equality to hold at all times, what must be true about the quantities $(k_{RX} - k_{IX})$ and $(k_{TX} - k_{IX})$? Briefly explain your reasoning.

C. According to the diagram:

$$\sin \theta_I = \frac{k_{IX}}{|\vec{k}_I|}$$

$$\sin \theta_R = \frac{k_{RX}}{|\vec{k}_R|}$$

$$\sin \theta_T = \frac{k_{TX}}{|\vec{k}_T|}$$



Keeping in mind that the incident and the reflected wave are both traveling in vacuum, what is the relationship between θ_I & θ_R ? Briefly explain your reasoning.

What is \vec{k}_R in terms of the components of \vec{k}_I ?

Determine the following ratio for the incident and transmitted waves:

$$\frac{\sin \theta_T}{\sin \theta_I} =$$

Keeping in mind that the transmitted wave is traveling slower than the speed of light in vacuum, which of the following is true? How does the angle of transmission compare with the angle of incidence in this case?

$$k_{TZ} < k_{IZ}$$

$$k_{TZ} = k_{IZ}$$

$$k_{TZ} > k_{IZ}$$